Evolution of rocky and icy planetary bodies in the Solar System

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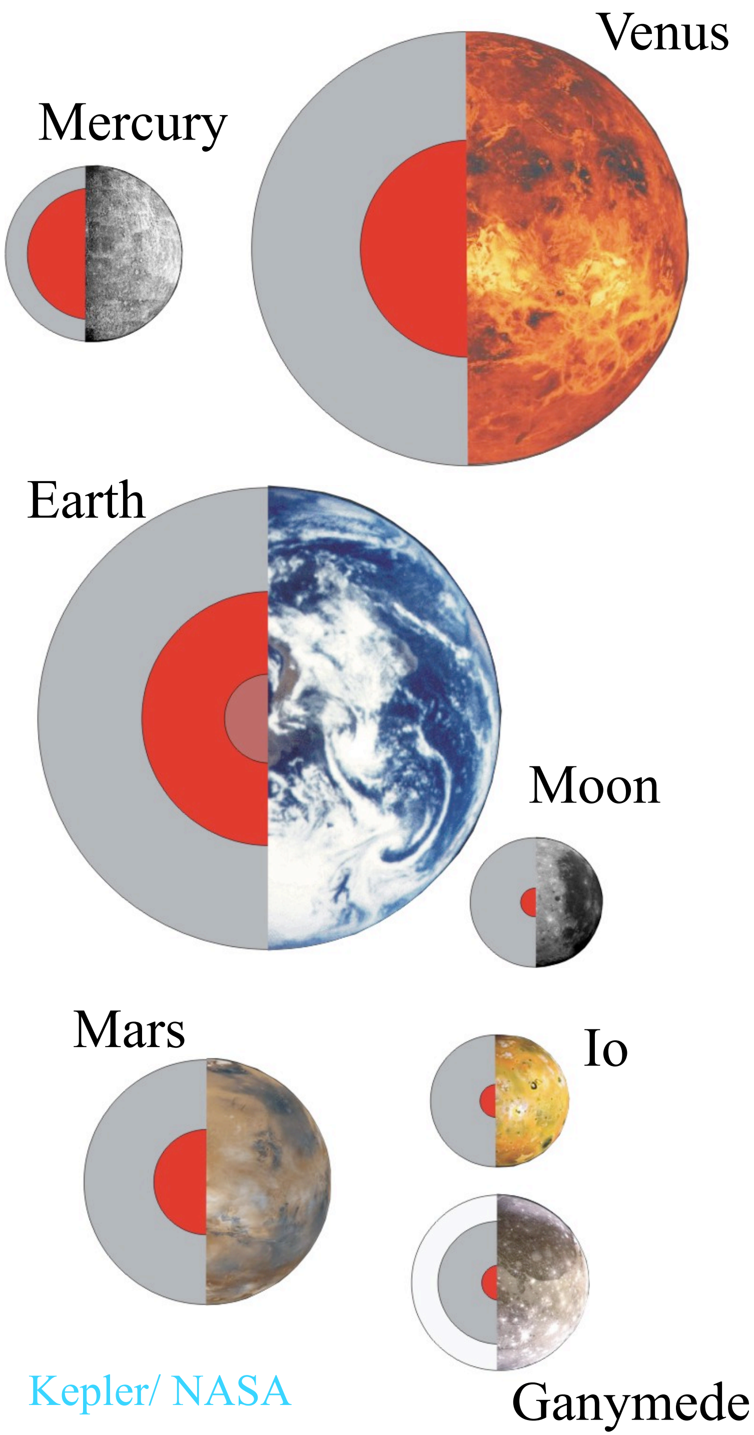
Approach & Resources

- Pedagogical, not research
- Generic, not specific, processes
- Order of magnitude arguments (+equations)

Planet formation & accretion:
http://www.es.ucsc.edu/~fnimmo/eart290c

Geophysics & heat transfer:
http://www.es.ucsc.edu/~fnimmo/eart290q_11

Satellites & tides:
http://www.es.ucsc.edu/~fnimmo/eart290q_09

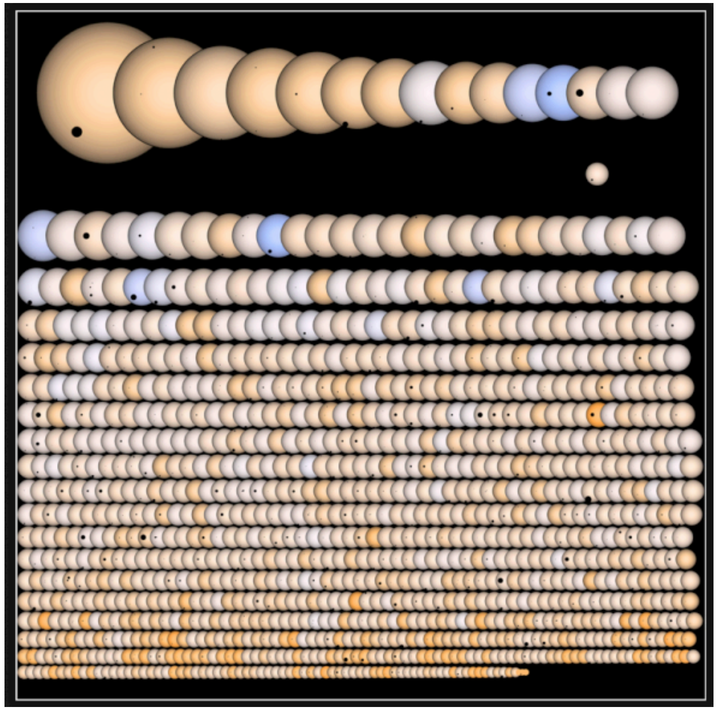


Kepler/ NASA

Small *N*, lots of information (e.g. chemistry)

Same processes operating

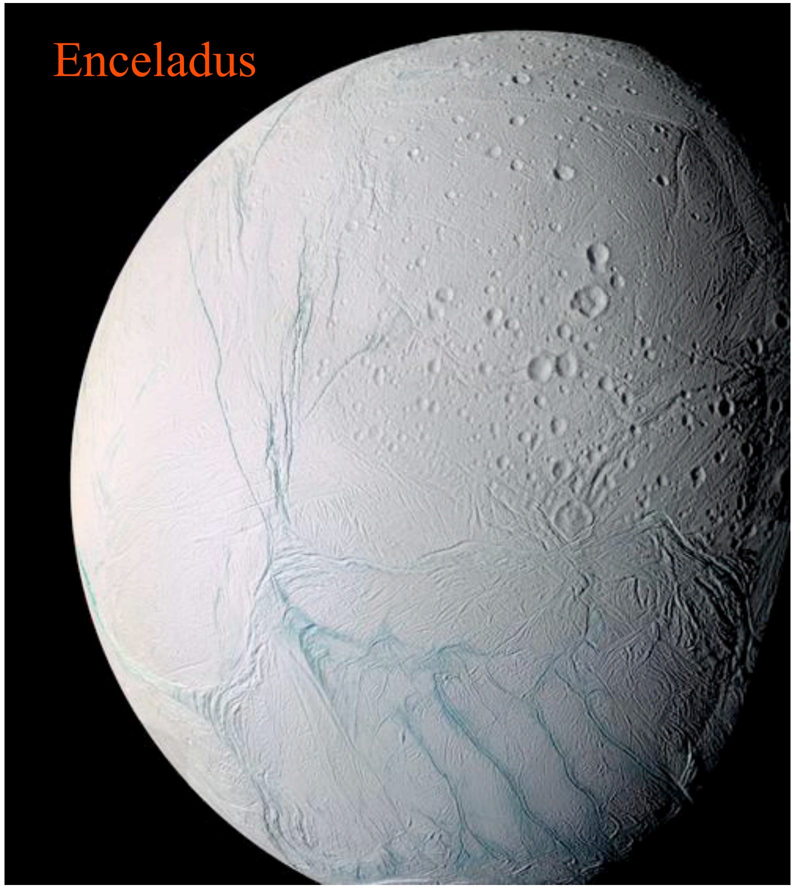
Large *N*, little information (especially for solid bodies!)



Kepler/ NASA

Why think about solid bodies?

- Habitability (sigh)
- Their surfaces, interiors, orbits and chemistry give clues to their *history* (and that of the solar system)
- Gas giants (mostly) don’t do this



Kepler/ NASA

Present-day state = Initial Conditions + Subsequent Evolution

Outline

I. Accretion (long)

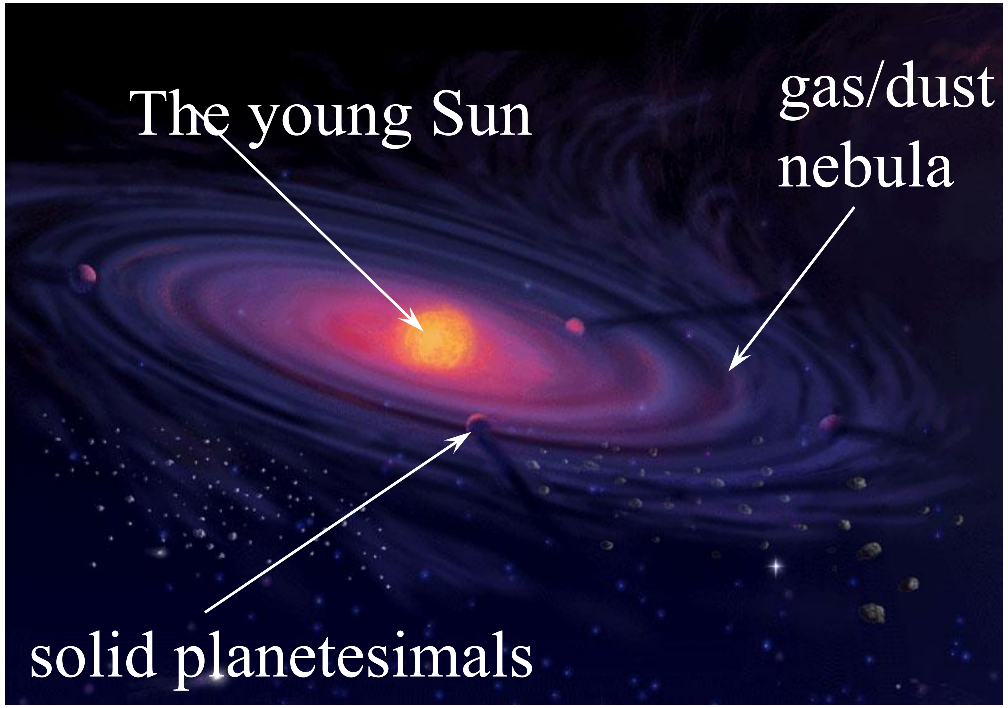
II. Heat Sources (short)

III. Heat Transfer (short)

IV. Tides (long)

Part I - Accretion

- Why is accretion important?:
  - Universal process
  - It sets the initial conditions from which the bodies evolve
  - It can yield diverse outcomes



Kepler/ NASA

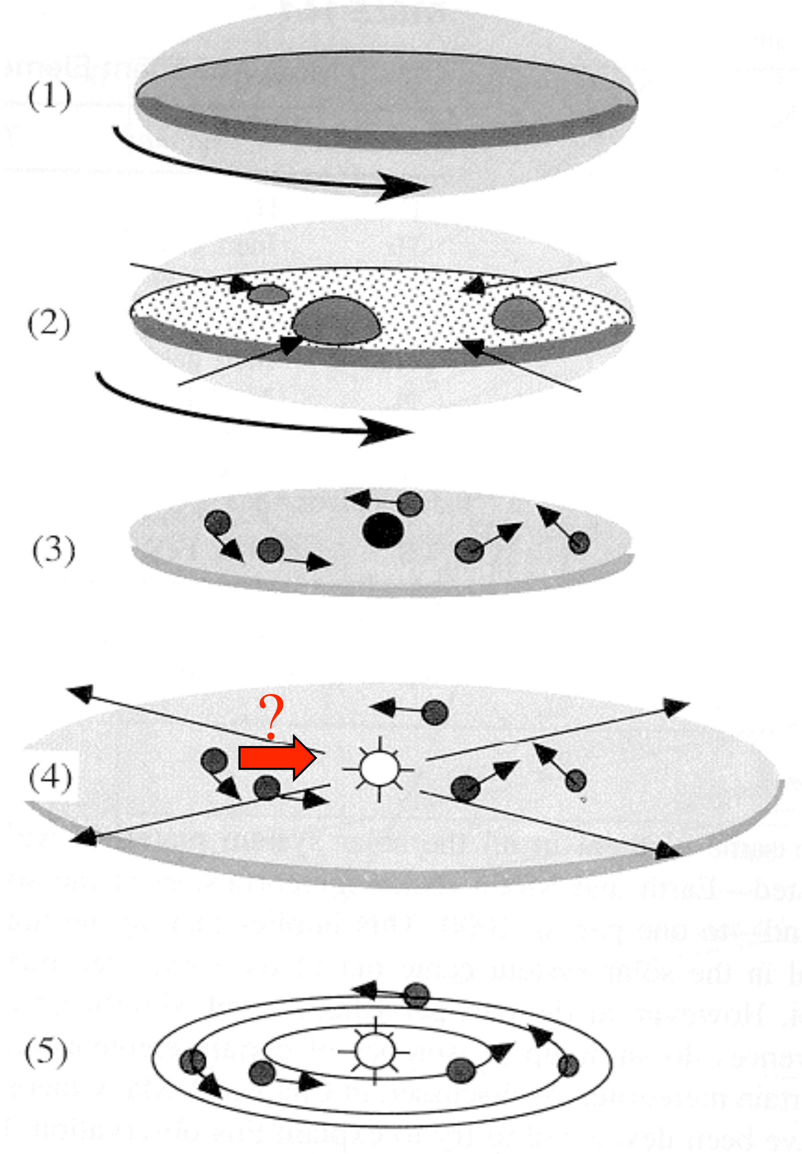
Accretion - Topics

- 0. Introduction
- 1. Bulk composition
- 2. Spin/orbit state
- 3. Later events
- 4. Energy delivery (see below)

Resources: J. Chambers, in *Exoplanets*, Sara Seager (ed.), U.Az. Press, 2010
P.J. Armitage, *Astrophysics of Planet Formation*, C.U.P., 2010
A. Morbidelli, in *Solar & Planetary Systems*, French & Kalas, eds.,
Springer, 2012

Sequence of events

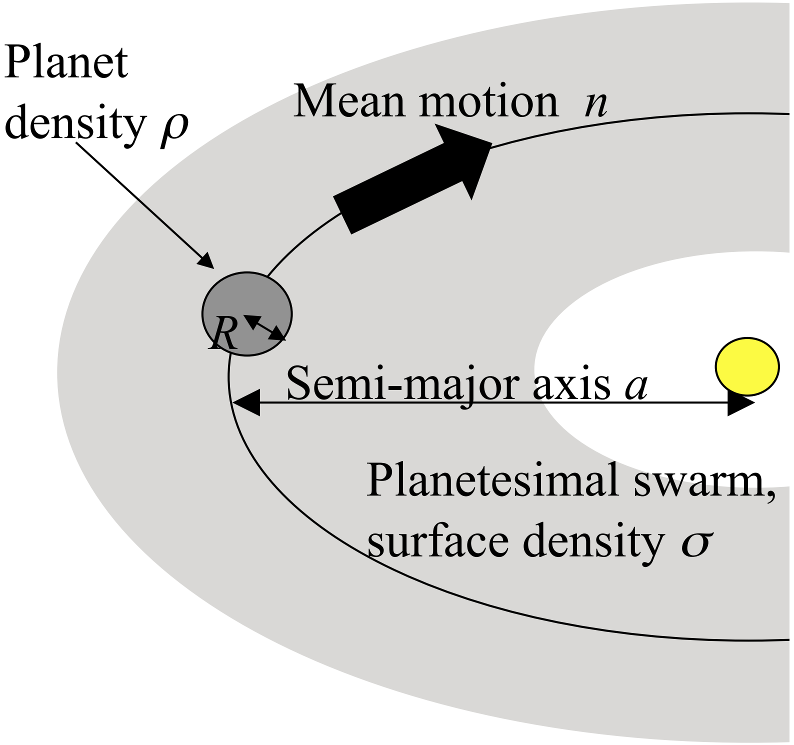
- 1. Nebular disk formation
- 2. Initial coagulation (~10km, ~10^4 yrs)
- 3. Runaway growth (to Moon size, ~10^5 yrs)
- 4. Oligarchic growth (to Mars size, ~10^6 yrs), migration (?), gas loss
- 5. Late-stage collisions (~$10^{7\text{-}8}$ yrs)



Accretion

$$\frac{dM}{dt} = (1+\theta)\, n\, \sigma\, \pi R^2$$

$$\theta = \frac{v_{esc}^2}{v_{rel}^2}$$



$\theta >> 1$: Runaway

$$\frac{dM}{dt} \sim \frac{n\sigma G}{\rho^{1/3} v_{rel}^2} M^{4/3}$$

$\theta << 1$: Oligarchic

$$\frac{dM}{dt} \sim \frac{n\sigma}{\rho^{2/3}} M^{2/3}$$

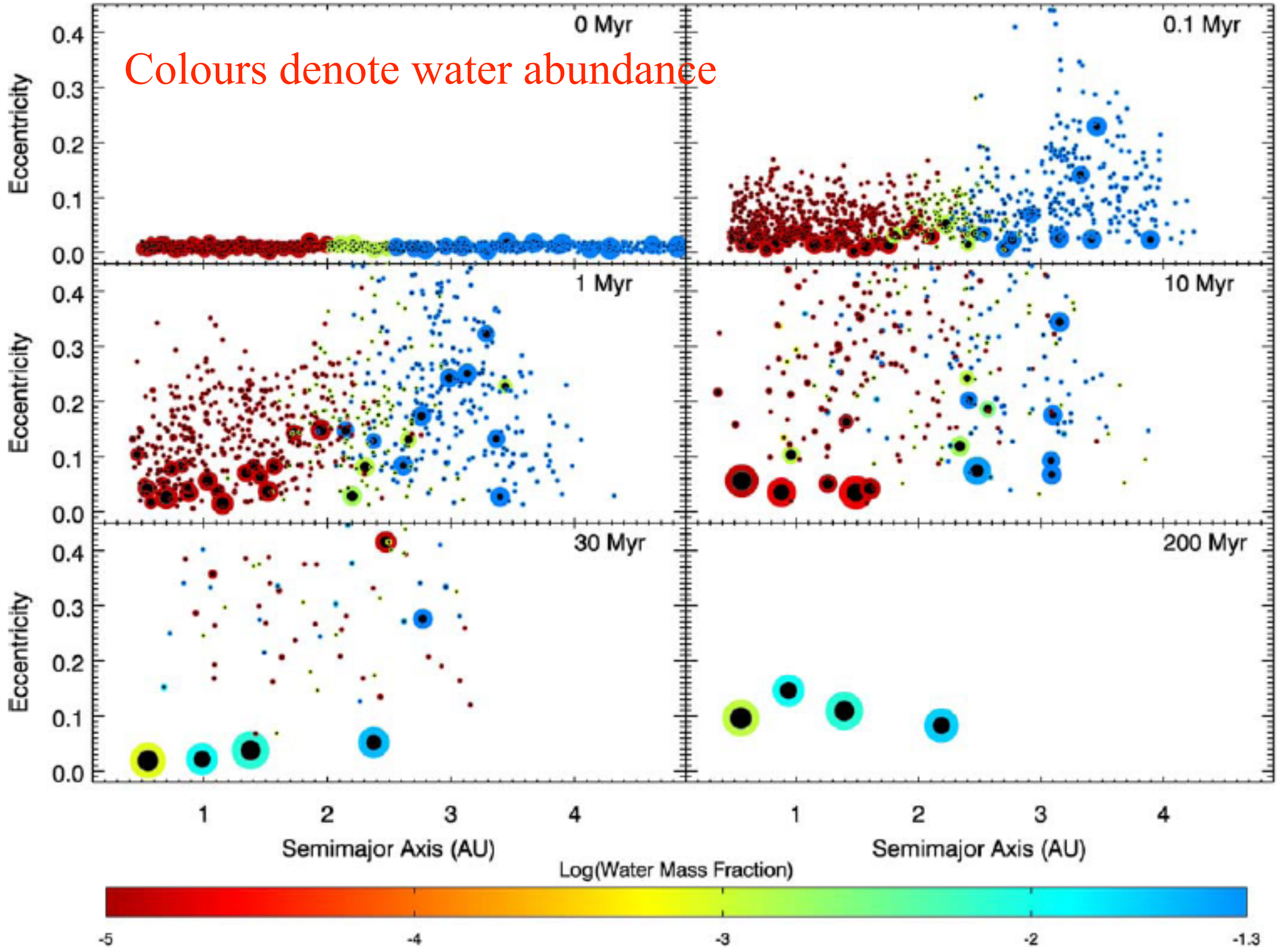
Accretion slows down once all material in a planet's Hill Sphere r_H has been accreted:

$$r_H \sim a(M / M_{star})^{1/3}$$

Warning!

- Our Solar System does *not* resemble many other planetary systems – high eccentricities & inclinations, “hot Jupiters” etc.
- Intuition developed by studying accretion of our (dynamically “cold”) Solar System may not apply to other planetary systems

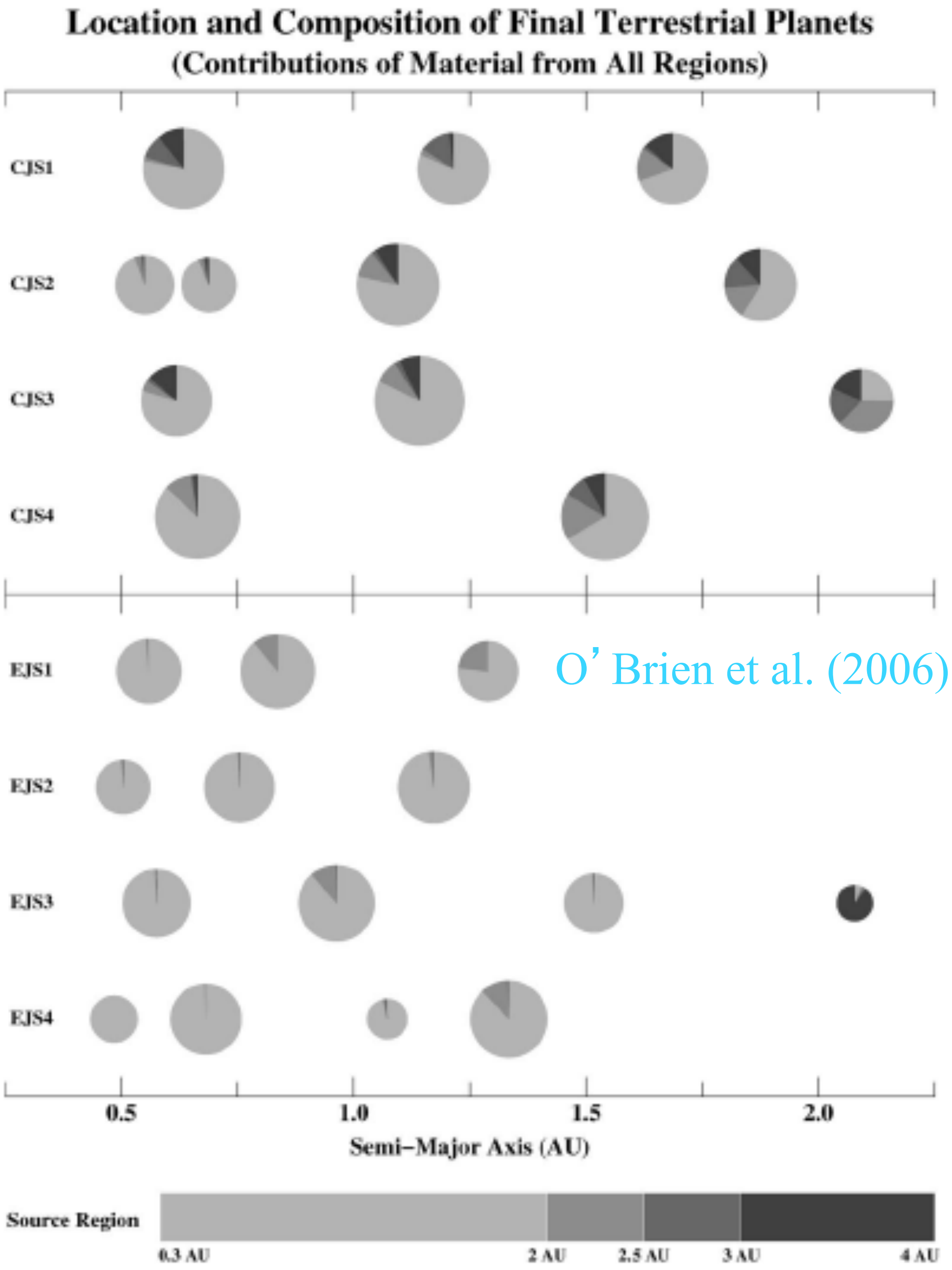
Late-stage Accretion



Colours denote water abundance

Raymond et al. 2006

- Volatiles arrive late – change in oxidation state?
- Chem. evidence (Schonbachler et al. 2010, Rubie et al. 2011)

