Theoretical Modelling of Stars
: Determination of Physical Parameters of μ Cas

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ABSTRACT.
We have determined physical dimensions of μ Cas astrometric binary system through a series of stellar modelling such as abundance analysis, evolutionary computation, asteroseismology and 3-D Radiation-Hydrodynamics (RHD). In spite of the well-defined parallax and astrometric orbit from *HIPPARCOS*, there has been a chronic mass ratio problem between components. Recently, the optical interferometric observation of the *CHARA* array has detected the radius of the primary star. Moreover, from the high resolution spectroscopic observation, we find that μ Cas have $\alpha-$element enhanced chemical composition with respect to the scaled solar abundance. Combining global properties from spectro-photometric observations, physical parameters for μ Cas have been calibrated within the frame work of standard stellar theory. Then, a reliable set of physical parameters has been defined through a statistical minimization between theoretical model grids. In addition, the mode oscillation frequency of the best model has been calculated in the context of the stellar evolution and structure theory. With a well-constrained initial configuration from 1-D analysis, the 3-D Radiation-hydrodynamical (RHD) numerical simulation for turbulent convection in atmospheres has been computed. The aim of this study is to constrain the physical dimensions and is to describe physical processes through a complete modelling of stars.

μ Cassiopeiae

Binary μ Cas is a nearby (~7.5 pc), old Population low-mass halo binary star. Astrometry revealed orbital elements (P ~ 22yr) for this binary. But the difficulties in detecting faint secondary make it extremely hard to determine the position angle, separation, magnitude difference, and mass ratio accurately. From these reasons, physical dimensions of this system have been poorly defined, and evolutionary status is also uncertain. In addition, it is believed that μ Cas has the primordial helium abundance.

Combining recent observations, we determine physical parameters of μ Cas within the framework of standard stellar theory.

Orbital Parameters

Parameters	α (")	P (yr)	T	ϵ	i (°)	Ω (°)	ω (°)	π (")
Values	0.19	22.1	1931.6	0.58	108.2	47.4	324.8	0.137

Russell & Gatewood (1984)

Reference Coding

Properties	Values	Reference
Spectral type	G5	1
V (mag)	5.17	1, 2
T_{eff} (K)	5297 ± 32	1, 2, 3
$L/L_{\odot}$	0.442 ± 0.014	1, 2, 3
Log g (cgs)	4.52 ± 0.04	1, 3
$R/R_{\odot}$	0.791 ± 0.008	2
[Fe/H]	-0.7 dex	2, 3
[α/Fe]	+0.4 dex	3

[1] Lebreton et al. (1999), [2] Boyajian et al. (2008), [3] This work

Abundance Analysis

Spectroscopy In order to determine the chemical composition of μ Cas, high-resolution echelle spectra have been obtained with the fiber sets of 300μm aperture (FWHM ~ 7.0 px) of Bohyun Optical Echelle Spectrograph (BOES). Model atmospheres were calculated using ATLAS 9 (Kurucz) with a new opacity distribution function (ODF).

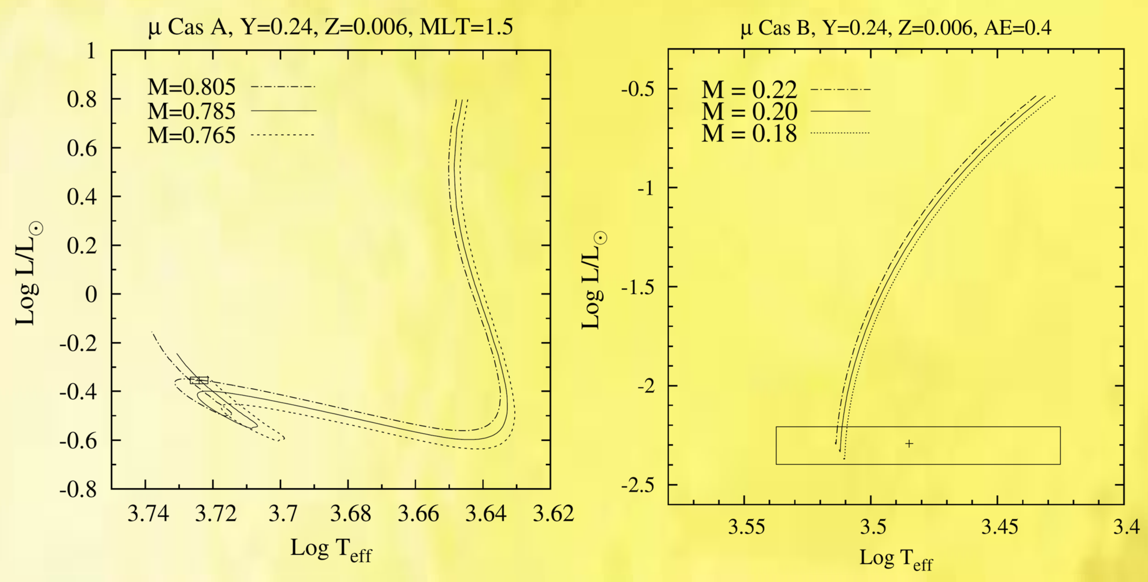
Element	[X/Fe]	Log ε	N_{line}
C I	0.45±0.00	8.11	1
O I	1.22±0.62	6.55	3
Na I	0.04±0.01	5.47	2
Mg I	0.48±0.00	7.16	2
Al I	0.30±0.01	5.87	3
Si I	0.29±0.06	6.94	13
Ca I	0.02±0.05	5.48	11
Sc II	0.38±0.05	2.58	3
Ti I	0.33±0.05	4.42	21
Ti II	0.33±0.03	4.42	5
V I	0.13±0.06	3.23	13
Cr I	0.00±0.03	4.77	3
Mn I	-0.26±0.00	4.23	1
Co I	0.17±0.03	4.19	5
Ni I	0.06±0.05	5.41	28
Zn I	0.28±0.00	3.98	1
Ba II	0.27±0.00	1.50	1