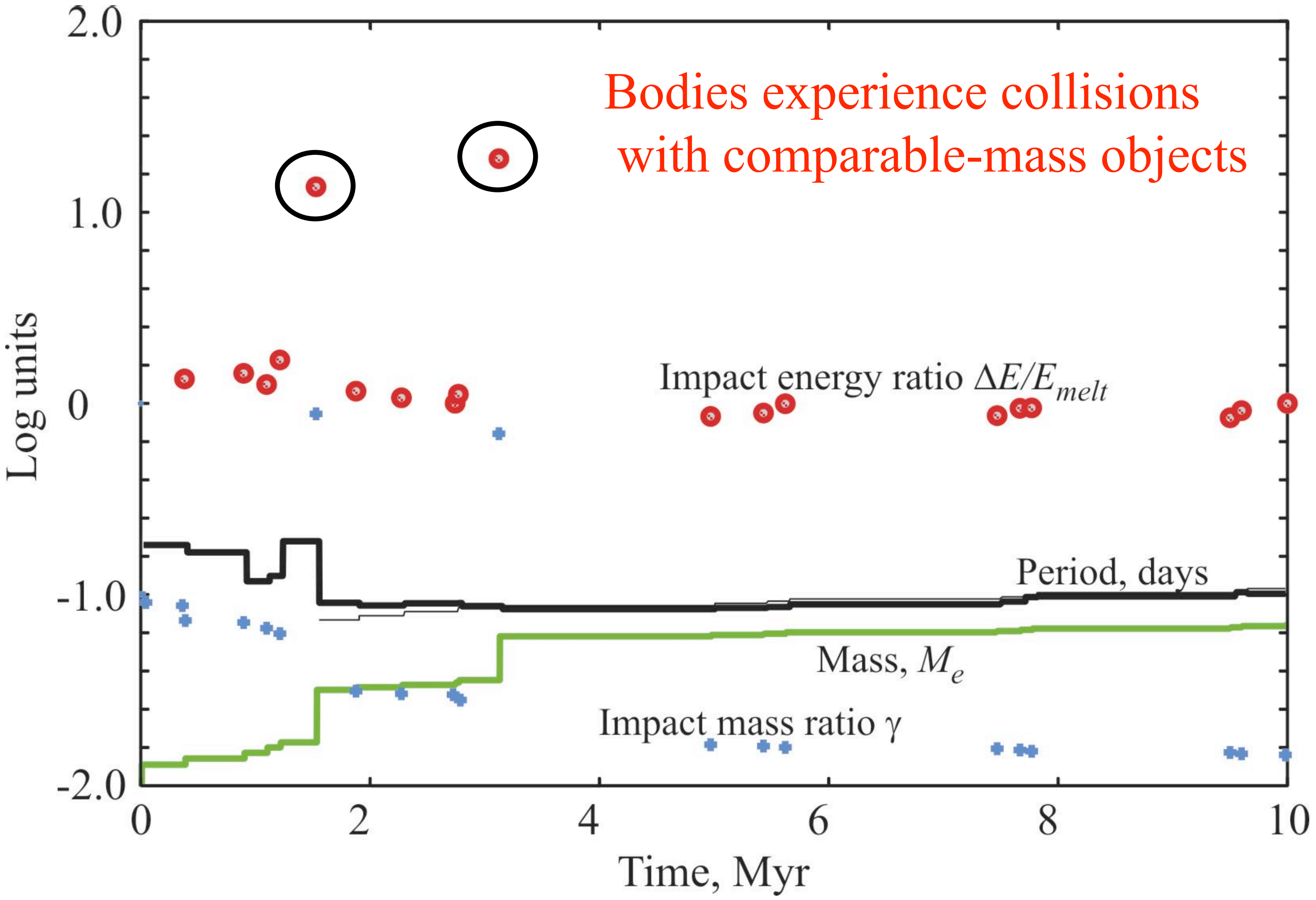


O’Brien et al. (2006)

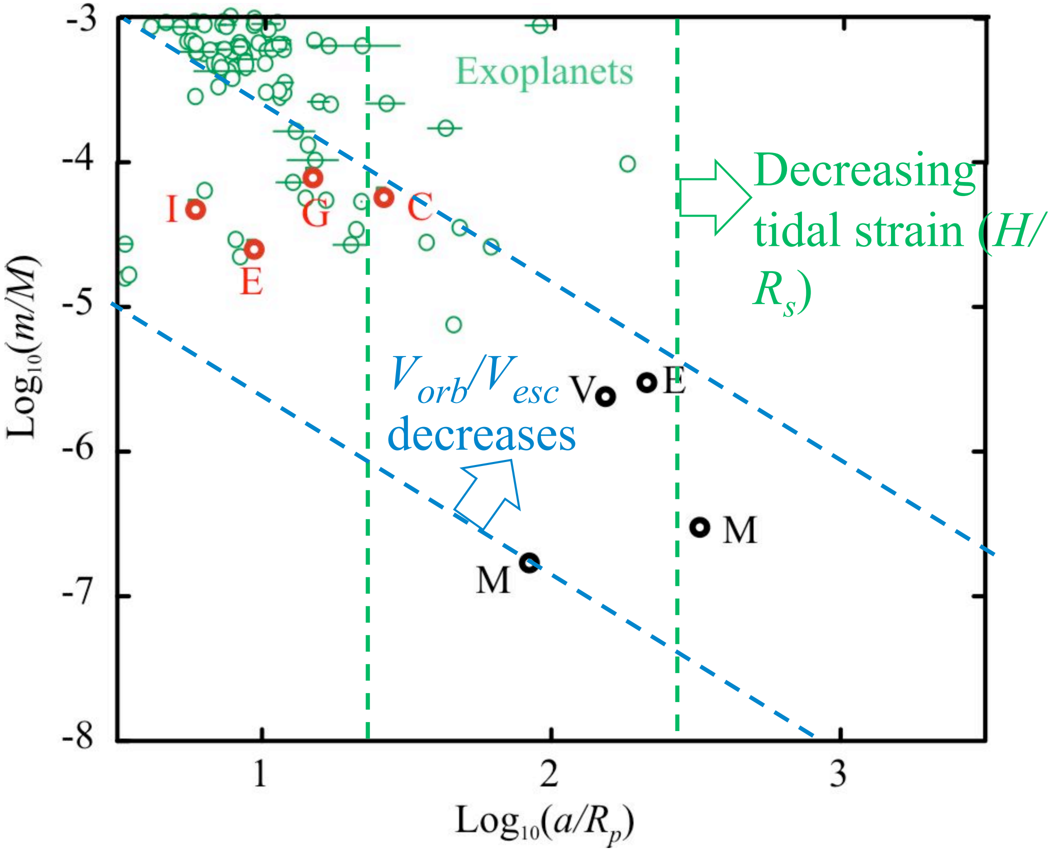
Ensemble Outcomes

- Stochastic process – diversity
- Some radial mixing
- Close-by gas giants can have a significant effect (e.g. asteroid belt)
- *Last, largest* impacts dominate

Individual Growth History



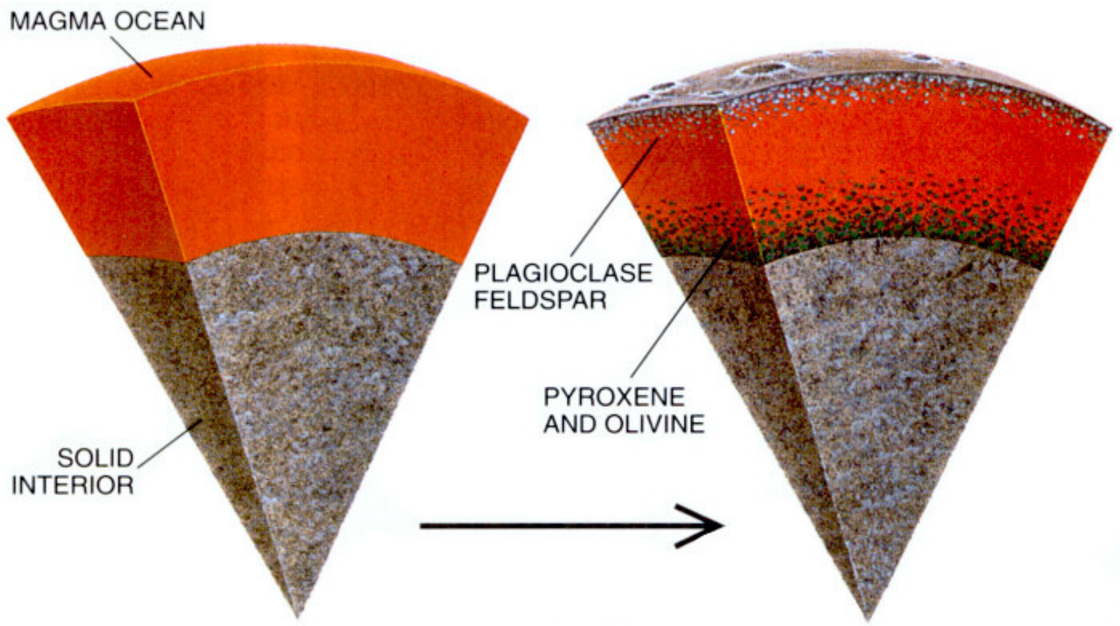
Planet vs. Satellite Accretion



- Satellites experience larger tides
- Impact velocities are comparable to planets
- Satellites require larger total surface densities (but not all material present at same time)

Consequences of Accretion

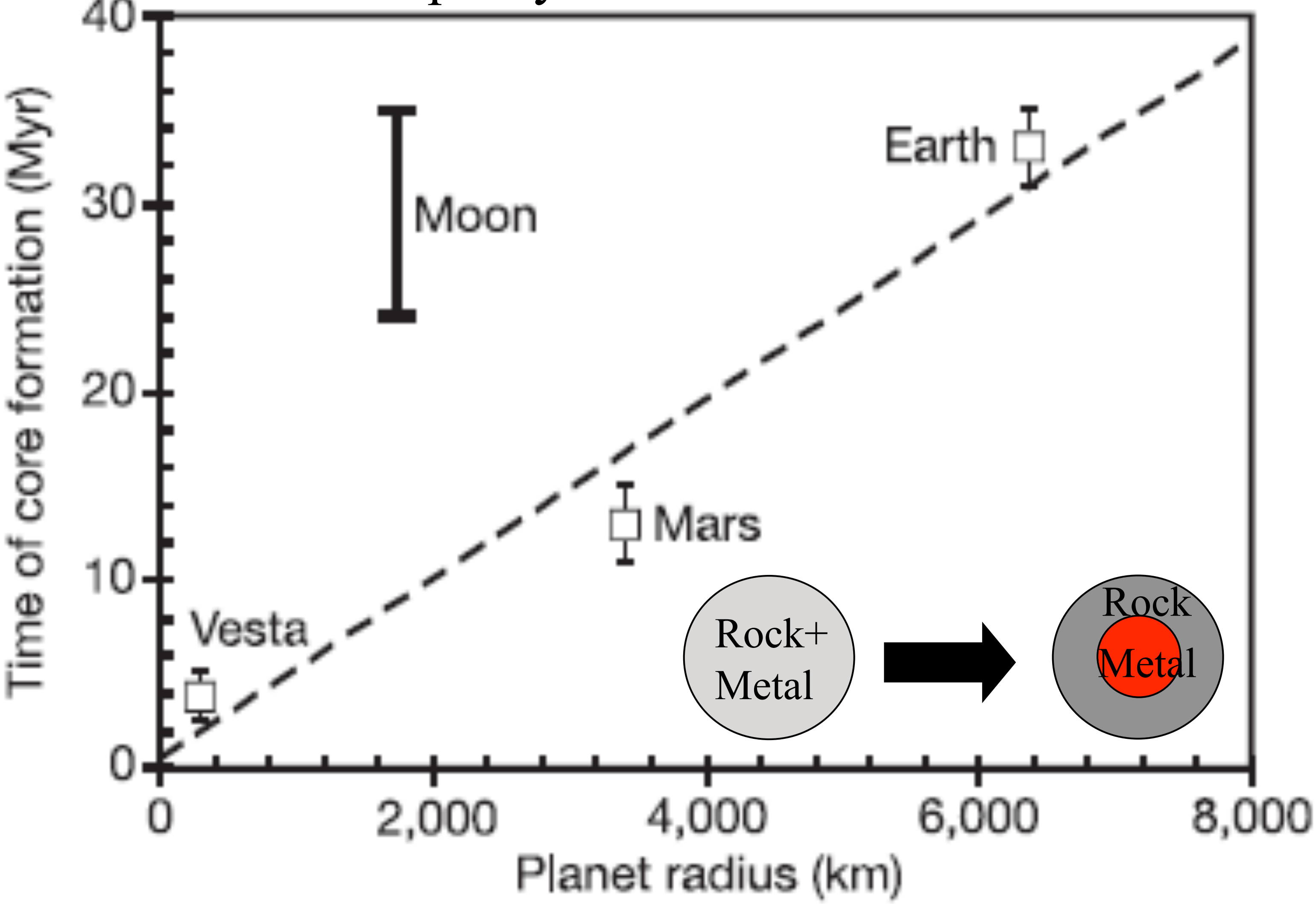
- Large amounts of energy delivered for bodies greater than ~ Mars-size (see below)
- Initially homogeneous body differentiates into core plus mantle
- Magma oceans develop, leading to further differentiation (e.g. lunar plagioclase crust)



Core formation chronometry

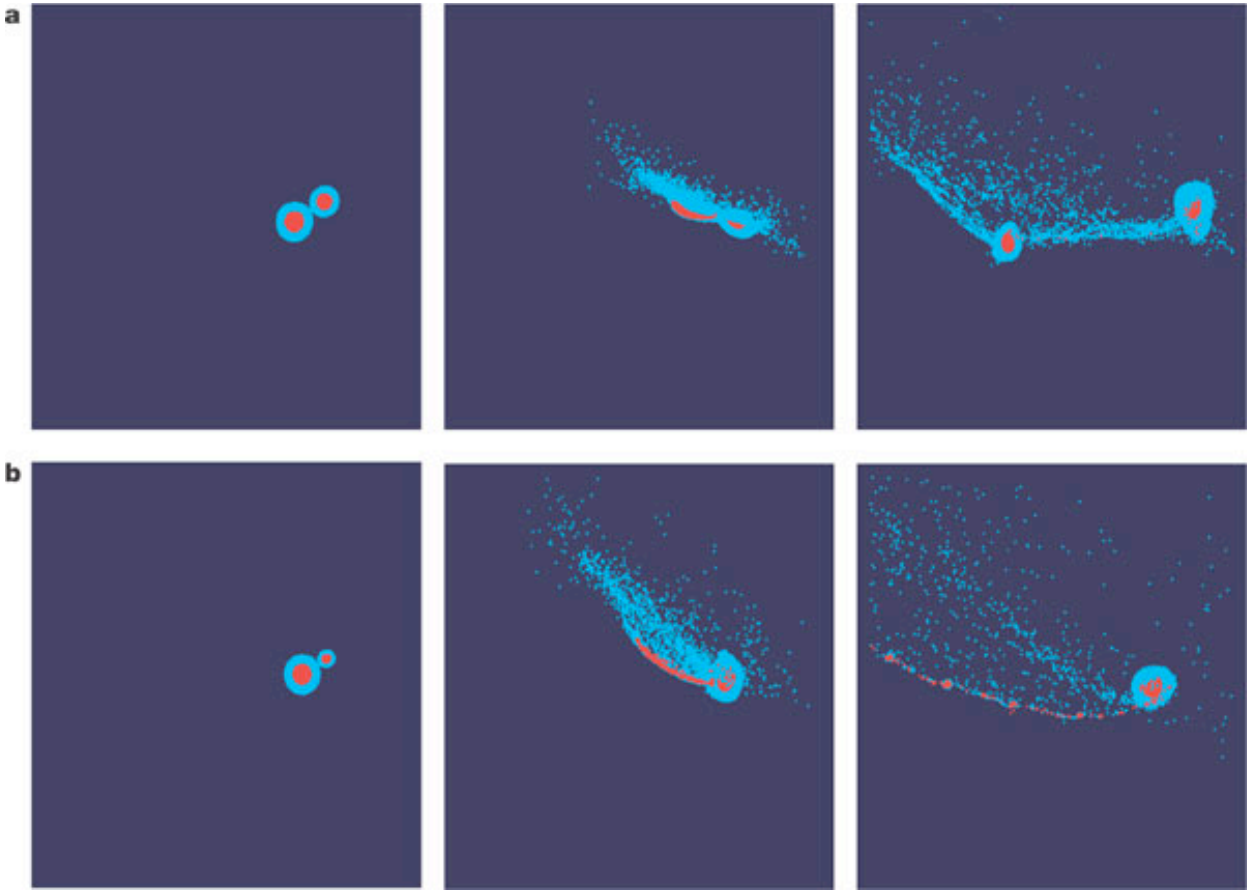
Hf-W isotopic system

Kleine et al. (2002)



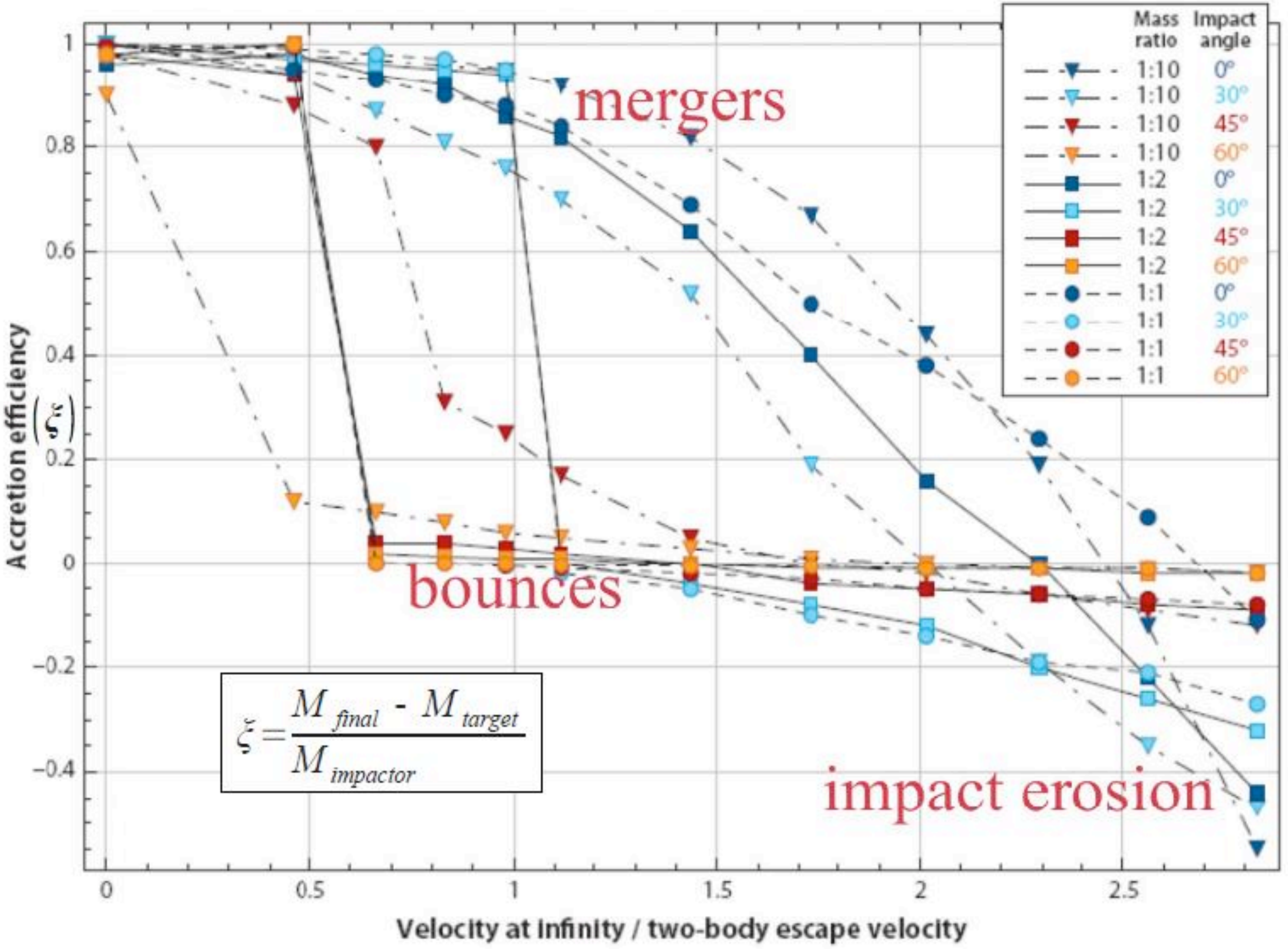
1. Bulk Composition

- Accretion is *not* 100% efficient!
- Examples: Earth/Moon, Mercury, asteroids . . .
- Chemical evidence? (non-chondritic Earth)



Asphaug et al. (2006)

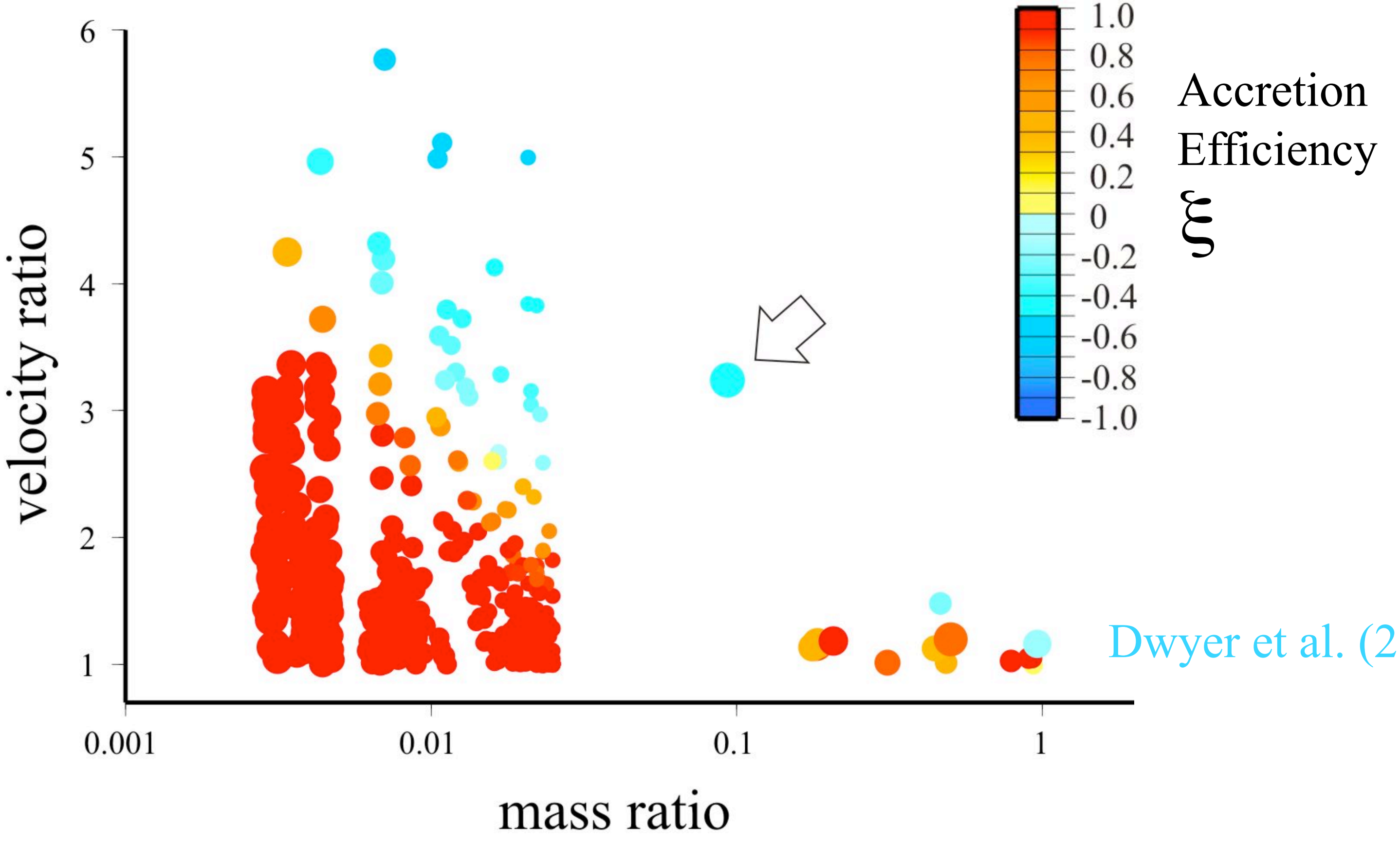
Diverse Outcomes



$$\xi = \frac{M_{final} - M_{target}}{M_{impactor}}$$

Asphaug (2009)

Not all impacts add mass

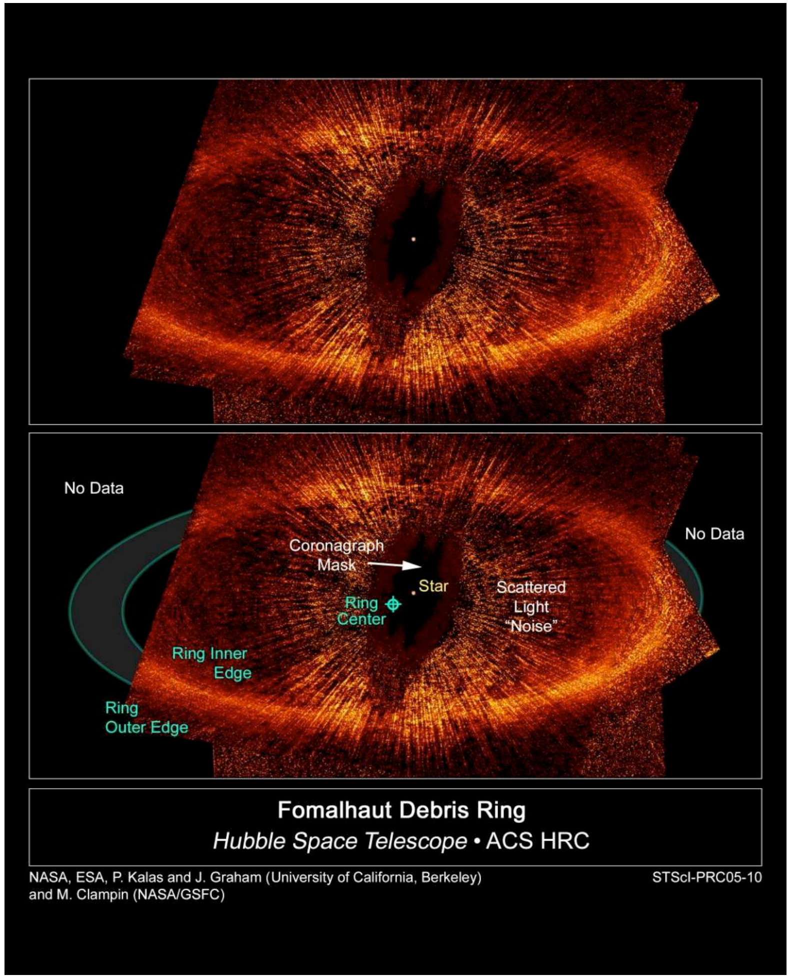


Dwyer et al. (2010)

- Can generate oddball outcomes (e.g. Mercury)

Debris Disks

- Are some debris disks the result of recent impacts?



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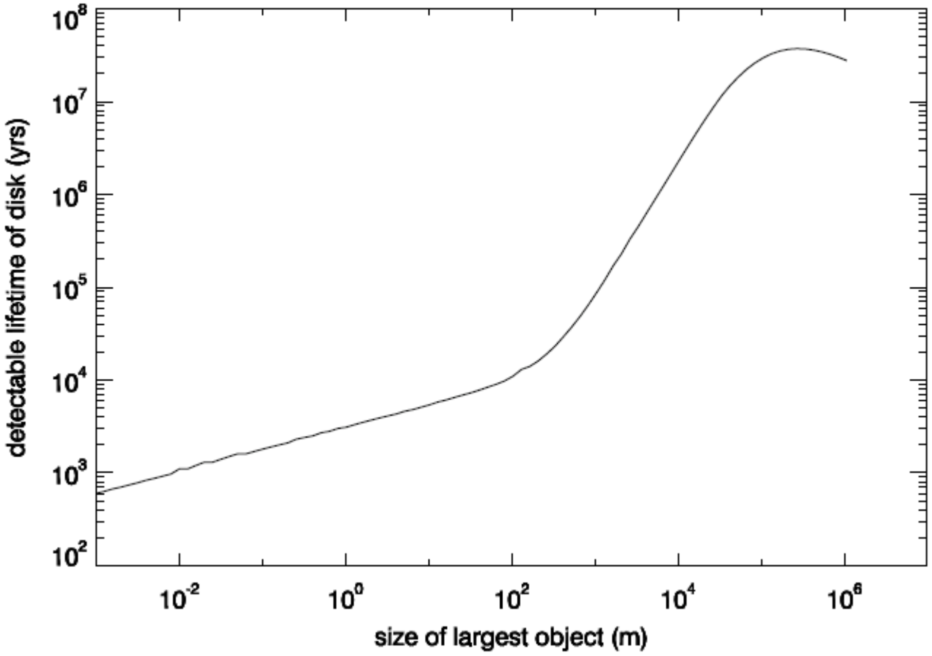
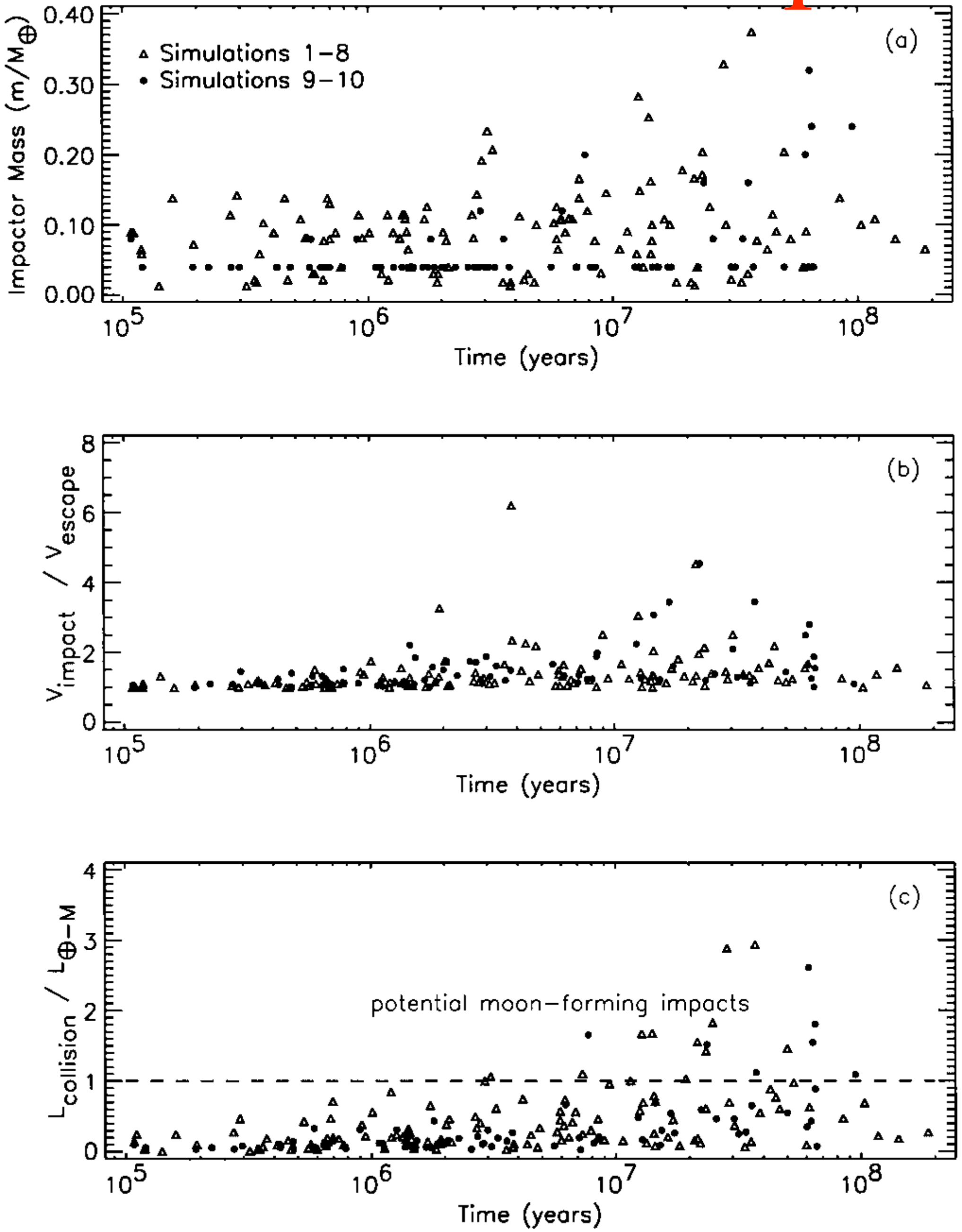
Jackson & Wyatt (2012)



2. Spin/Orbit

Agnor et al. (1999)



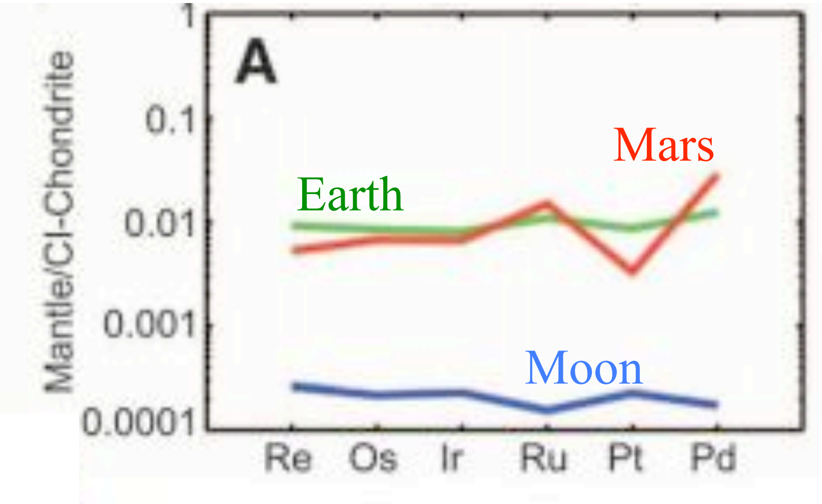
- Spin *rate* close to break-up (on average)
- Spin *orientation* close to random (Uranus?)
- Are these results due to simplified models?
- Tides can modify subsequently (see below)

- Eccentricities/inclinations can be perturbed (but larger angular momentum budget)

3. Waning Accretion

- Earth suffered declining impact flux:
  - Moon-forming impact (~10% M_E, ~4.4 Ga)
  - "Late veneer" (~1% M_E, 4.4-3.9 Ga)
  - "Late Heavy Bombardment" (0.001% M_E, 3.9 Ga)

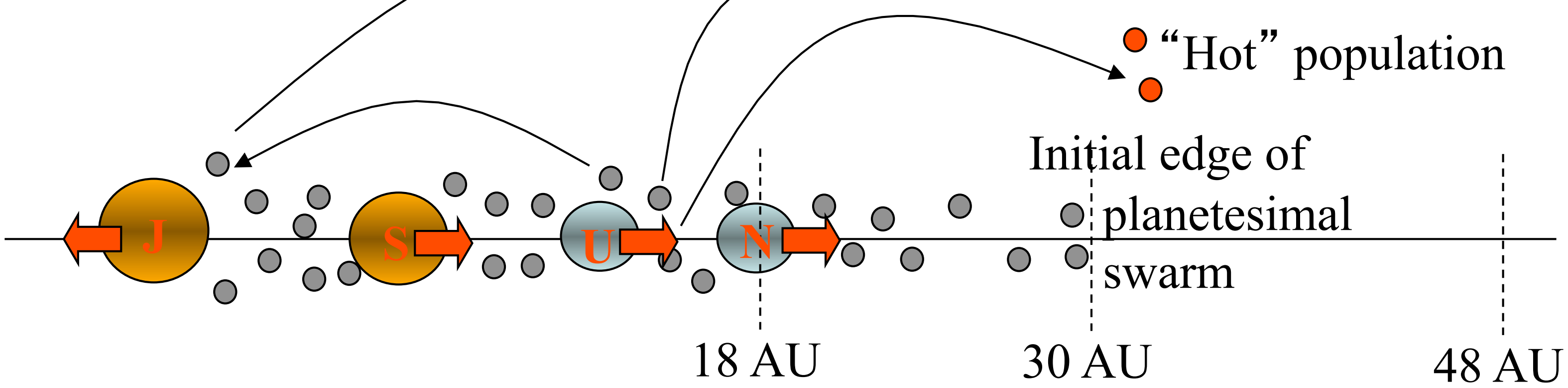
- The LHB may have represented a "spike" due to reorganization of gas giant orbits ("Nice model")
- Consequences for volatiles unclear – addition or blowoff?



Bottke et al. 2010

Nice Model

Early in solar system

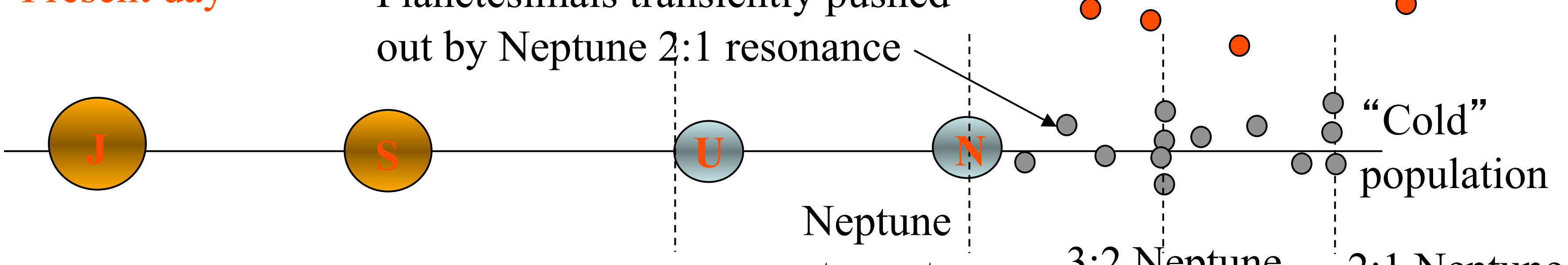


Ejected planetesimals (Oort cloud)

“Hot” population

Initial edge of planetesimal swarm

Present day



Planetesimals transiently pushed out by Neptune 2:1 resonance

“Hot” population

“Cold” population

Neptune stops at original edge

3:2 Neptune resonance (Pluto)

2:1 Neptune resonance

See Gomes et al., *Nature* 2005

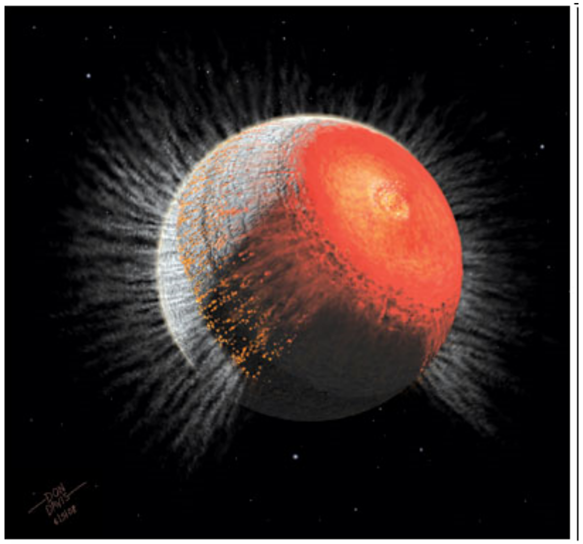
Part I (Accretion) - Summary

- Late-stage planetary growth involves collisions between like-size objects
- Collisions are stochastic events - diversity
- Accretion is *not* 100% efficient (though it is usually modelled as such)

- Geochemical constraints on growth process exist
- The *last, large* impact determines the initial boundary conditions

Part II – Heat Sources

- Why are heat sources important?:
  - High temperatures cause observable effects (differentiation, melting, dynamos etc.)
  - Initial heating can influence long-term evolution
  - Long-term (Gyr) evolution controlled by balance between heat sources and heat loss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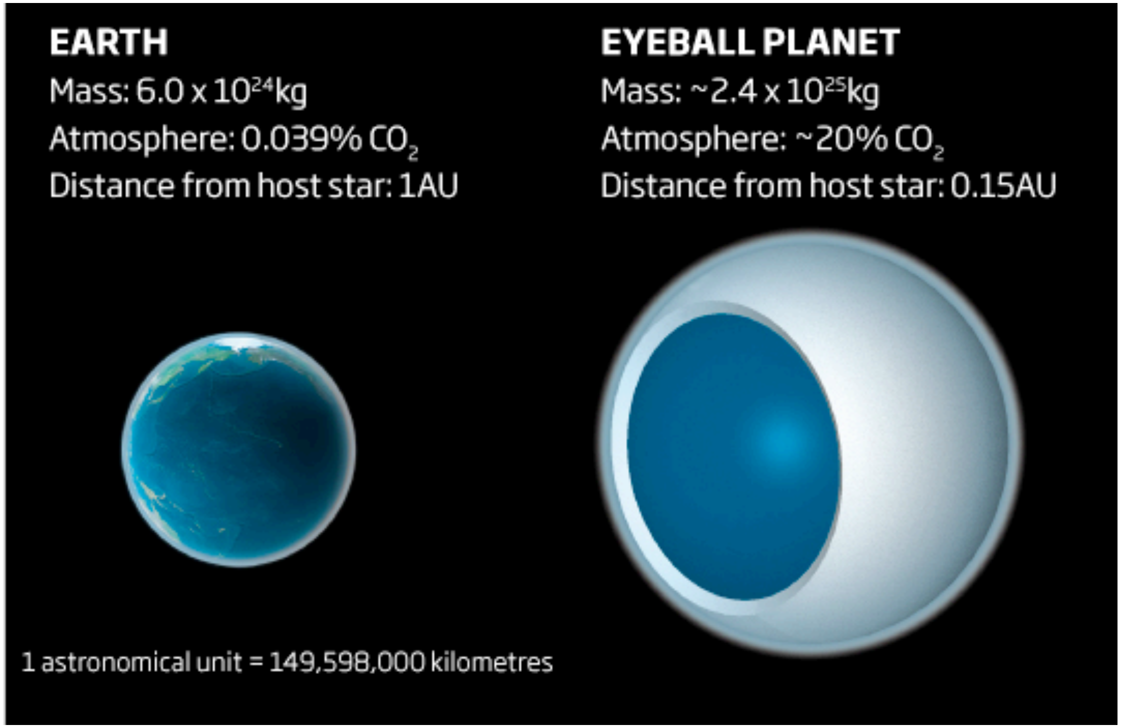
Topics

- 1. Insolation
- 2. Radioactive decay
- 3. Gravitational energy (impacts)
- 4. Tides (see below)
- 5. Induction heating (not covered)

Resources: Rubie et al.,*Treatise Geophys.,* 2007

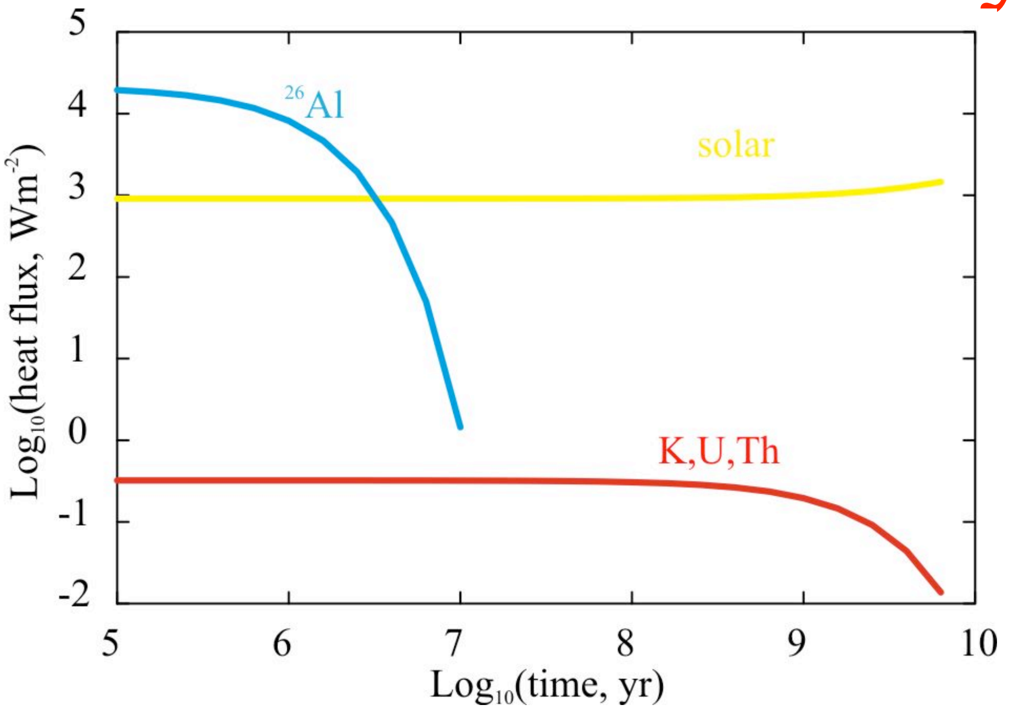
1. Insolation

- Determines surface temperature
- Greenhouse effect, runaways (Venus)
- Lava-ocean planets & "Eyeball Earths"



Pierrehumbert (2011) and Leger et al. (2011)

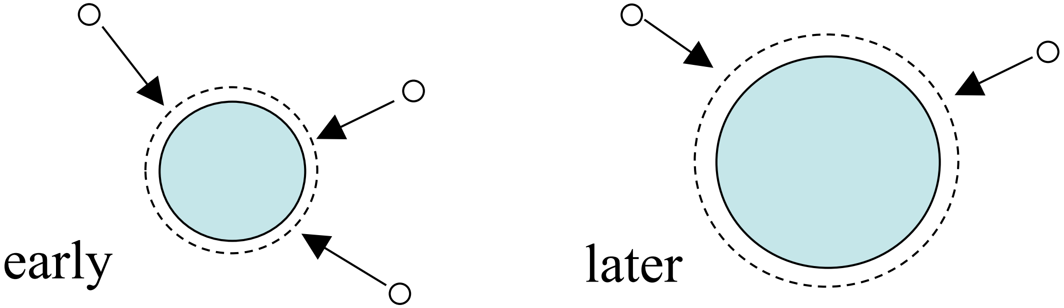
2. Radioactive decay



- ^{26}Al decay ($t_{1/2}$=0.7 Myr) is *extremely* energetic
- Planet growth time relative to ^{26}Al decay time matters
- ^{26}Al was definitely present when some asteroids (and perhaps Mars) formed and melted
- K,U,Th provide main long-lived (Gyr) energy source

3. Accretion

- “Onion-shell model”, assuming no radiative losses



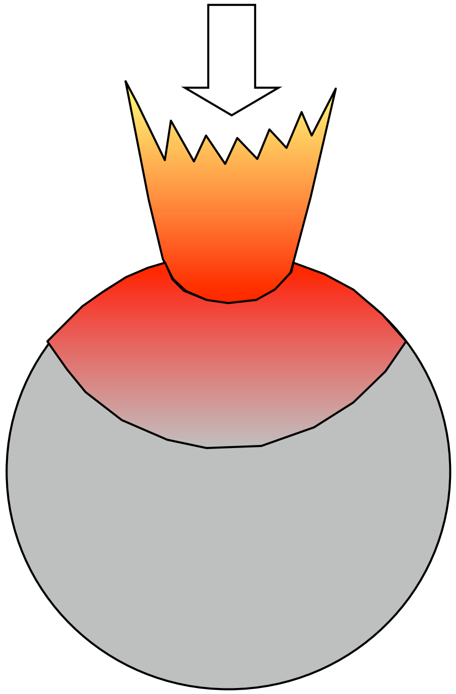
$$\Delta E = \frac{3}{5}\frac{GM^2}{R}$$

$$\Delta T \approx \frac{GM^{2/3}\rho^{1/3}}{C_p} \approx 40{,}000\,\mathrm{K}\left(\frac{M}{M_E}\right)^{2/3}$$

- CAVEATS!:
- For slow accretion and small impactors , radiation may be important
- Impacts are large, discrete and stochastic, not continuous and small
- Spatial heterogeneity may be important