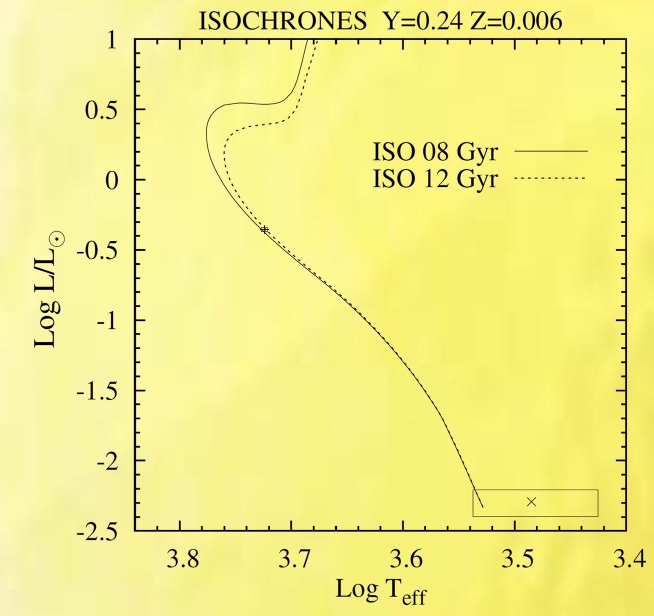
Mixture For a consistent modelling, equation of state (OPAL) and opacities (OPAL, Alexander) have been generated with the newly determined mixture. All our theoretical models such as evolutionary computation and numerical simulation have been constructed based on this chemical mix.

Evolution & Structure

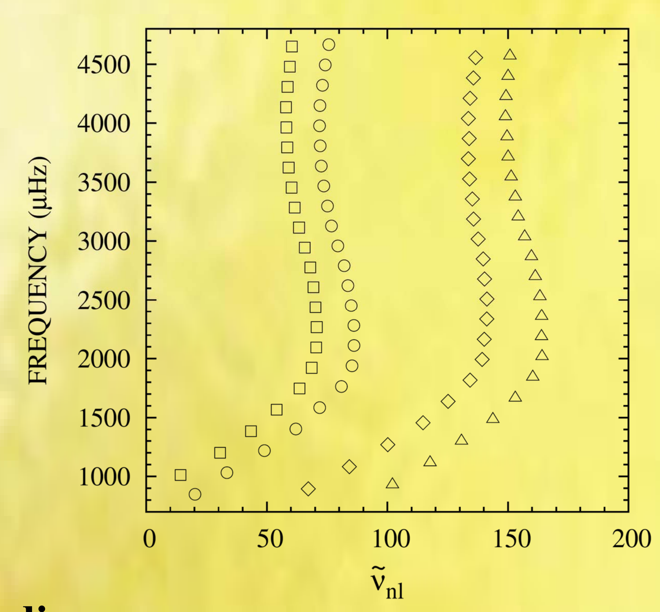
Calibration Stellar parameters have been calibrated in the context of standard stellar modelling (1-D analysis). Basic assumption for calibration of binary stars is that chemical composition and age of each component is identical. First, model grids for the primary have been generated using the Yale stellar evolution code (YREC). And then, the best solution which minimizes observational errors has been determined. With the well-defined parameter set of the primary, models for the secondary have been computed.

$$(M_1, M_2, Y_0, Z_0, \alpha_{MLT};\ t_{age}) = (0.785, 0.20, 0.24, 0.006, 1.5;\ 10Gyr)$$





Asteroseismology has been a powerful tool for investigation of stellar interiors. Due to its detectable surface amplitude of oscillation, μ Cas A may be considered favorable to seismological studies. From the evolutionary solution, the acoustic eigen frequencies of the primary have been estimated.