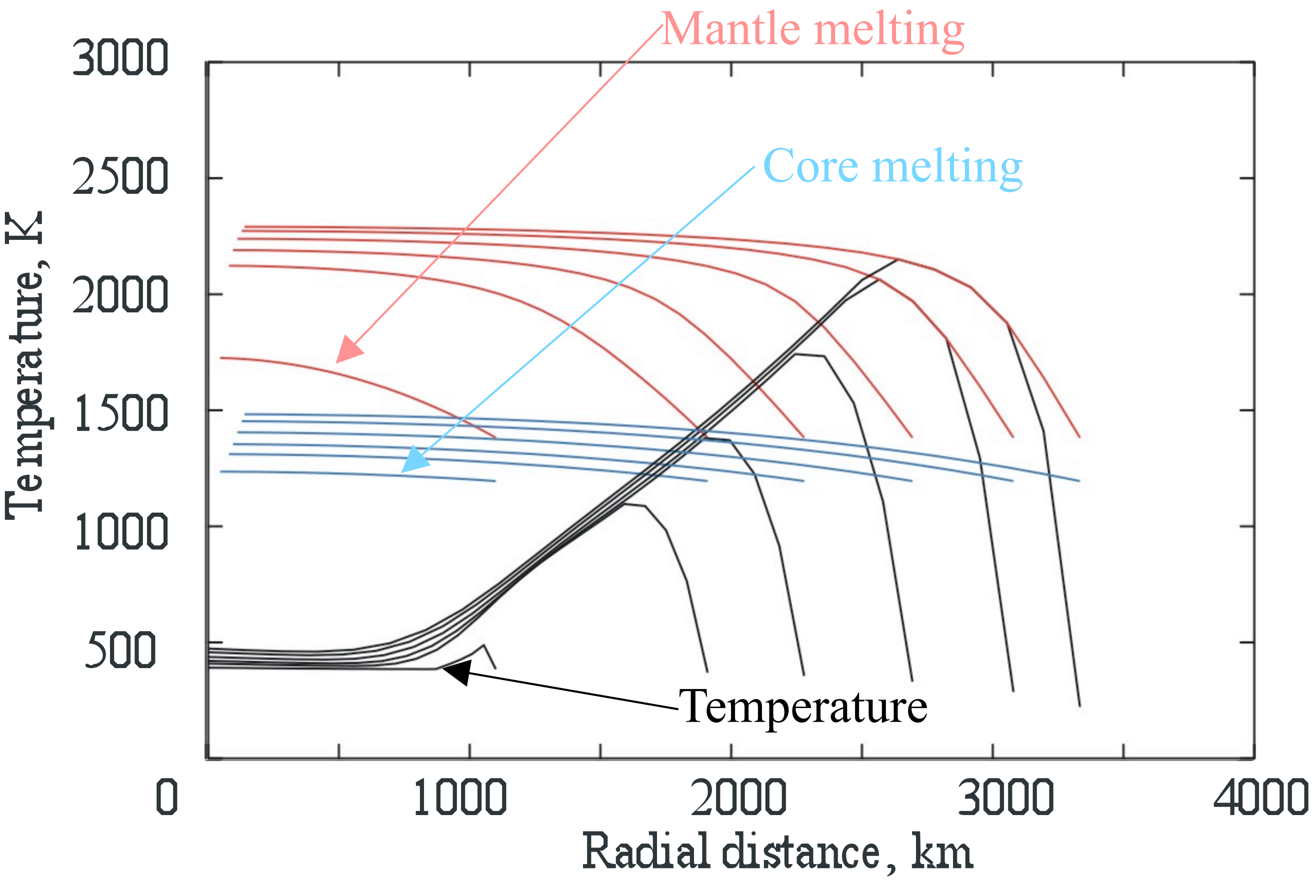
None the less, Earth-mass bodies almost certainly started life *molten*

Impacts



- “Small” impacts (very roughly < 1% of target mass) cause local heating
- “Large” impacts have global effects
- *Size spectrum* of impactors is very important

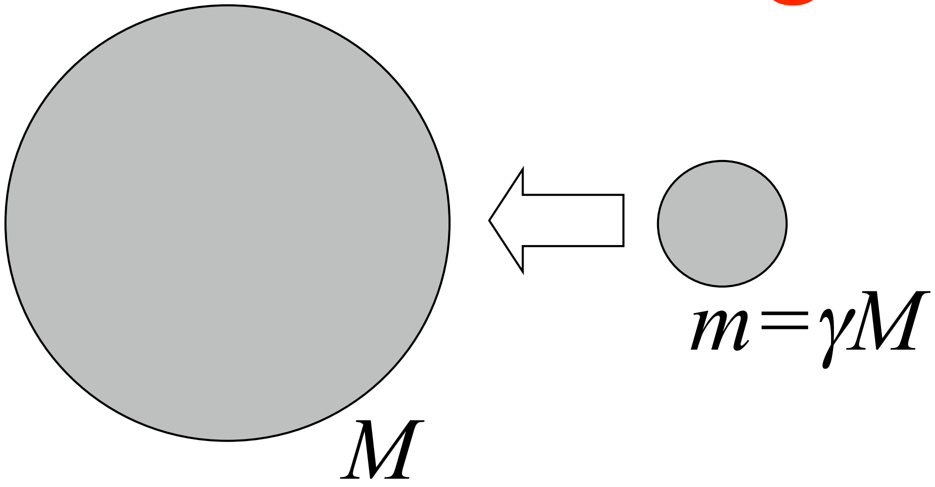
“Small” Impacts



Method of Squyres et al. (1988)

- Temperatures highest near surface
- Melting only at shallow depths for Mars-sized object
- Accretion entirely from small bodies makes *cold* bodies

“Big” Impacts



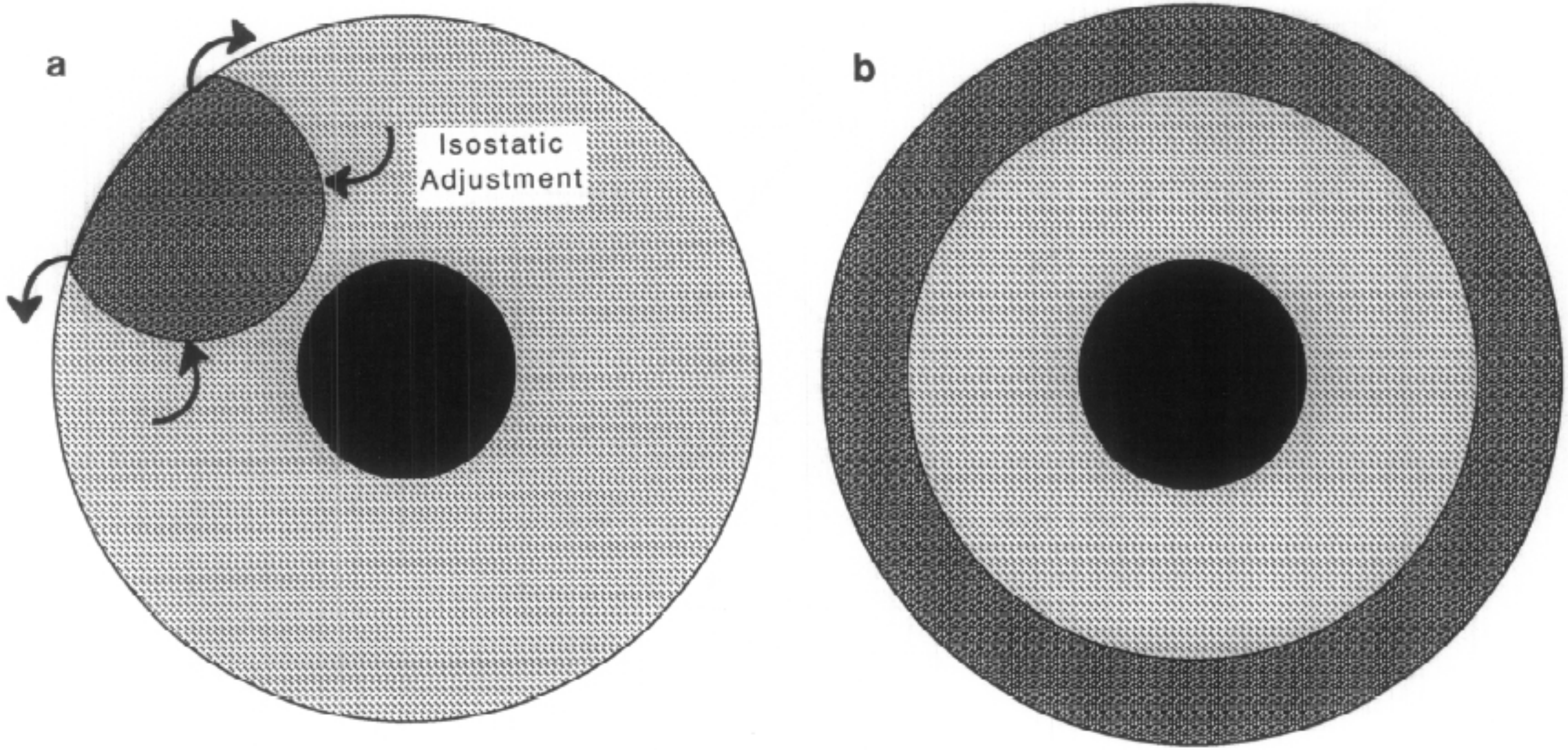
- Accretion involves collisions between comparable-mass objects
- Assume all energy deposited into interior

$$\Delta T \approx 6{,}000\,\mathrm{K}\left(\frac{\gamma}{0.1}\right)\left(\frac{M}{M_E}\right)^{2/3}$$

Rubie et al. 2007

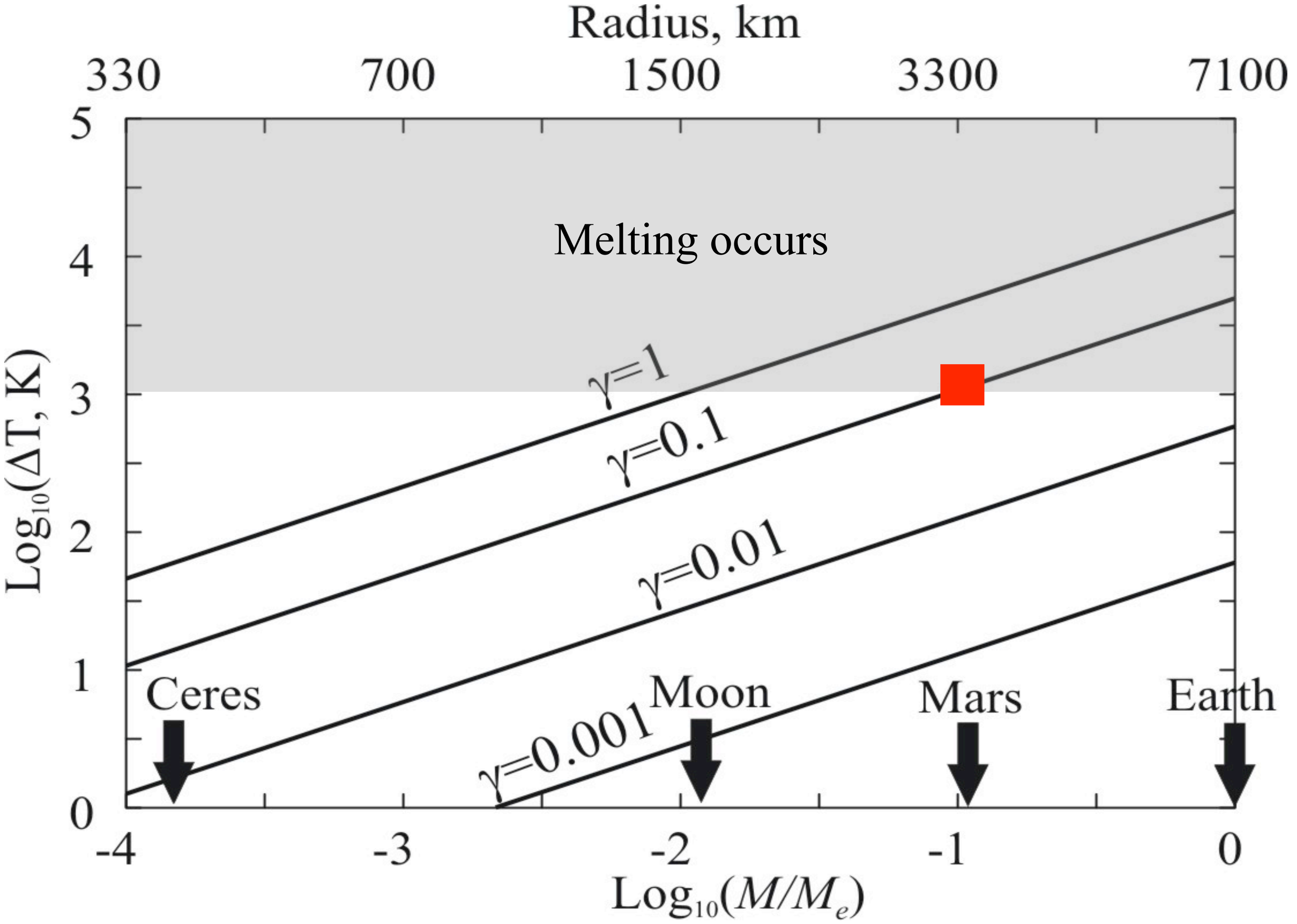
- This is *averaged* temperature increase
- Heating will in reality be (initially) spatially variable

Magma “sea” readjustment

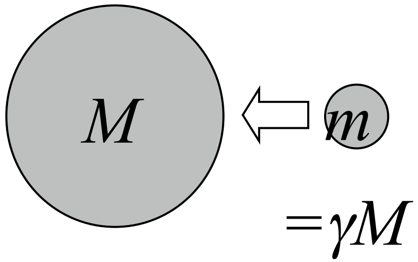


Tonks and Melosh 1992

"Big" Impacts

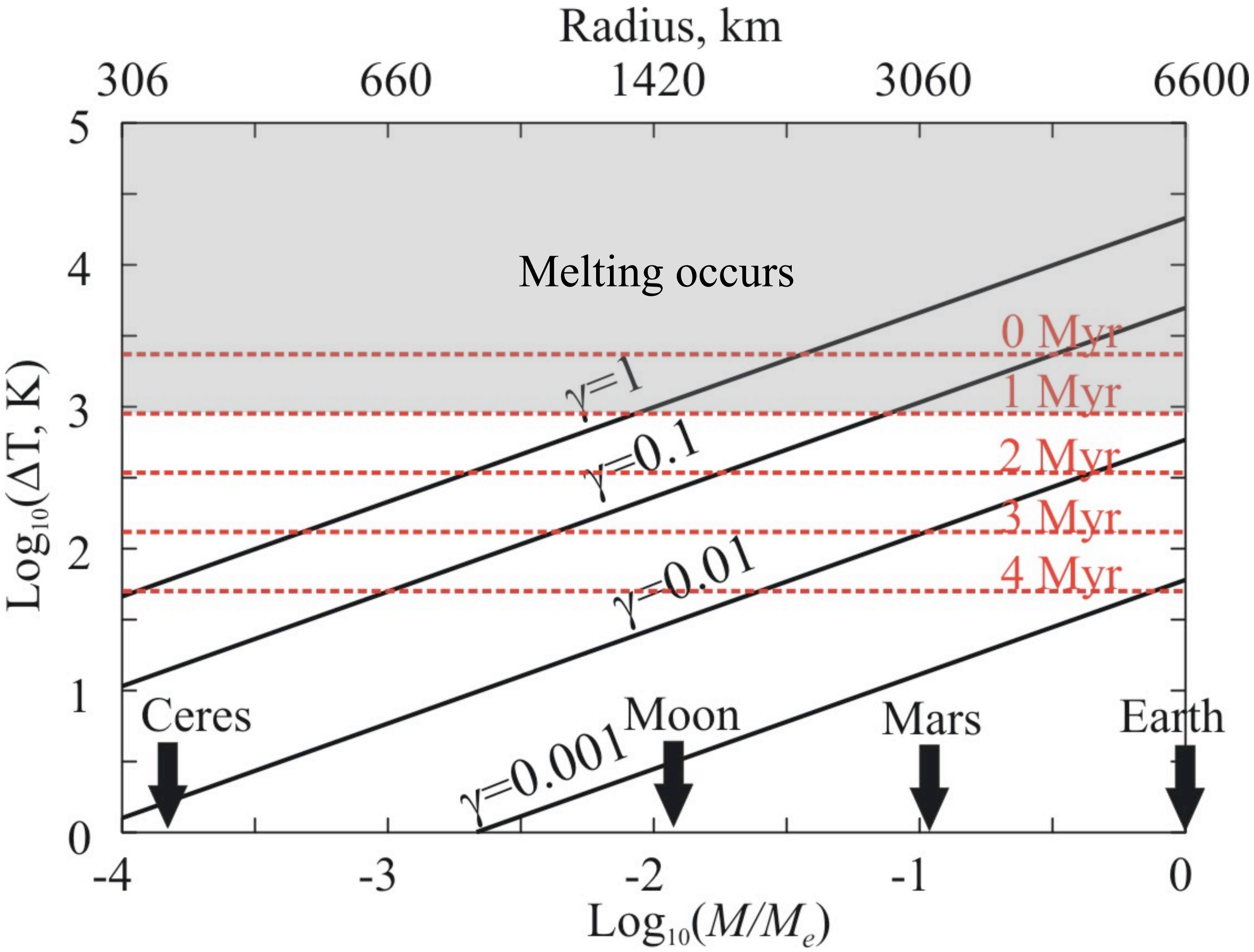


Rubie et al. 2007



- Whether melting occurs depends on both M and γ – impactor size spectrum is important
- 0.1 M_E body suffering a single giant impact (γ=0.1) will be *hot*

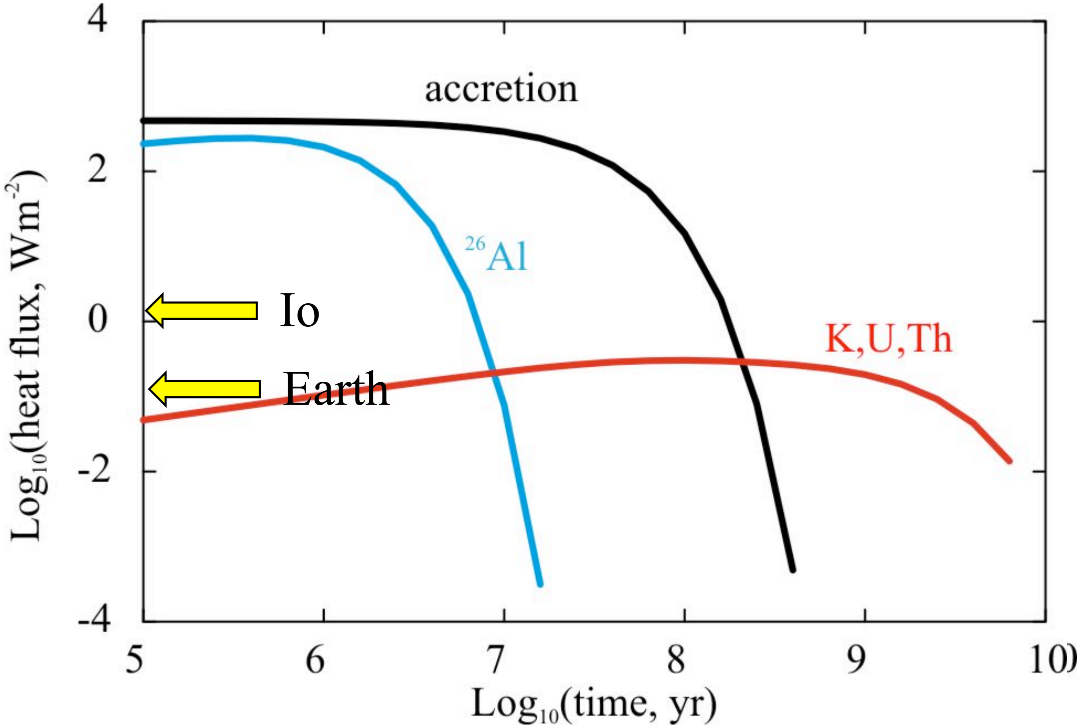
Combined Effects



- Relative importance of impacts and radioactivity depends on body mass, impactor size and timescale
- Melting unavoidable for Earth-sized objects

Evolution with time

$$\dot{E} \sim G\rho^{1/3} M^{2/3} \frac{dM}{dt}$$

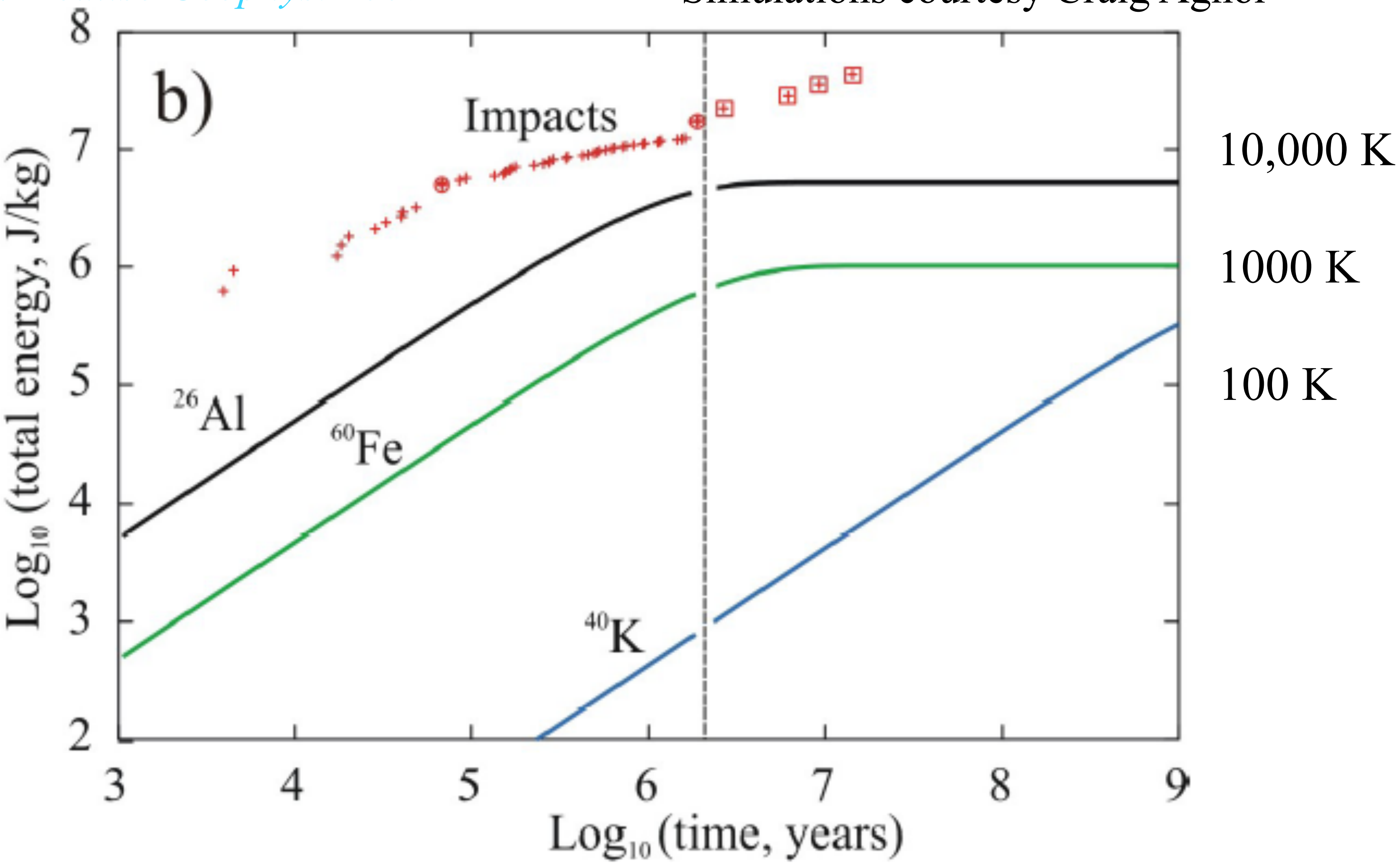


Accretion dominates for ~100 Myr
Continuum approximation to discrete, stochastic, spatially variable process!

N-body simulations

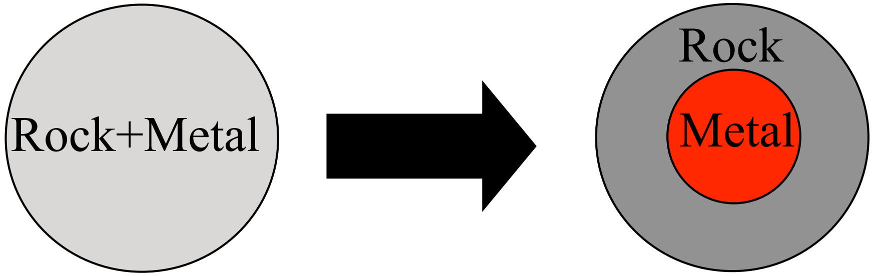
Rubie et al. *Treatise Geophys. 2007*

Simulations courtesy Craig Agnor



- Gravity is the dominant heat source

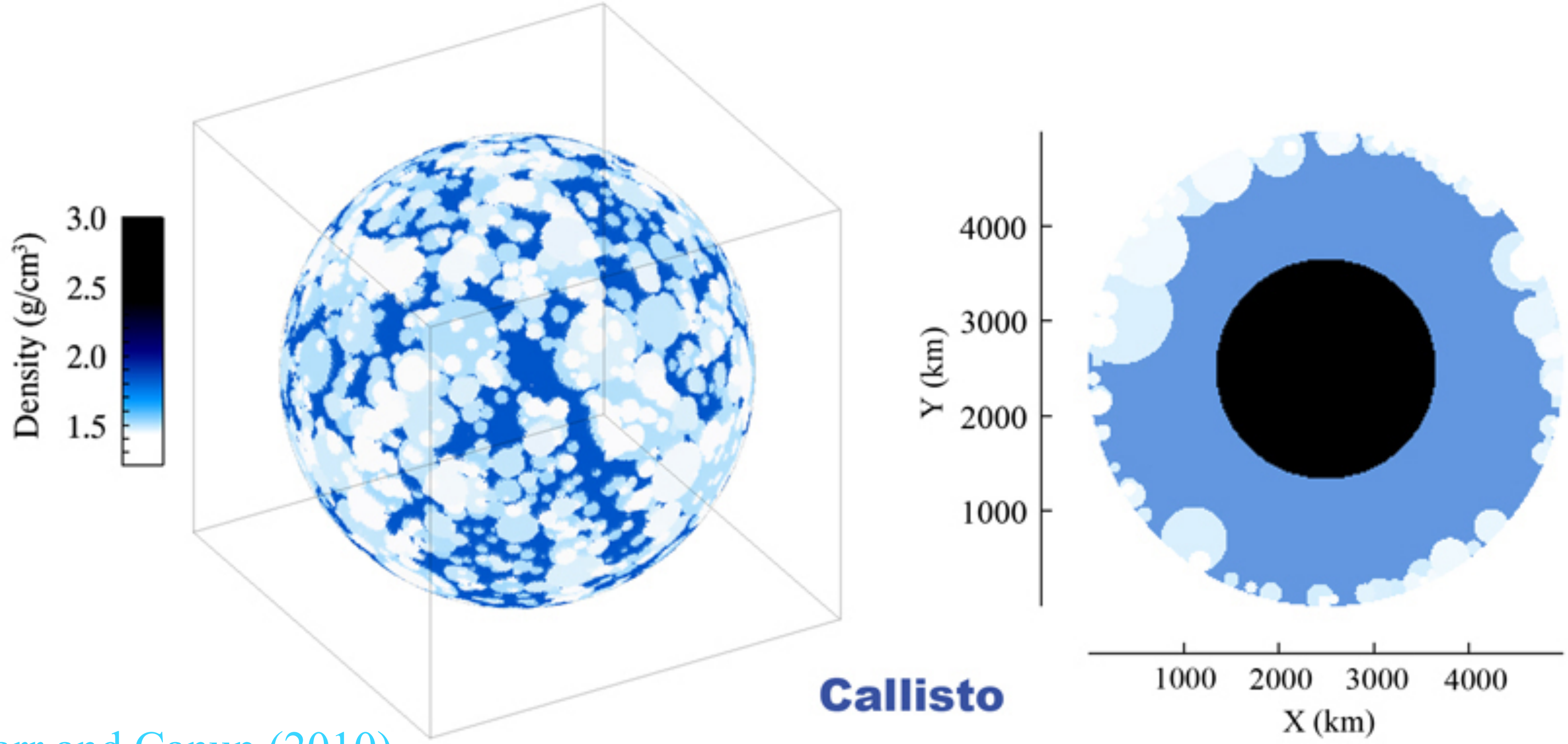
Differentiation and Core Formation



- Differentiation occurs when temperatures get high enough for melting to occur
- Differentiation releases further potential energy
- Cores of Earth-size bodies start life hot (assuming rapid transport of core material)
- Hot cores are good for driving planetary dynamos (Earth, Mars, Mercury?)
- Differentiation leaves isotopic signatures (Hf/W)
- Similar arguments apply to rock/ice mixtures

Incomplete differentiation (?)

- Titan (likely*) and Callisto (possibly) have not completely differentiated – requires low *T*
- This implies they were put together slowly, out of small objects – constraint on accretion process



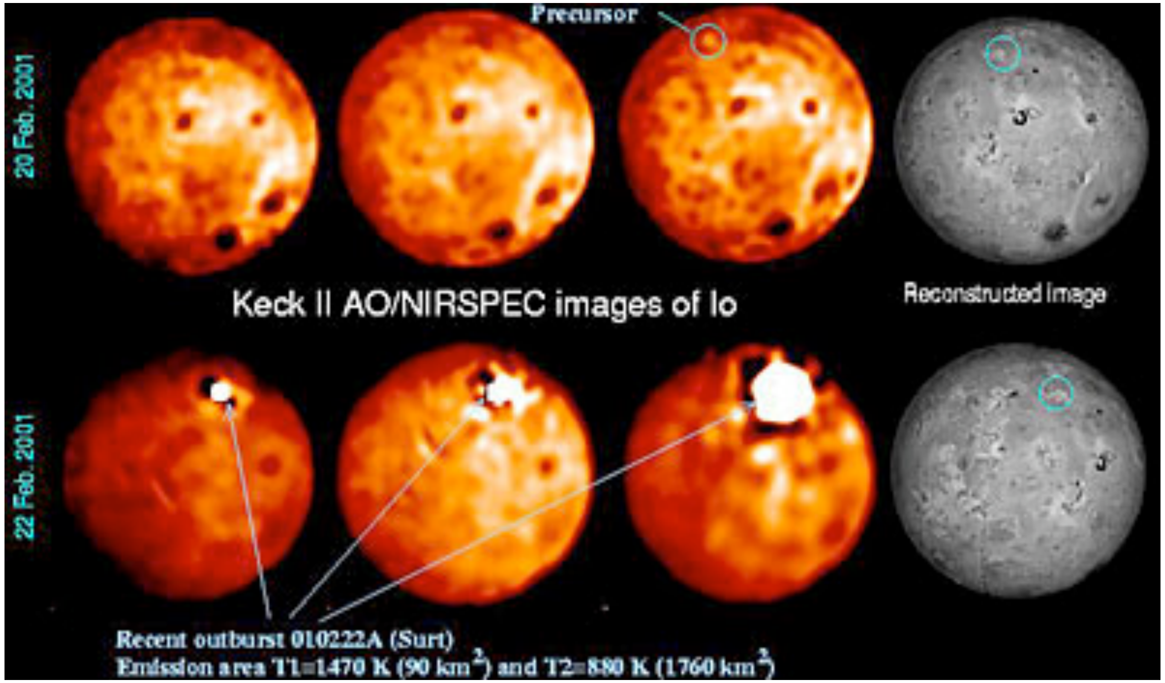
Barr and Canup (2010)

Part II (Heat Sources) - Summary

- Insolation only sets boundary conditions
- Grav. energy depends on size-spectrum of bodies
- Global melting is *inevitable* for Earth-sized objects (magma oceans)
- Radioactivity most important for small objects
- For Earth-mass bodies, two epochs:
  - Early (~100 Myr): accretion dominates
  - Later: long-lived radionuclides
- Melting leads to differentiation (core formation)

Part III – Heat transfer

- Why is heat transfer important?:
  - It controls the duration and magnitude of a body’s geological activity (outgassing, dynamo etc.)
  - It can (potentially) be remotely measured



NASA

Topics

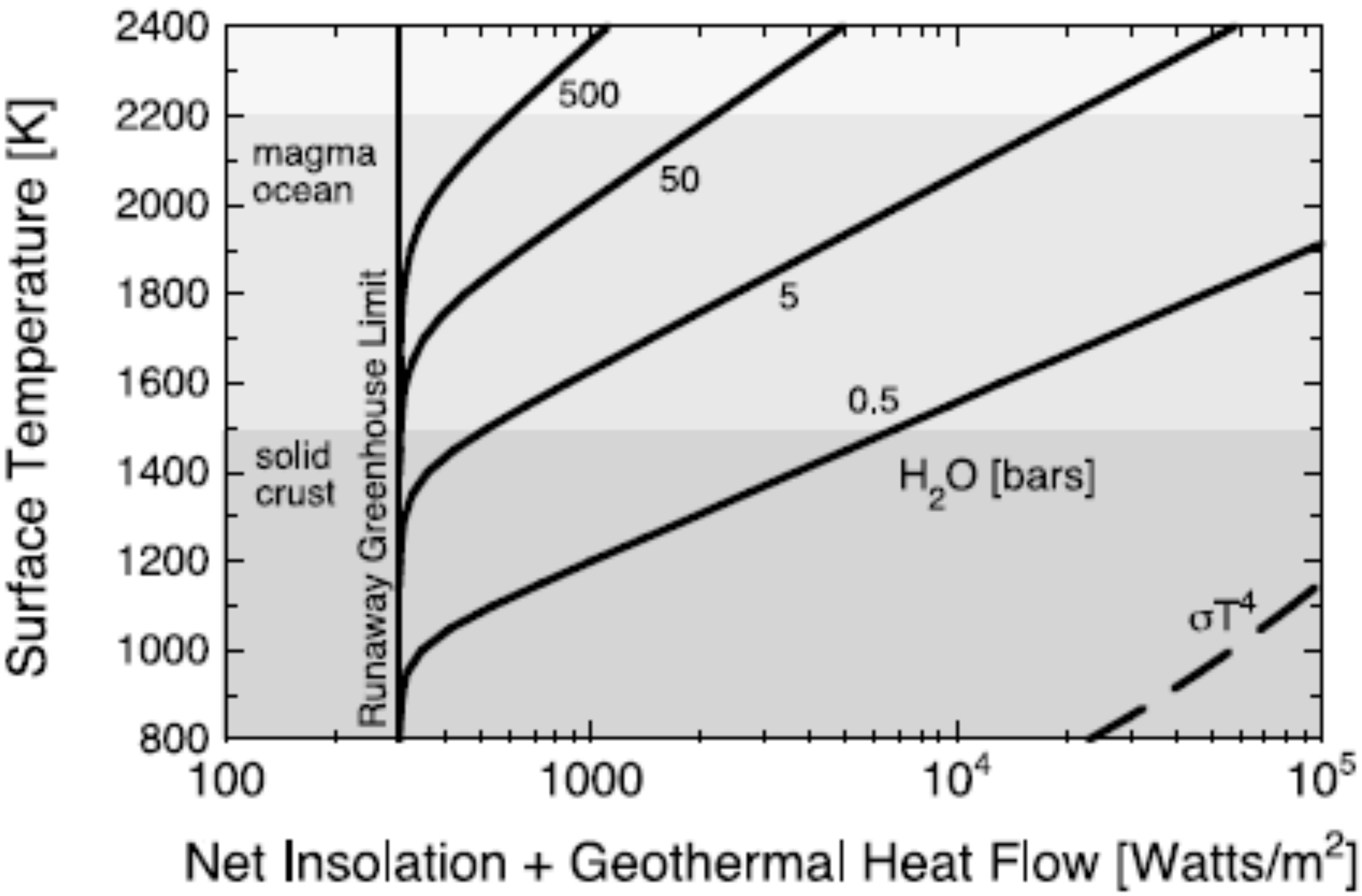
- 1. Magma oceans
- 2. Solid state convection
- 3. Advection (melt)

Resources: Rubie et al., *Treatise Geophys.*, 2007

Magma ocean evolution

- Large bodies started life pervasively molten
- Magma ocean lifetimes highly uncertain:
  - Convective/radiative: few kyr (e.g. Solomatov 2000)
  - Conductive: tens of Myr (flotation crust, small bodies *only*)
  - Thick steam atmosphere: ~100 Myr (Zahnle et al. 2007)
- Is lifetime long or short compared to interval between "big" impacts?
- For how long would the IR emission be visible?
- Magma oceans can produce unstable density structures (subsequent overturn)

Early thick atmosphere?



Zahnle et al. 2007

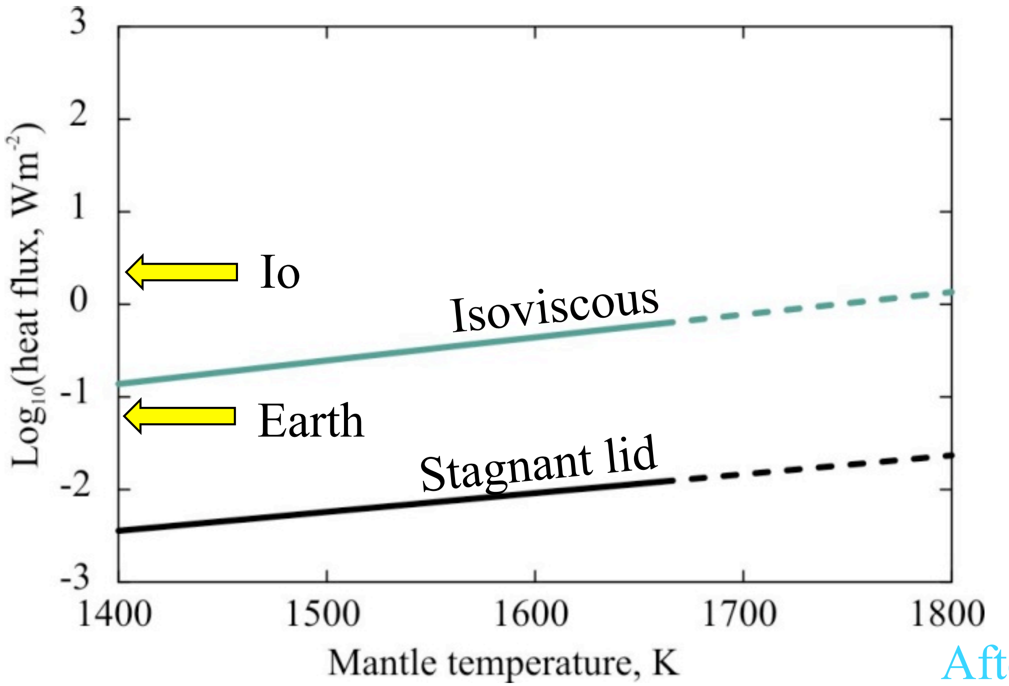
Mantle convection

Mantle viscosity temperature-dependent $\eta=\eta_0\exp(-\gamma T)$

$$F_{sl} \sim k\left(\frac{GR\alpha\rho^2}{\kappa\eta_b}\right)^{1/3}\gamma^{-4/3} \sim 4\ \mathrm{mWm^{-2}}\left(\frac{10^{21}\ \mathrm{Pa\ s}}{\eta_b}\right)^{1/3}\left(\frac{\gamma}{0.01\ \mathrm{K^{-1}}}\right)^{-4/3}$$

$$F_{pt} \sim k\left(\frac{GR\alpha\rho^2}{\kappa\eta_b}\right)^{1/3}\Delta T^{4/3} \sim 150\ \mathrm{mWm^{-2}}\left(\frac{10^{21}\ \mathrm{Pa\ s}}{\eta_b}\right)^{1/3}\left(\frac{R}{6000\ \mathrm{km}}\right)^{1/3}$$

Solomatov (1995)

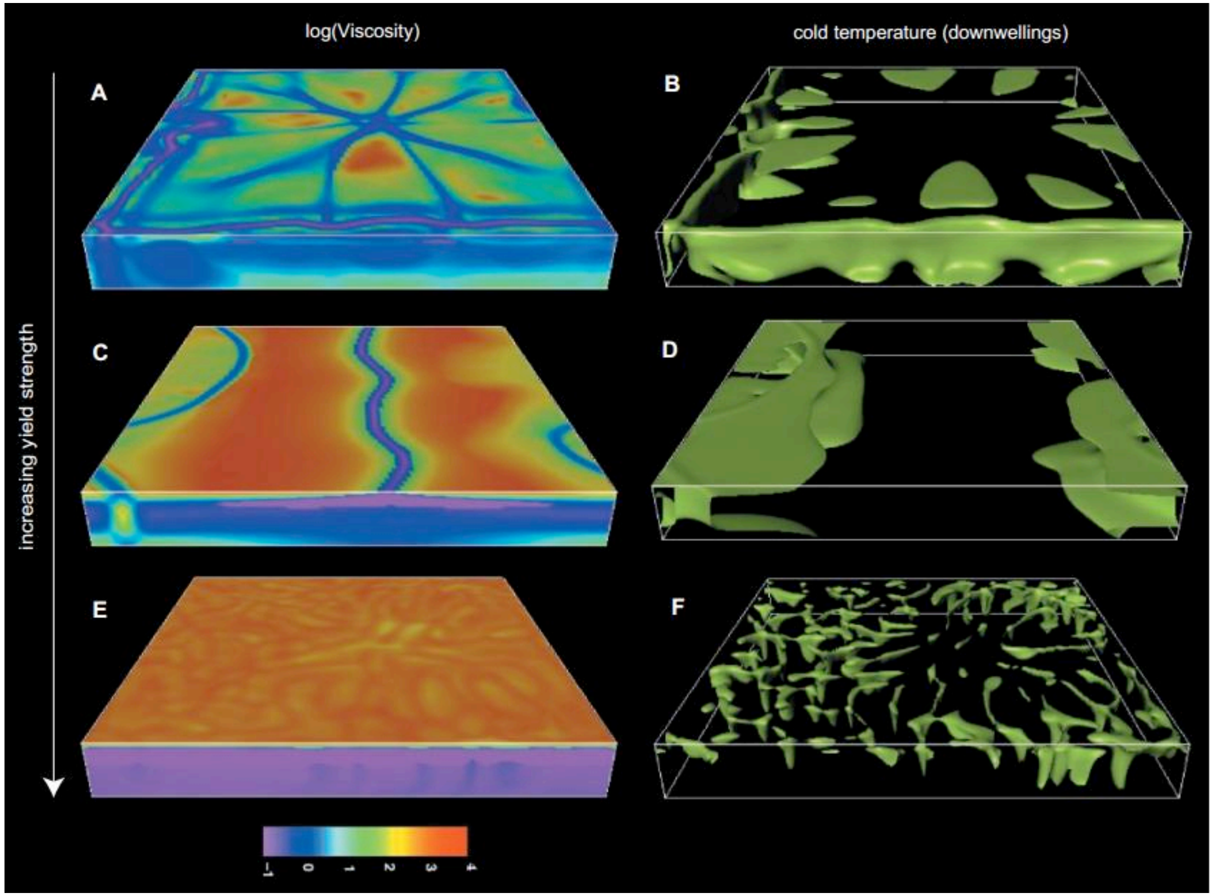


After Moore, *Icarus*, 2001

Stagnant lid vs. plate tectonics

Low yield strength

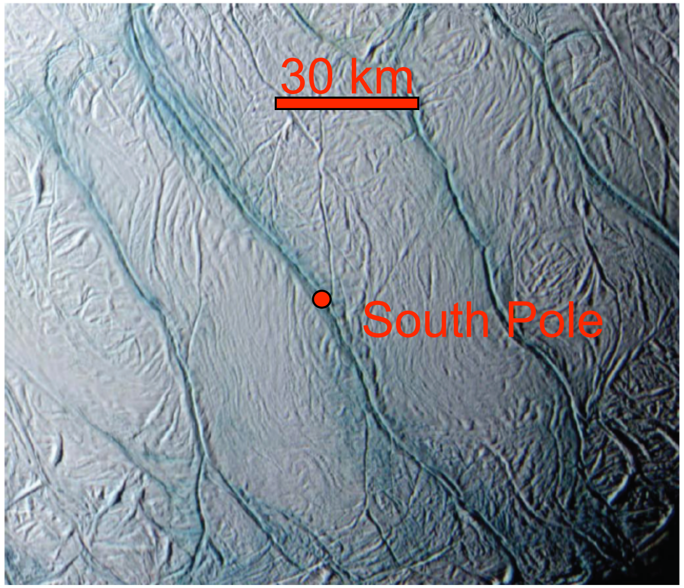
High yield strength

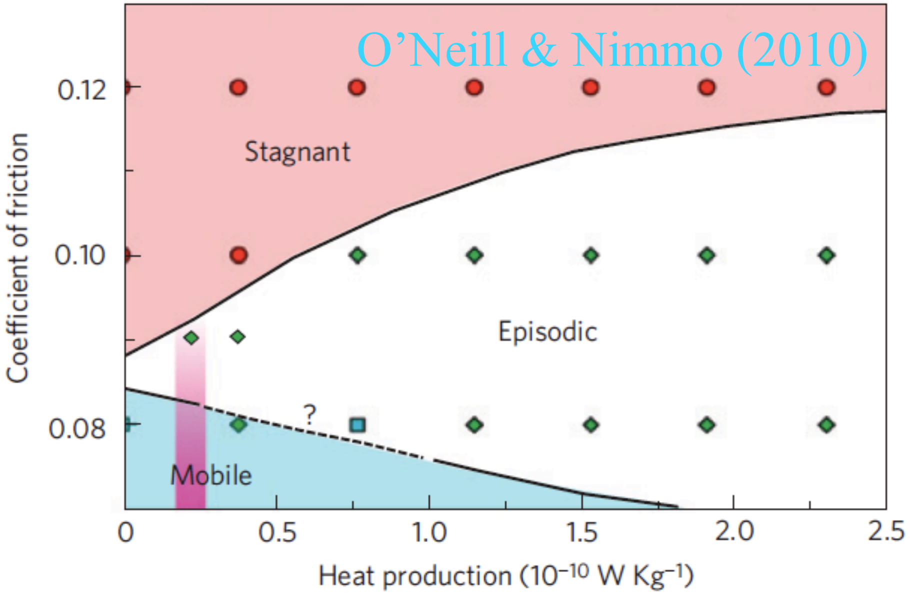


Tackley (2002)

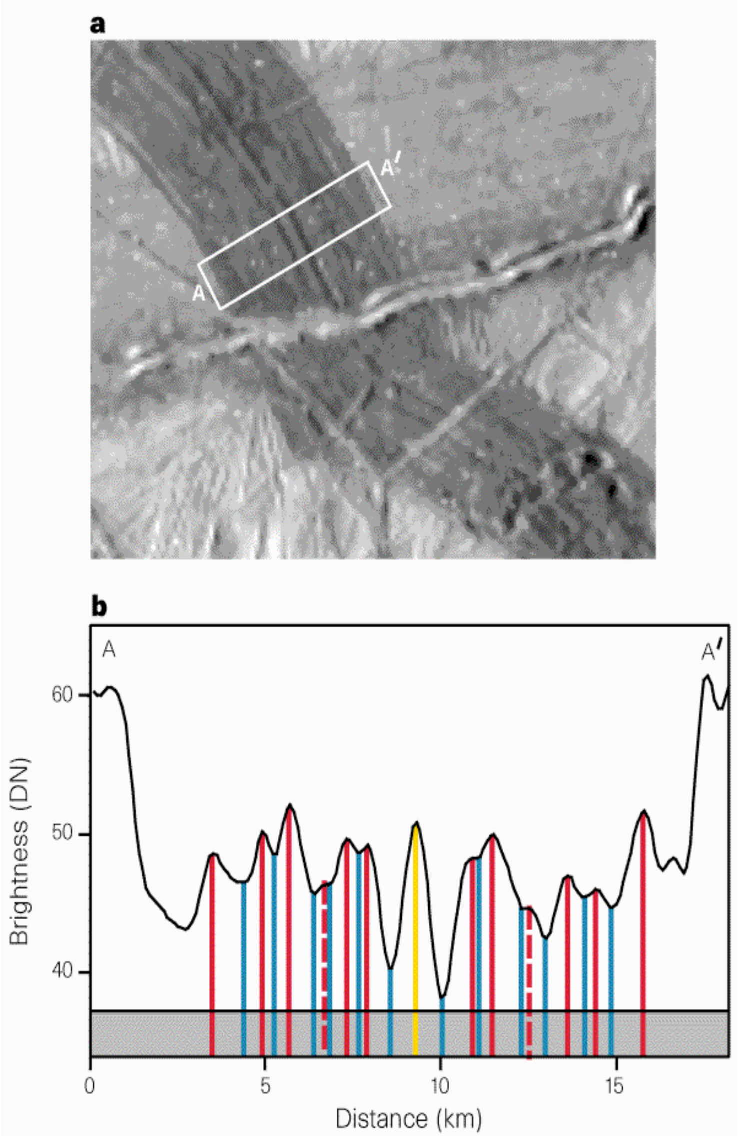
- Yield strength (compared to convective stress)
- Earth vs. Venus – water is important!
- What does “yield strength” really mean?

Icy satellite plate tectonics?





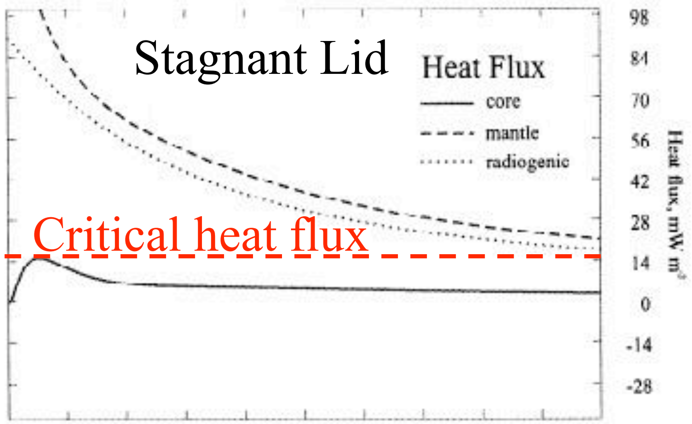
O'Neill & Nimmo (2010)



Sullivan et al., (1998)

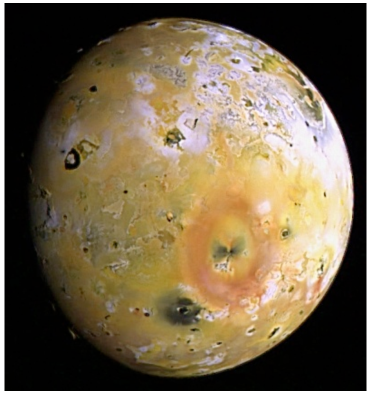
Dynamos

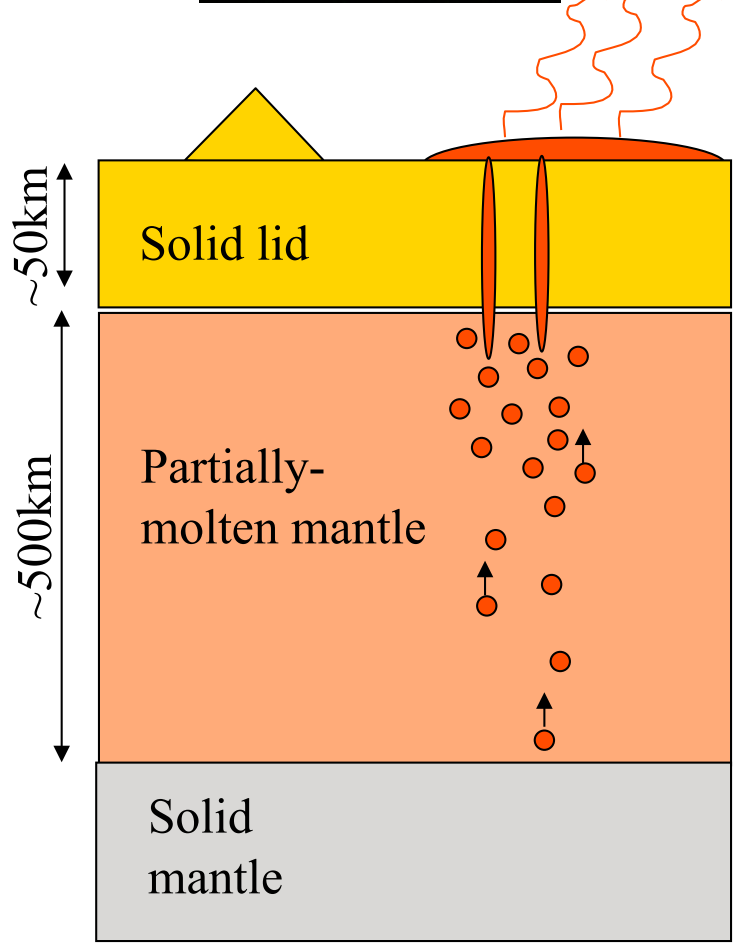
- Dynamos (usually) depend on how rapidly heat is being extracted *by the mantle*
- Whether or not plate tectonics operates can control dynamo activity (e.g. Earth vs. Venus)
- Early dynamos (Moon, Mars) are affected by initial hot core
- So initial conditions (accretion) may control dynamo operation
- Mechanically-stirred dynamos? (Dwyer et al. 2011)

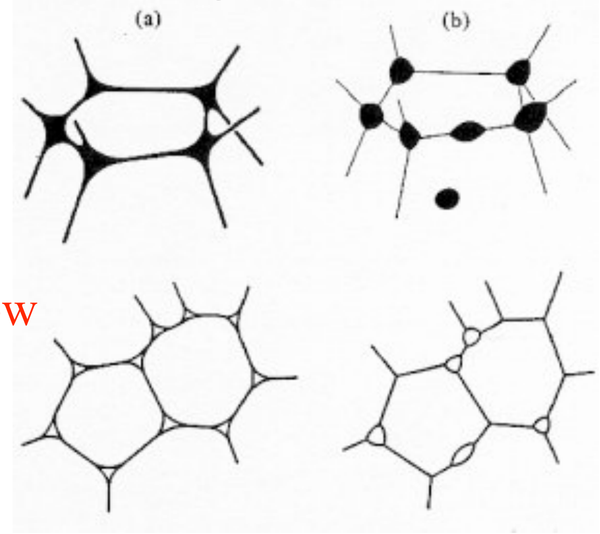


Melting

- Advection can be an efficient heat transfer mechanism
- E.g. Io 2 Wm^{-2} (!)
- Near-surface melt transfer is macroscopic (e.g. dikes)
- Mantle melt transfer is microscopic (porous flow)
- Dihedral angle matters!





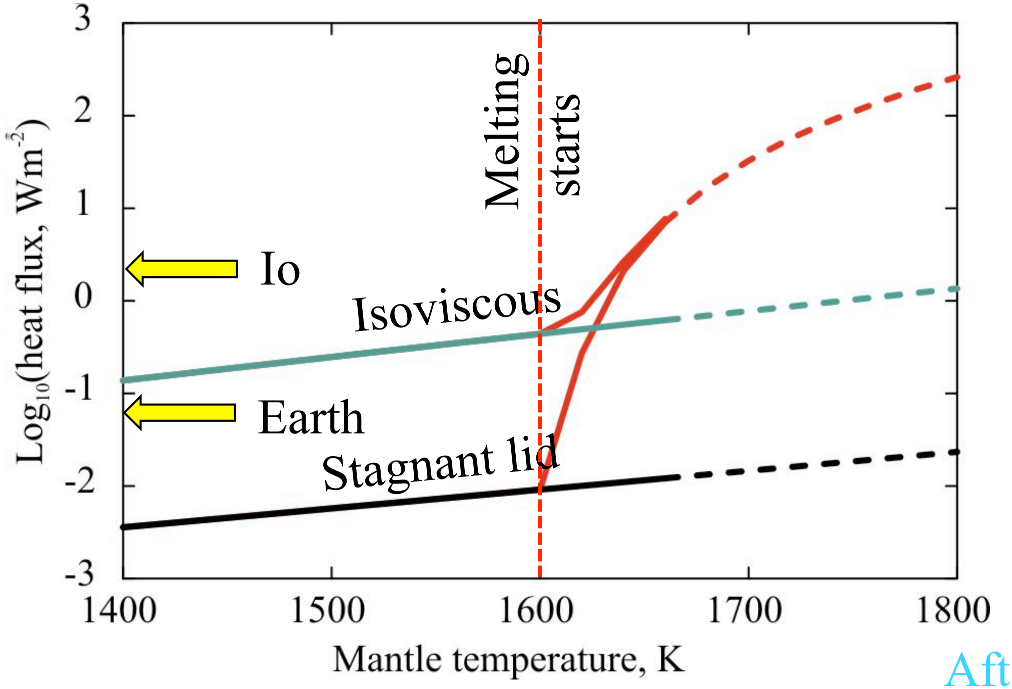


θ<60º
Rapid flow

θ>60º
No flow

Melting

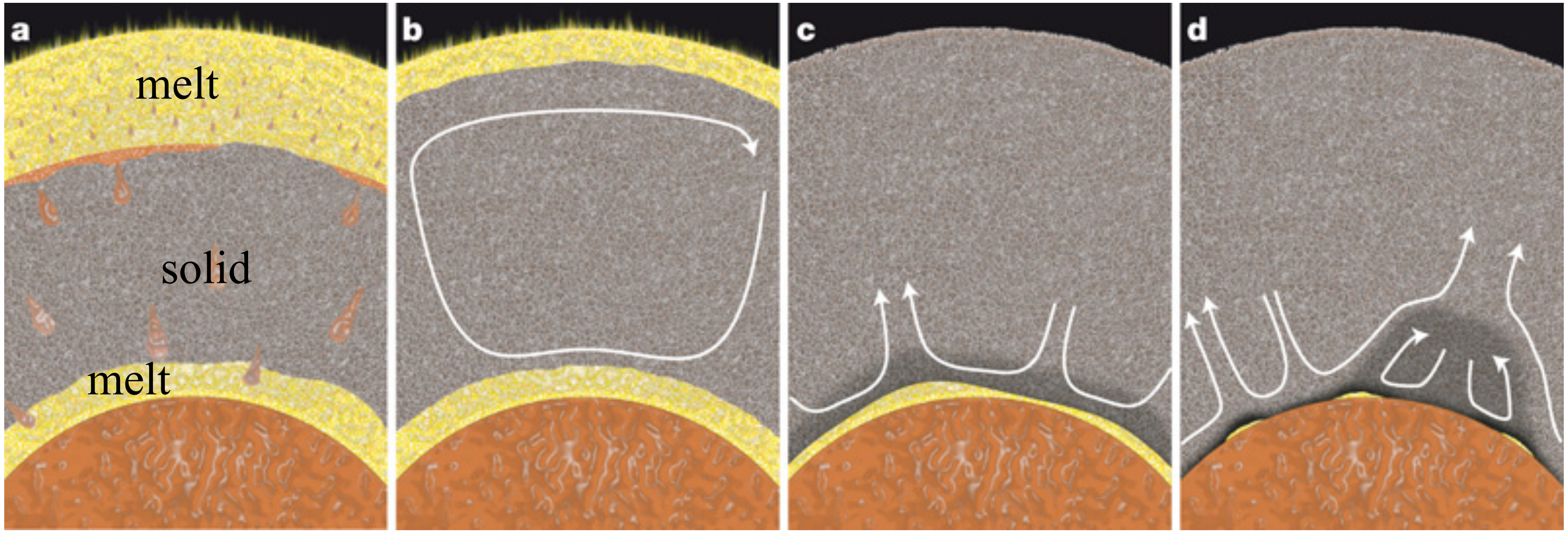
$$F_{melt} \sim \underbrace{cd^2\phi^n}_{\text{permeability}} \underbrace{\frac{g\Delta\rho}{\eta_{melt}}}_{\text{driving/resistive stresses}} \underbrace{\rho L_H\phi}_{\text{advected heat}}$$



After Moore, *Icarus*, 2001

Melting/Density

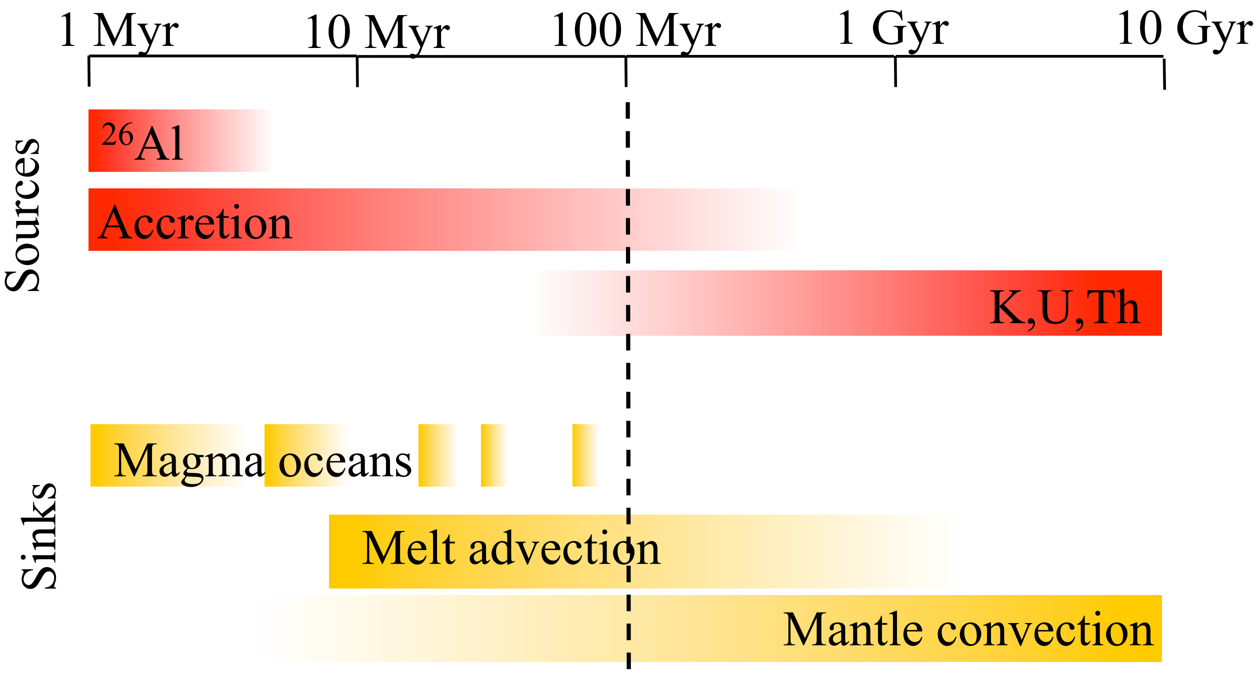
- Deep mantle melting behaviour controls whether magma ocean solidifies from top or bottom – important!
- Melt-solid density contrast controls whether magma can move upwards or not – affects e.g. CMB heat flux
- E.g. "Deep magma ocean" on Earth



Labrosse et al. (2007)

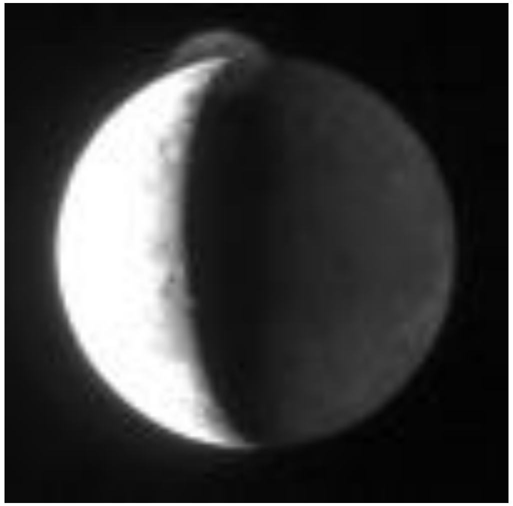
Part III (Heat Transfer) - Summary

- Molten or partially-molten mantles cool rapidly
- Solid-state mantles cool slowly
- Mantles spend a long time close to the melting point

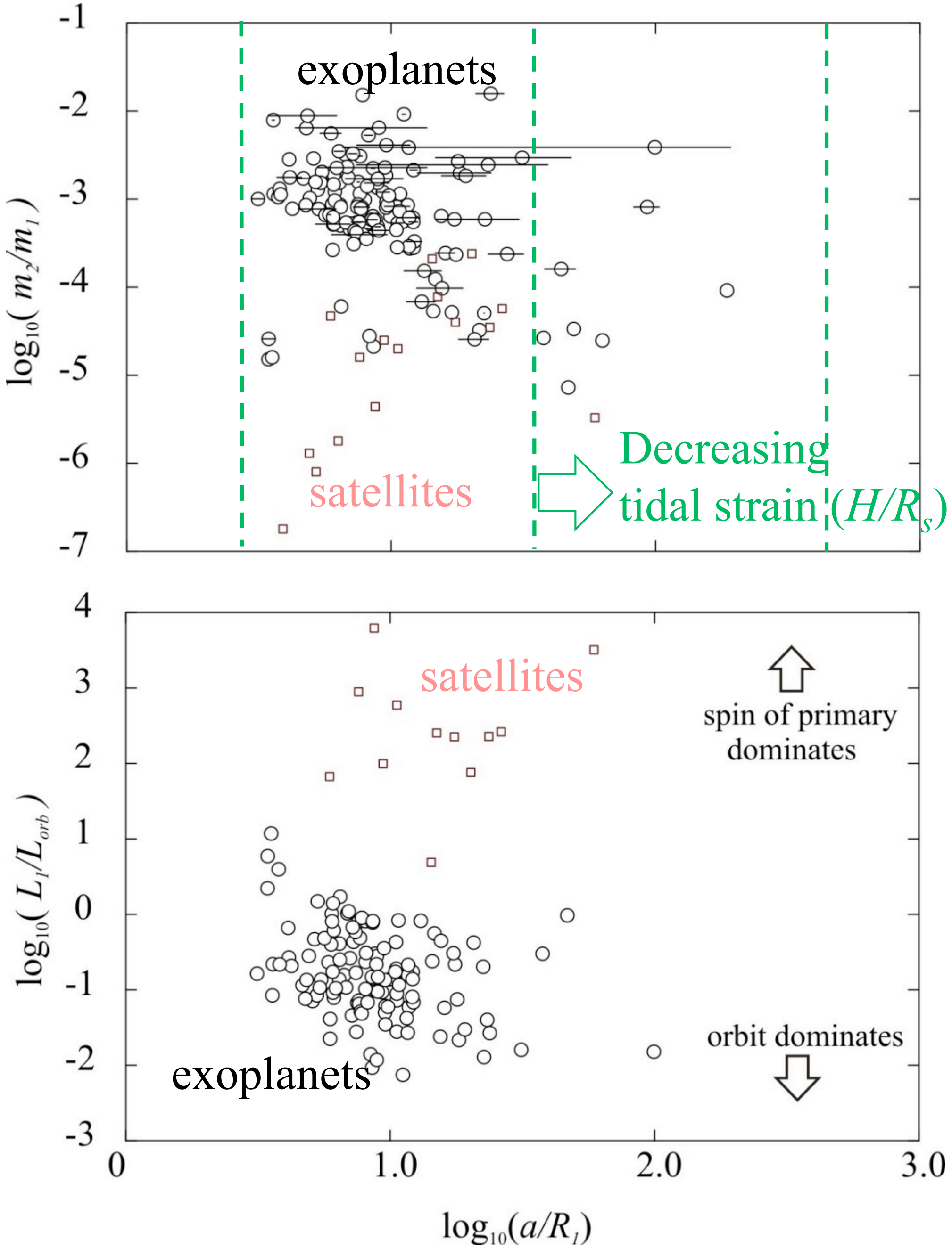


Part IV – Tides

- Planetary tides are important for two reasons:
  - We can use observations of tidal effects to constrain the internal structures of planetary bodies
  - Tides play an important role in the orbital (and thermal) evolution of some bodies



NASA


exoplanets
satellites
Decreasing
tidal strain (H/R_s)
$\log_{10}(m_2/m_1)$
-1
-2
-3
-4
-5
-6
-7
satellites
spin of primary
dominates
orbit dominates
exoplanets
$\log_{10}(L_1/L_{orb})$
4
3
2
1
0
-1
-2
-3
0
1.0
2.0
3.0
$\log_{10}(a/R_1)$

Topics

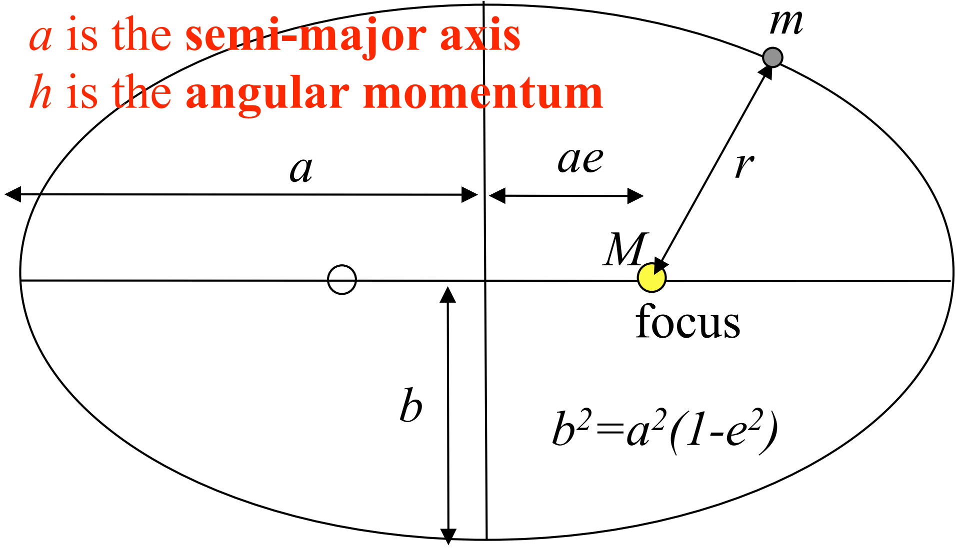
- 0. Introduction
- 1. k_2 and Q
- 2. Despinning
- 3. Tidal heating
- 4. Inclination and obliquity

Resources: Murray & Dermott, *Solar System Dynamics*, CUP, 1999

Basics

$$n^2 a^3 = GM$$

$$E = -\frac{GMm}{2a}$$



e is the **eccentricity**,
a is the **semi-major axis**
h is the **angular momentum**

$$\boxed{h = na^2\sqrt{1-e^2}}$$

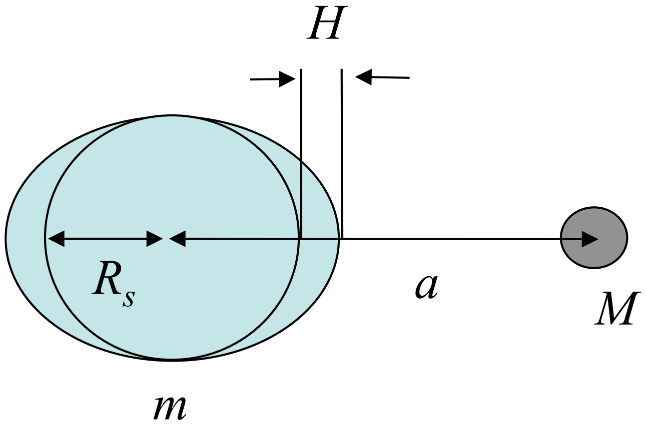
Angular momentum per unit mass.
Compare with na^2 for a circular orbit

An elliptical orbit has a *smaller* angular momentum than a circular orbit with the same value of *a*

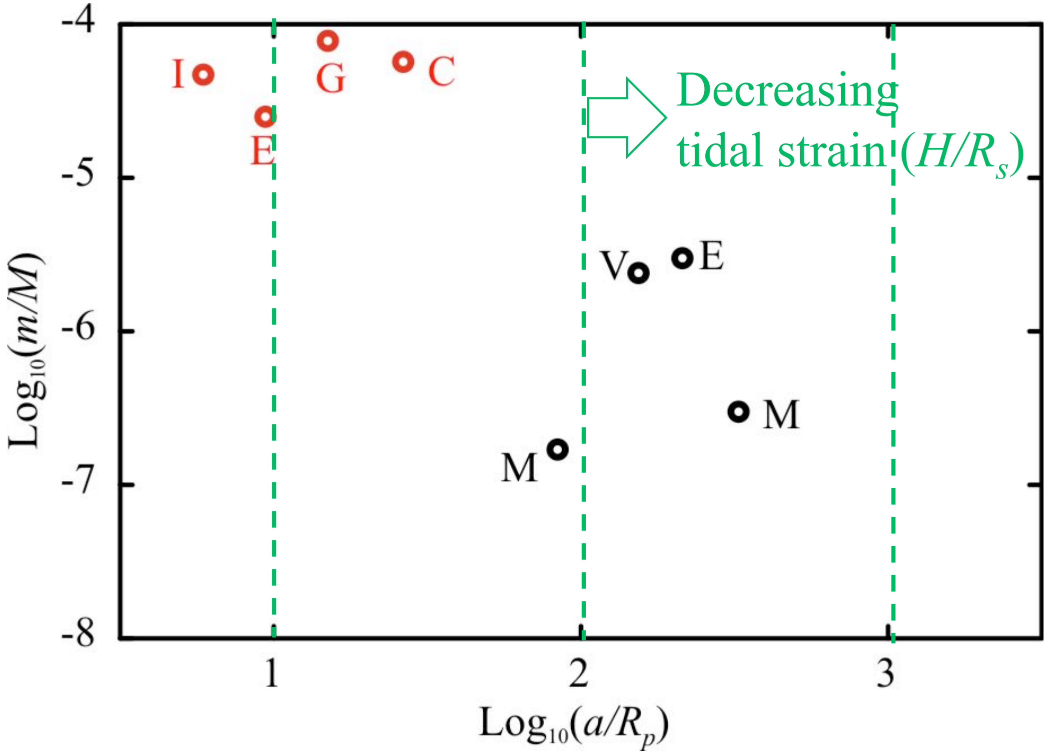
Orbital angular momentum is *conserved* unless an external torque is acting upon the body

Tides

$$H = h_2 R_s \left(\frac{M}{m}\right)\left(\frac{R_s}{a}\right)^3$$



H strongly influences tidal *torques* and tidal *dissipation*



Rigidity

- *Reduces* the tidal amplitude
- Gravity competes with rigidity μ:

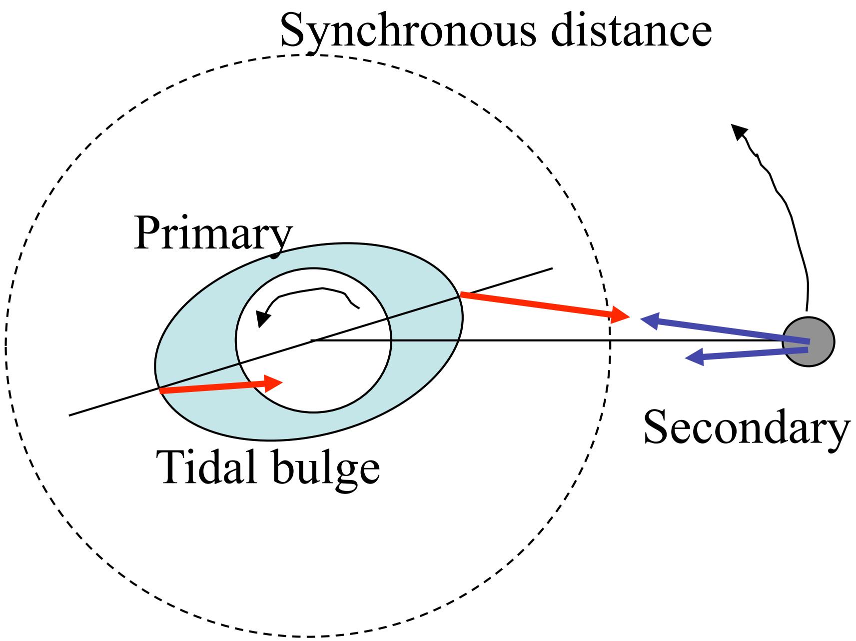
$$\tilde{\mu} = \frac{\mu}{\rho g R_s} \sim 0.7 \left(\frac{\mu}{100\,\mathrm{GPa}} \right) \left(\frac{M_E}{M} \right)^{2/3}$$

- E.g. Love number h_2 for a uniform body:

$$h_2 = \frac{5}{2} \frac{1}{(1 + \frac{19}{2}\tilde{\mu})}$$

- Rigidity dominant for small bodies, moderate for Earth-mass bodies, small(?) for larger bodies

Tidal torques on the primary



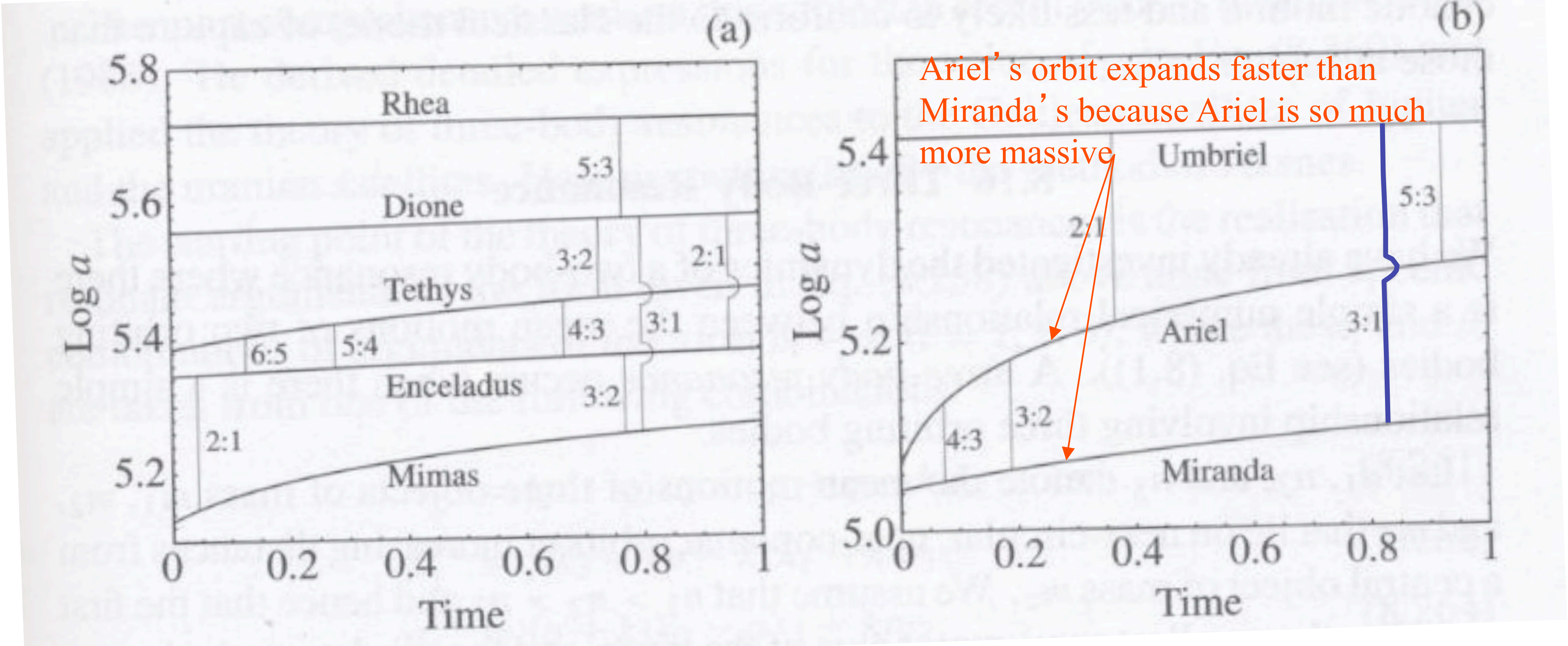
Torque spins down primary and moves secondary outwards

(Reversed if within synchronous distance – exoplanets!)

The Moon has moved outwards from ~5 R_E to 60 R_E over 4.5 billion years.

The current measured recession rate (4 cm/yr) tells us how large the torques are, and thus how dissipative the Earth is, at present.

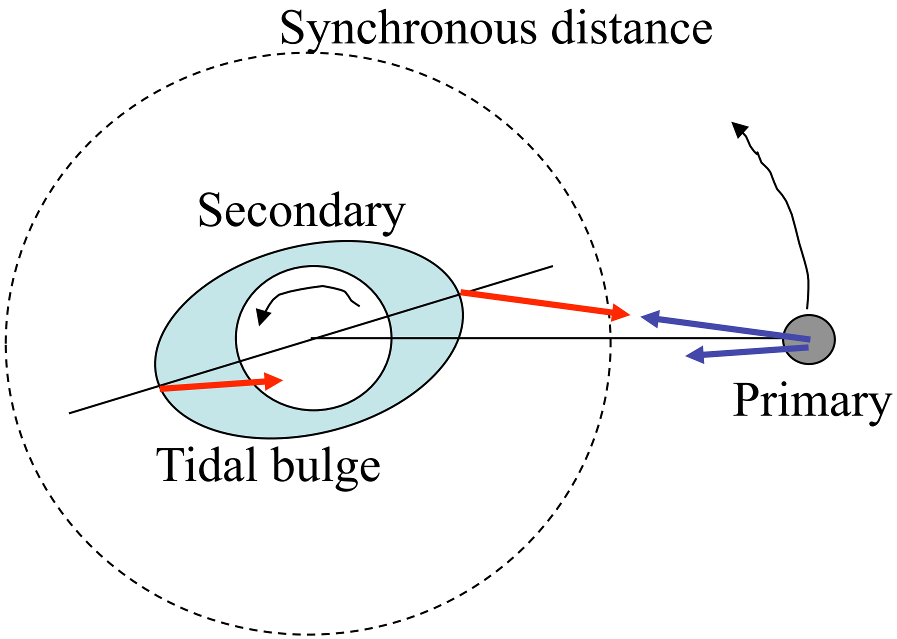
Orbital evolution



Ariel's orbit expands faster than Miranda's because Ariel is so much more massive

- (Murray and Dermott 1999)
- Passage through resonance may have led to transient eccentricities and heating
- Note that diverging paths do not allow capture into resonance (though they allow passage through it), while converging paths do.

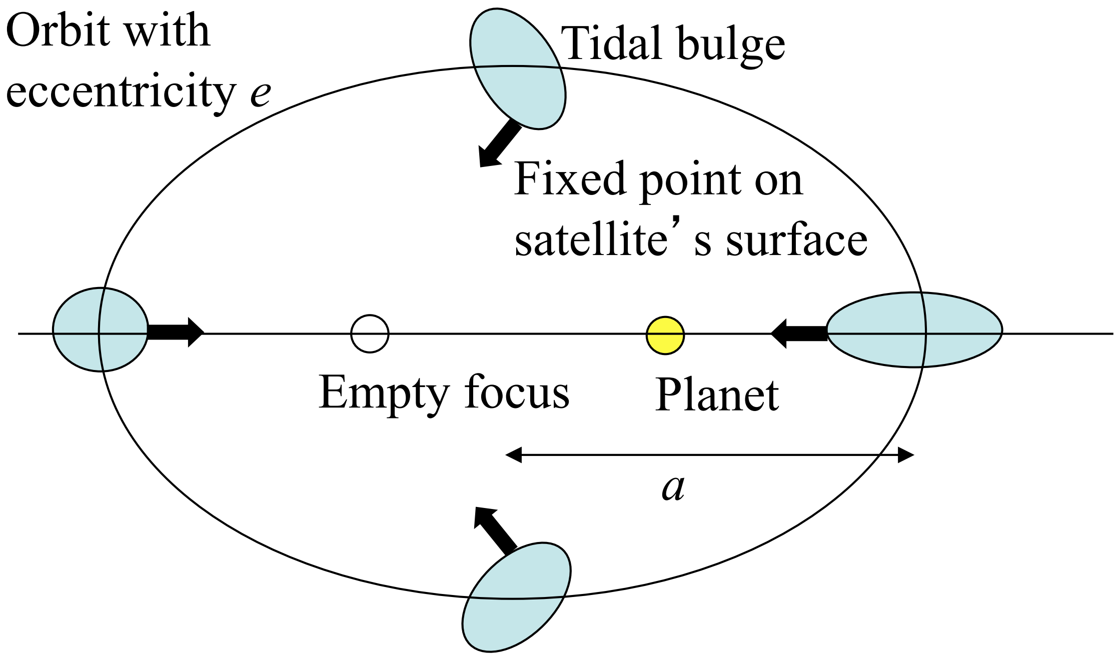
Tidal torques on the secondary



Tide raised by primary on secondary is large, so torque is large

- Synchronization is *rapid* for close-in objects (see later)
- Rotation period may not *exactly* equal orbit period (see later)
- Even synchronous objects generally experience tides . . .

Diurnal Tides



- From a fixed point *on the satellite*, the resulting tidal pattern can be represented as a static tide (permanent) plus a much smaller component that oscillates (the diurnal tide)

N.B. it's often helpful to think about tides from the satellite's viewpoint

$$H_d = 3eH$$

1. k_2 and Q

- Torques and dissipation both depend on k_2/Q

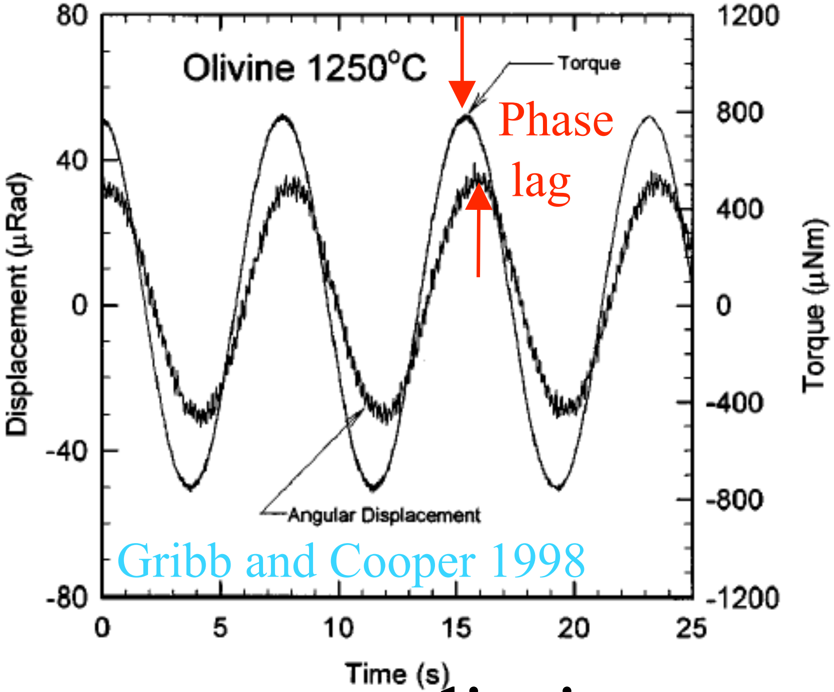
$$\frac{k_2}{Q}$$

k_2 → Tidal bulge *amplitude*.

Depends on rigidity and density.

Q → Tidal bulge *phase*.

Depends on viscosity & rigidity.



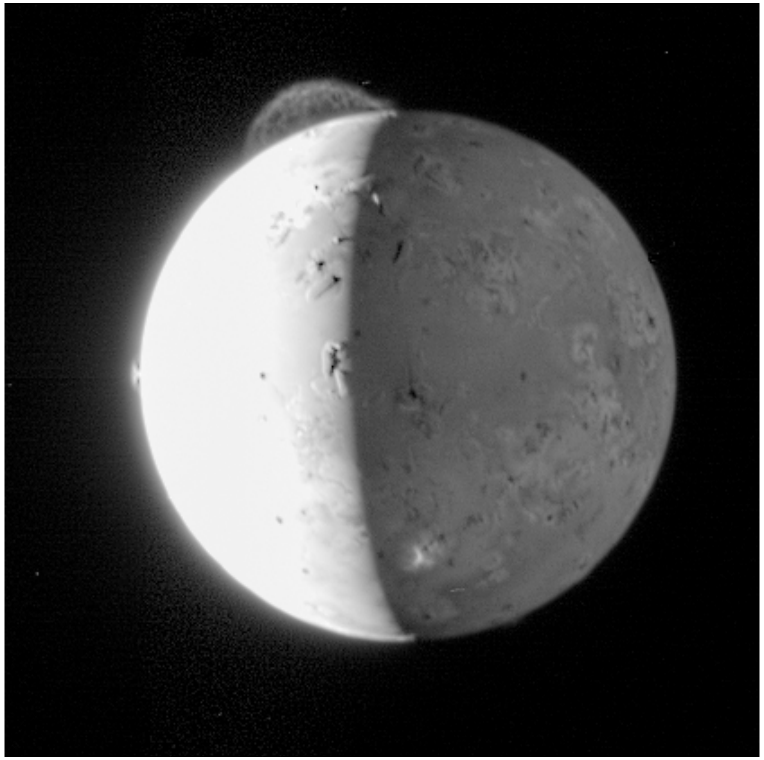
Phase lag

Gribb and Cooper 1998

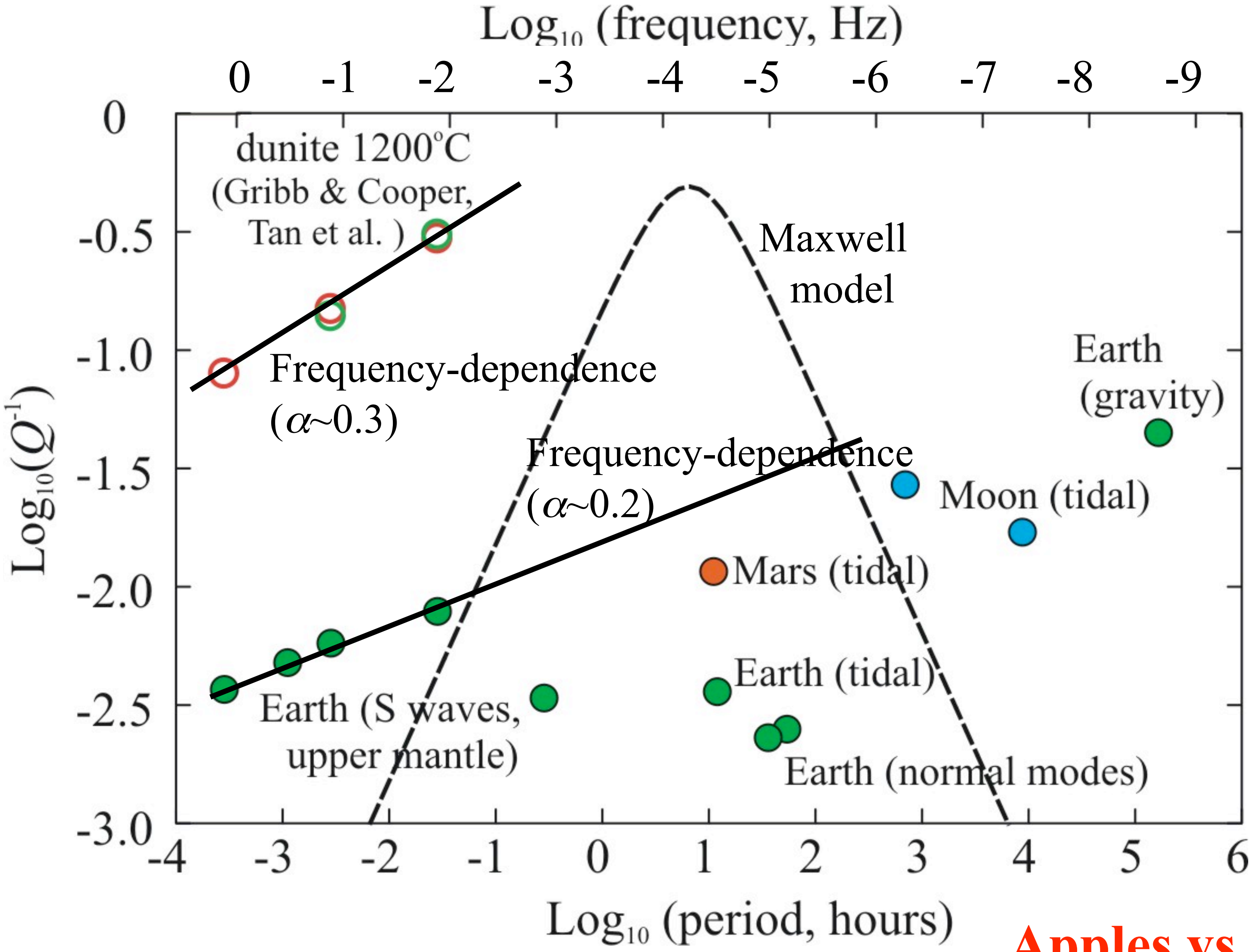
- Q is ~ number of cycles for energy to dissipate
- Large Q means small phase lag/torque (!)
- Q depends *strongly* on mechanical properties

Observational constraints

- Earth as a whole has a Q of 12 (oceans)
- The solid Earth is *not* very dissipative (Q~300)
- Mars is dissipative (Q~80)
- So is the Moon (Q~30 at tidal periods)
- Io and Enceladus are generating observable heat, so we can infer k_2/Q directly
- Gas giants (Saturn, Jupiter) have astrometrically-determined Q~10^4-10^5 (Lainey et al. 2009)
- Q is frequency-dependent!


NASA

Observations of *Q*



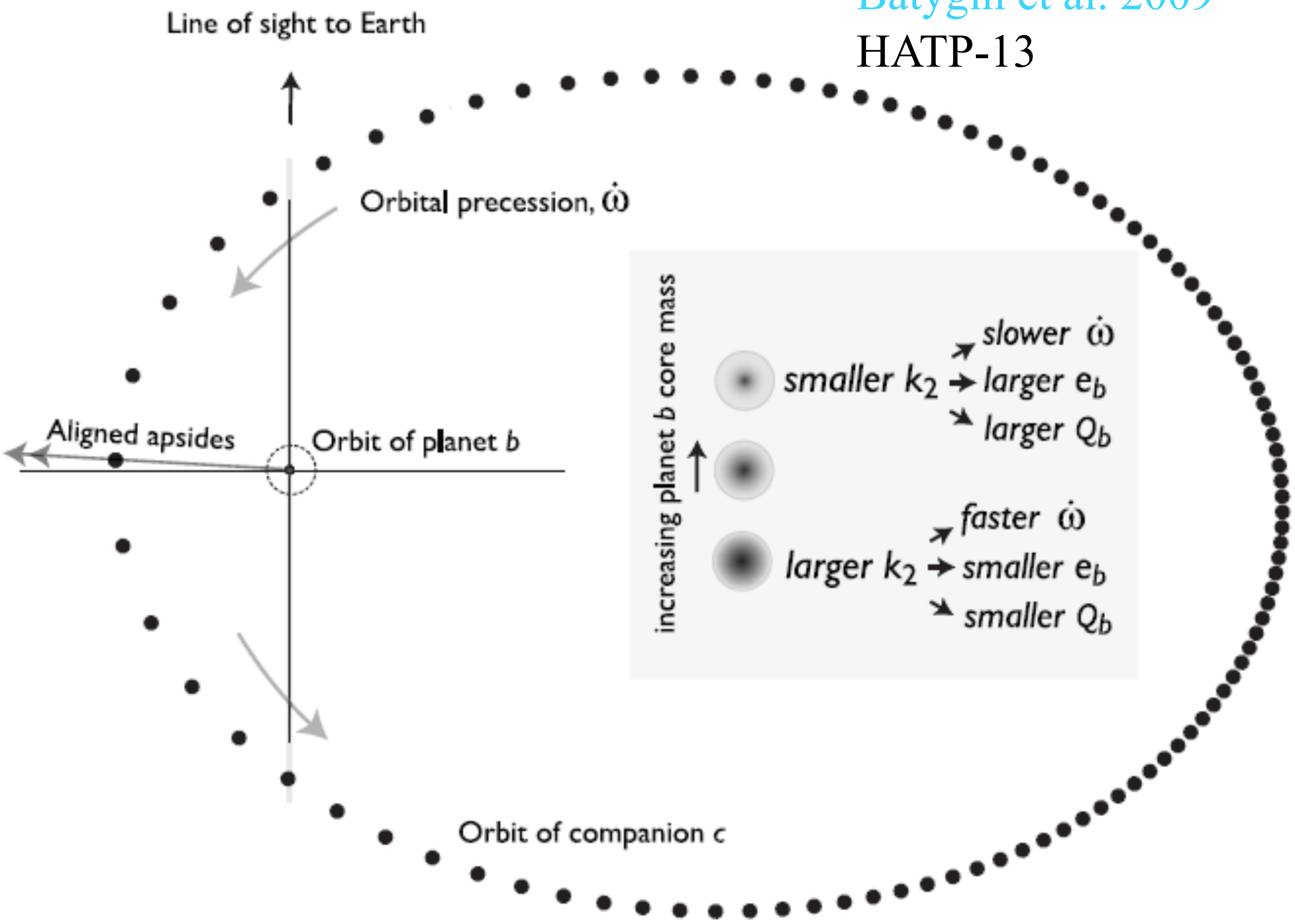
Increasing
dissipation

Apples vs. oranges?

An observational constraint!

Batygin et al. 2009

HATP-13



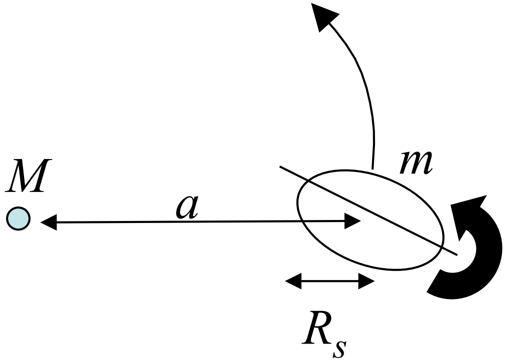
- Both k_2 (and Q) have been inferred

Tidal torques

Size of (static) tidal bulge: $H = h_2 R_s \left(\frac{M}{m}\right)\left(\frac{R_s}{a}\right)^3$

Torque on *non-synchronous* satellite by primary: $\langle T \rangle_{ns} \approx \frac{k_{2s}}{Q_s}\left(\frac{R_s}{a}\right)^6 \frac{GM^2}{R_s}$

Torque on *synchronous* satellite by primary: $\langle T \rangle_{synch} \approx \langle T \rangle_{ns} 3e$

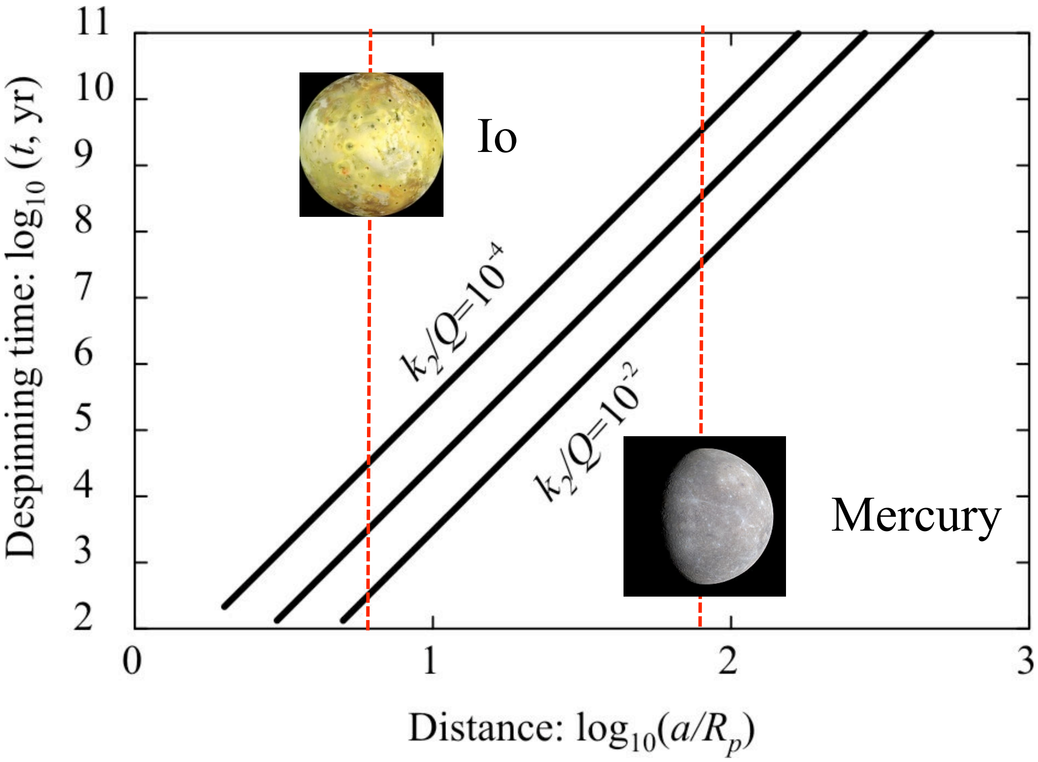


Torque *on primary by satellite* can be calculated using symmetry

2. Despinning to synchronous

- *Fast* for close-in non-synchronous objects
- Subsequent evolution (synchronous) is slower

$$\tau \sim \frac{Q}{k_2} \frac{\rho_s}{G^{1/2} \rho_p^{3/2}} \left(\frac{a}{R_p} \right)^{9/2} \sim 30\,\mathrm{kyr} \left(\frac{a/R_p}{10} \right)^{9/2} \left(\frac{10^{-3}}{k_2/Q} \right)$$

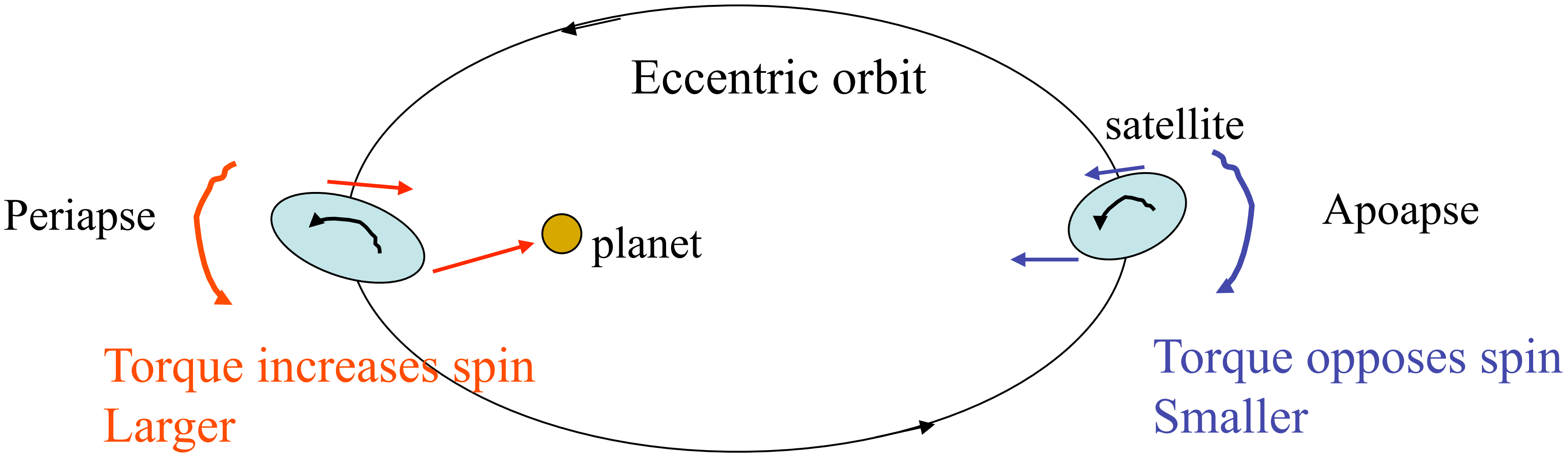


Non-synchronous rotation

- Torque on a synchronous satellite is given by:

$$\langle T \rangle_{synch} \approx \langle T \rangle_{ns} 3e$$

- This torque should increase the satellite's rotation rate slightly above synchronous (Greenberg & Weidenschilling 1984) . . .
- . . . As long as there are no permanent mass asymmetries
- Potentially *very* important for eccentric close-in exoplanets

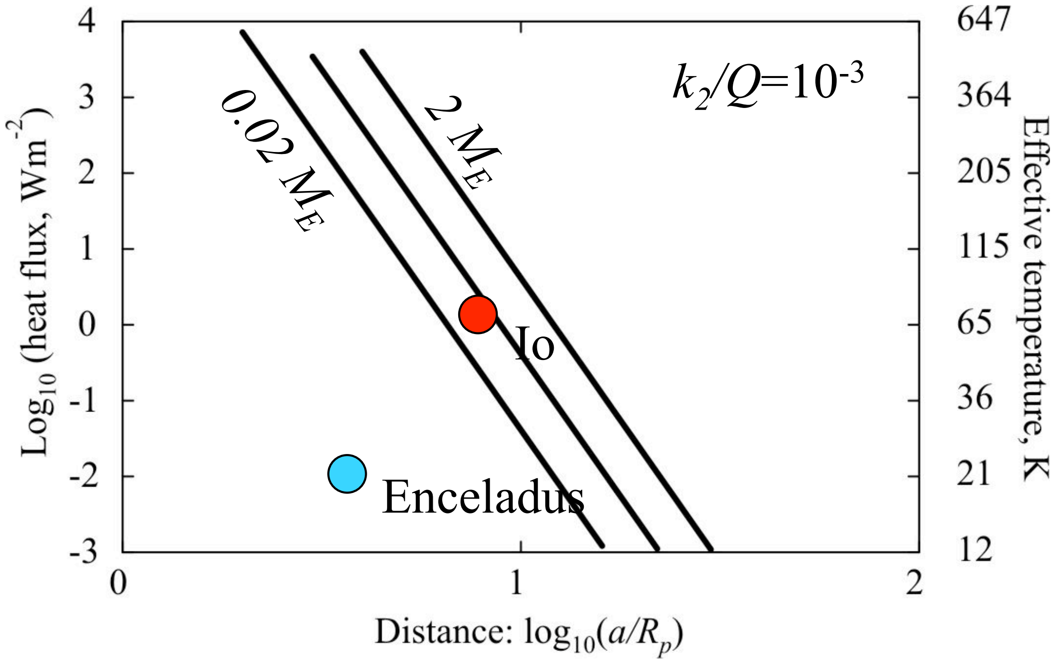


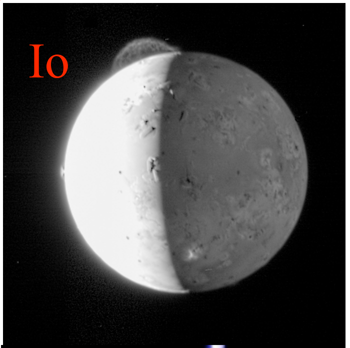
3. Tidal Heating

- Diurnal tides – deformation -> heating
- Heat output allows k_2/Q determination (~0.01)

Synchronous rotator.
Valid for small *e*.

$$\dot{E} \sim \boxed{\frac{k_2}{Q}} \frac{G^{3/2}\rho_p^{5/2}}{\rho_s^{5/3}} \boxed{\left(\frac{a}{R_p}\right)^{-15/2}} m^{5/3} \boxed{e^2}$$

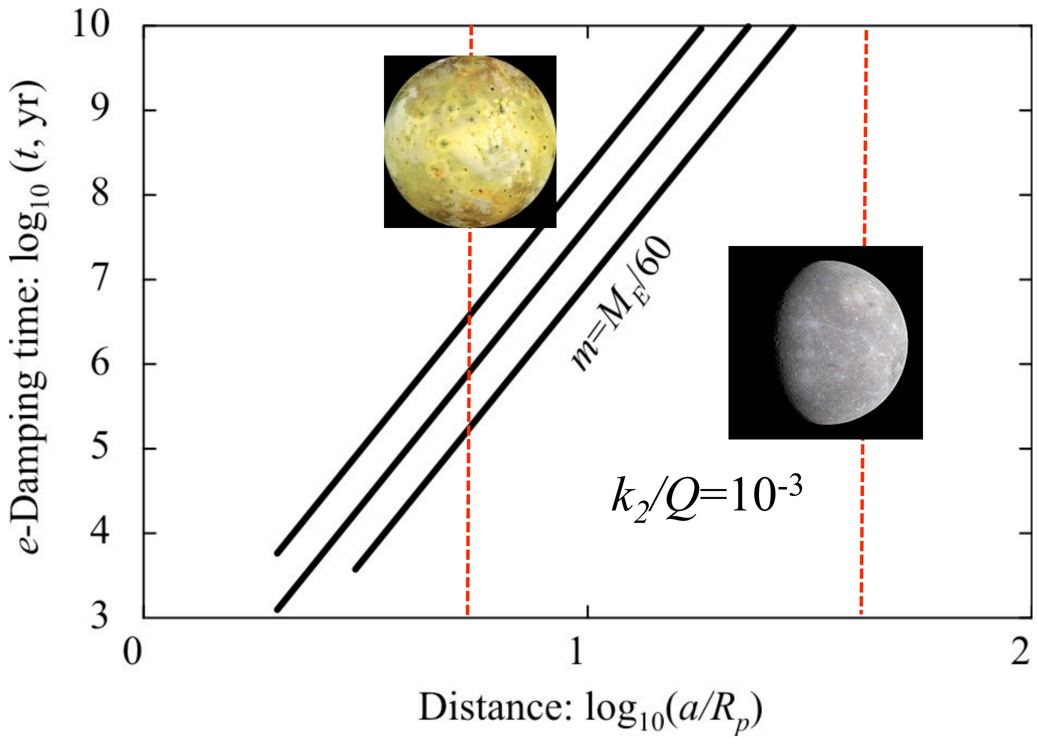




Eccentricity Damping

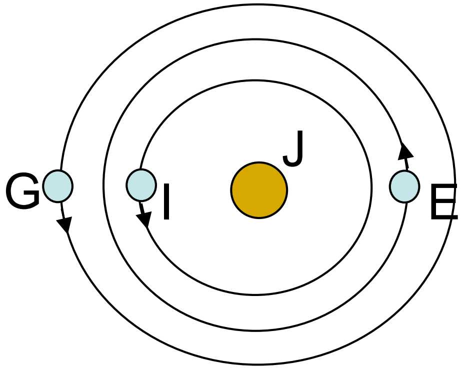
- Energy from orbit, *e* should damp to zero
- *e*-damping time *long* compared to despin time

$$\tau \sim \frac{Q}{k_2}\frac{\rho_s^{10/6}}{G^{1/2}\rho_p^{13/6}}\left(\frac{a}{R_p}\right)^{13/2}\left(\frac{M}{m}\right)^{2/3} \sim 30\,\mathrm{Myr}\left(\frac{a/R_p}{10}\right)^{13/2}\left(\frac{10^{-3}}{k_2/Q}\right)\left(\frac{M/m}{10^3}\right)^{2/3}$$



Resonances

- Eccentricities will damp, unless they are being excited
- *Mean-motion resonances* can excite eccentricities:



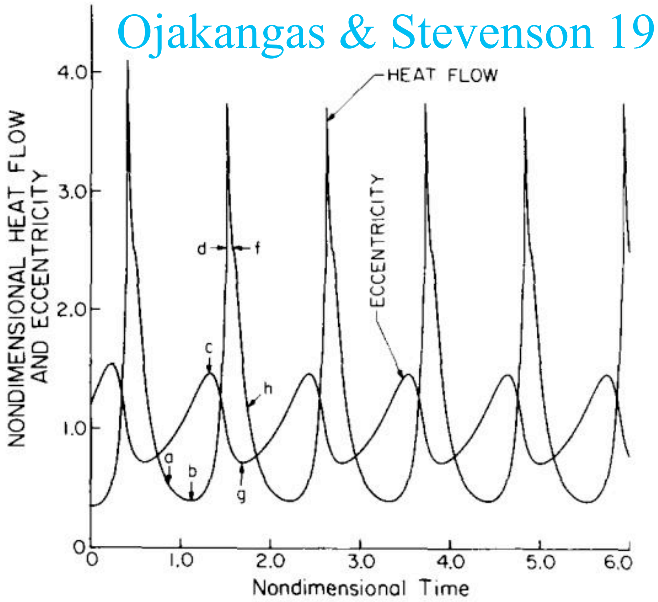
One of the conjunctions occurring due to the Laplace resonance. Note that there is never a triple conjunction.

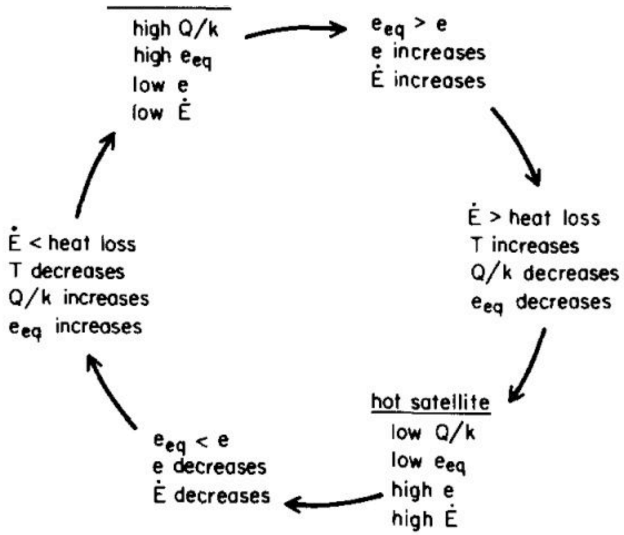
- These ultimately involve transfer of (rotational) angular momentum from the primary to the secondaries
- In steady-state ($de/dt=0$), the dissipation rate in the secondaries depends only on k_2/Q of the *primary*

$$\dot{E}_{steady} \sim n \frac{k_{2p}}{Q_p} \left(\frac{R_p}{a} \right)^6 \frac{Gm^2}{R_p} \sim n \langle T \rangle_{ns}$$

A possible observational constraint . . .

Feedbacks and coupling


Ojakangas & Stevenson 1986


high Q/k
high e_{eq}
low e
low $\dot{E}$

$e_{eq} > e$
e increases
$\dot{E}$ increases

$\dot{E} >$ heat loss
T increases
Q/k decreases
e_{eq} decreases

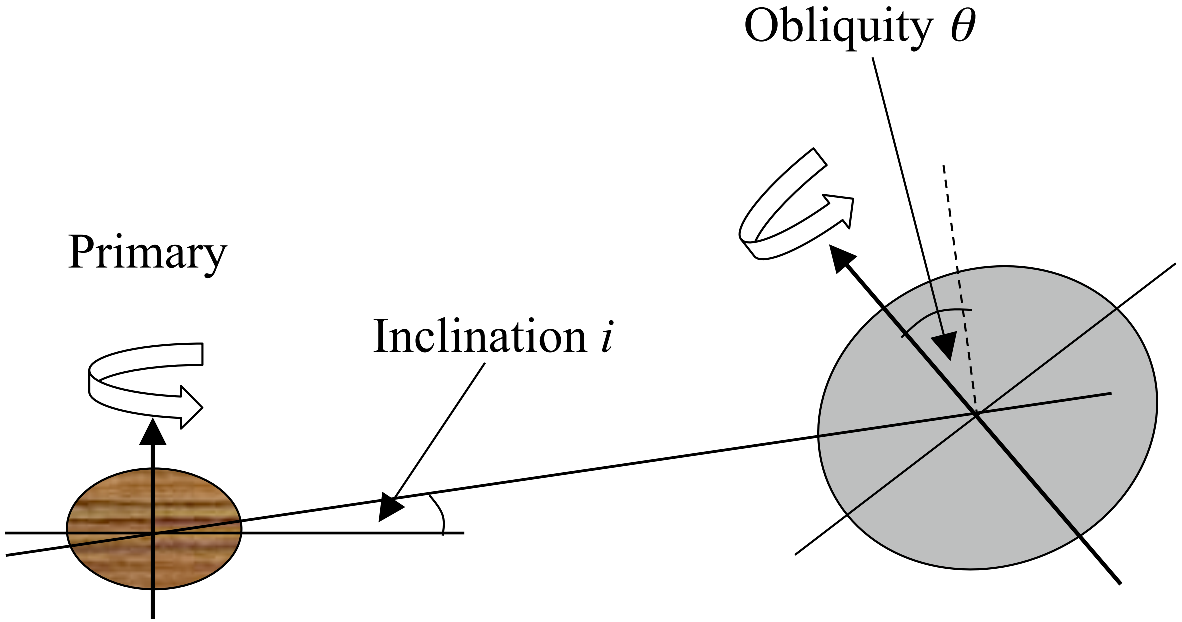
hot satellite
low Q/k
low e_{eq}
high e
high $\dot{E}$

$e_{eq} < e$
e decreases
$\dot{E}$ decreases

$\dot{E} <$ heat loss
T decreases
Q/k increases
e_{eq} increases

- Dissipation in primary increases eccentricity
- Dissipation in satellite decreases eccentricity
- Heat transfer, dissipation and *e*-damping are coupled, because *Q* is strongly temperature-dependent
- *Complex* (periodic?) behaviour can result

4. Inclination and obliquity



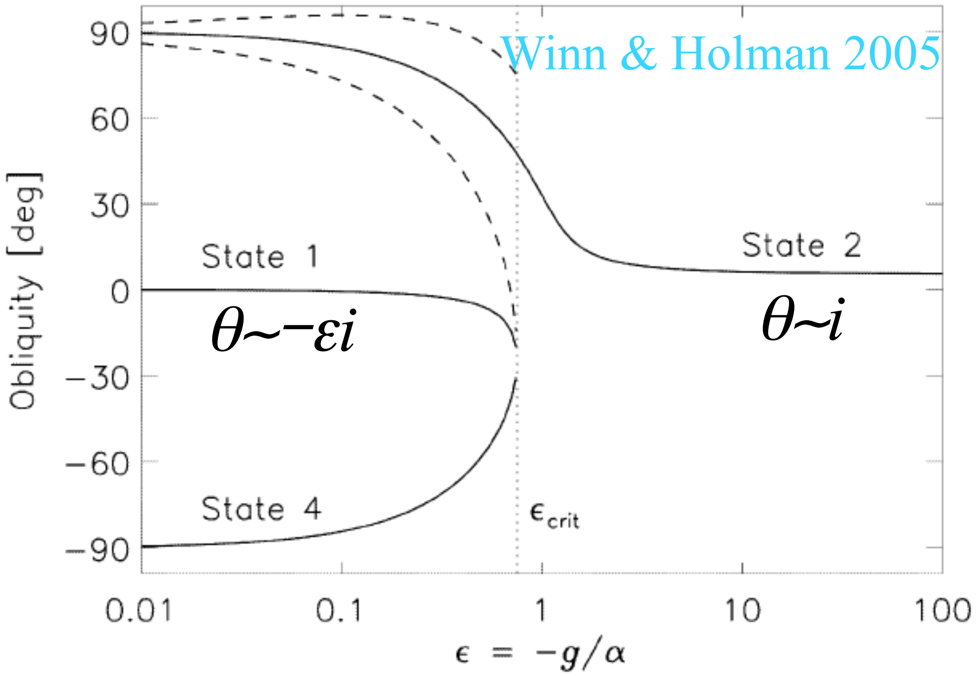
- Inclination damping is slow $\frac{1}{i}\frac{di}{dt} = \frac{1}{4a}\frac{da}{dt}$ $\frac{da}{dt} = \frac{2a^2}{GMm}\dot{E}$
- $\tau_{\mathrm{despin}} << \tau_{\mathrm{ecc}} << \tau_{\mathrm{inc}}$
- Many satellites occupy a Cassini state, in which the obliquity is controlled by the inclination

Cassini state

$$\varepsilon = \frac{\text{orbit precession rate}}{\text{spin precession rate}} \approx \frac{R_p^2 a}{R^3}\,\frac{m}{M}\,\boxed{\frac{cJ_{2p}}{k_2}}$$

θ and i are not independent, related via ε

Things we would like to know



Obliquity tides & heating

- Bulge moves "up-and-down", rather than "side-to-side"
- Otherwise heating effect same as eccentricity tides:

$$\dot{E} \sim \frac{k_2}{Q} \frac{G^{3/2} \rho_p^{5/2}}{\rho_s^{5/3}} \left(\frac{a}{R_p} \right)^{-15/2} m^{5/3} \theta^2$$

- Crucial distinction: obliquity damps *much more slowly* than eccentricity (because controlled by inclination)
- So obliquity tides can be a good long-term source of heat for bodies in Cassini states (within limits – see Fabrycky et al. 2007)

Summary

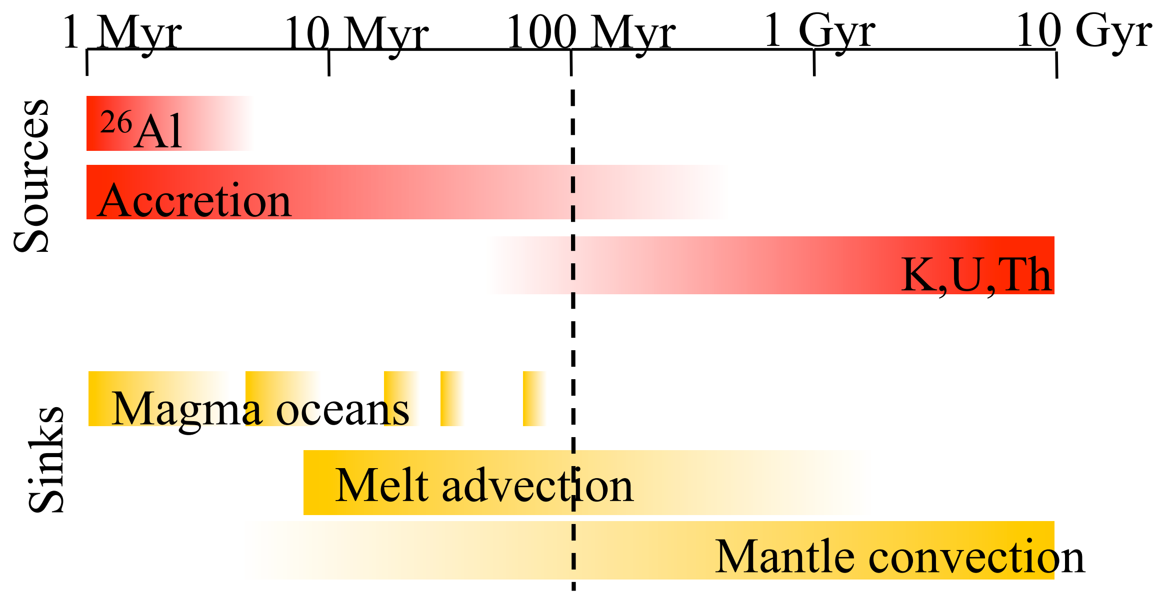
- Tides depend strongly on a/R_p – important for our satellites and many exoplanets
- Tidal processes happen at different rates:

$$\tau_{\mathrm{despin}} \ll \tau_{\mathrm{ecc}} \ll \tau_{\mathrm{inc}}$$

- Tidal heating important in our solar system and likely elsewhere (resonances, inclinations)
- Orbital observations can constrain k_2 etc.
- Coupling between thermal and orbital evolution – complicated problem . . .
- . . . But may allow us to use orbital observations to constrain interior state, or vice versa

What have we learnt?

- Late-stage impacts: generate initial diversity; dominate the thermal budget for ~100 Myr
- Tides and radioactivity: longer-term energy sources
- Planets start hot; stay “slightly molten” for Gyrs
- Orbit-interior coupling: challenge and opportunity



Lessons

- Observations of exoplanets will be limited, but:
  - Young solid planets are good targets (luminous)
  - Bulk densities may be diagnostic of impact history
  - Tidal/interior coupling (e.g. HATP-13)
  - Likewise atmosphere/interior coupling
- Solid bodies are complex systems which defy simple predictions (Earth vs. Venus, Mimas vs. Enceladus)
- Chemistry helps! (in this Solar System)
- Our Solar System is likely *not* typical - biases

Eccentricity damping

- Damping releases a lot of energy:

$$\frac{\Delta E_{ecc}}{E_{grav}} \sim 0.3\left(\frac{M/m}{10^4}\right)^{2/3}\left(\frac{10}{a/R_p}\right)\left(\frac{\Delta e}{0.1}\right)^2$$

Planetary Growth & Accretion

- Early growth – from dust/gas to ~ 1 km (e.g. Weidenschilling 1997)
  - Occurs over ~10^4 yrs at 1 AU
- Runaway growth (e.g.Wetherill & Stewart 1989)
  - $dM/dt \sim M^{4/3}$
  - Terminates when $v \sim v_{esc}$, ~10^5 yrs at 1AU
- Oligarchic growth (e.g. Kokubo & Ida 1998)
  - $dM/dt \sim M^{2/3}$
  - Terminates at ~0.1 M_E , ~10^6 yrs at 1 AU
- Late-stage accretion (e.g. Agnor et al. 1999)
  - Stochastic, large impacts, 10^7-10^8 yrs

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