Echelle diagram of the frequency spectrum for μ Cas A is presented. Circles, triangles, squares, and diamonds correspond to modes l = 0, 1 , 2, and 3, respectively. The approximated large separation is $\Delta\nu$ ~ 171 μHz at the reference frequency ν_0 = 828 μHz.

Numerical Simulation

Large-Eddy Simulation as a numerical tool for turbulent flows of stellar convection has been applied to a fully compressible Newtonian fluid. In order to describe stellar turbulent convection, the full set of Navier-Stokes equations should be solved.

$$\frac{\partial \rho}{\partial t} = -\nabla\cdot\rho\mathbf{v}$$

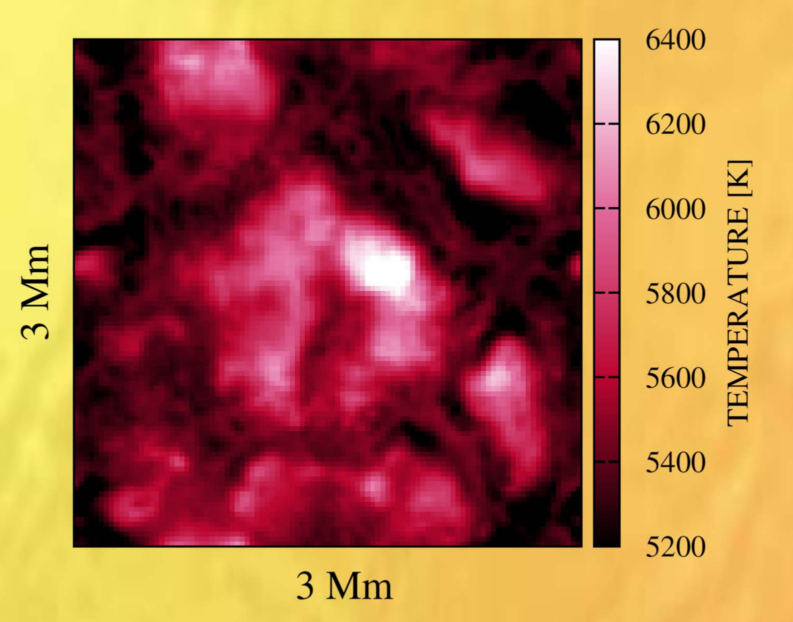
$$\frac{\partial \rho\mathbf{v}}{\partial t} = -\nabla\cdot\rho\mathbf{v}\mathbf{v} - \nabla P + \nabla\cdot\mathbf{\Sigma} + \rho\mathbf{g}$$

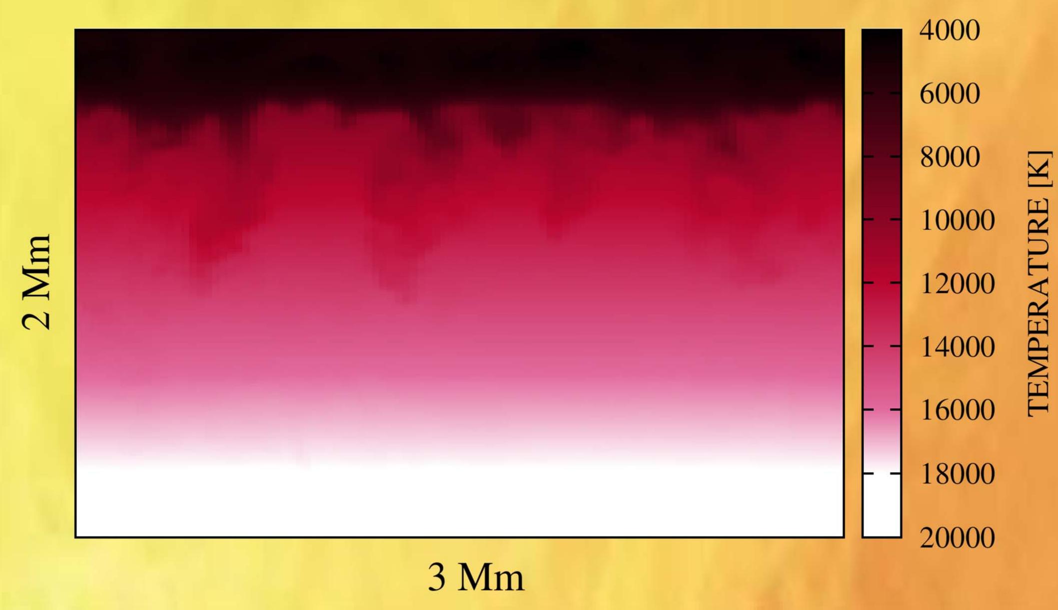
$$\frac{\partial E}{\partial t} = -\nabla\cdot[(E+P)\mathbf{v} - \mathbf{v}\cdot\mathbf{\Sigma} + f] + \rho\mathbf{v}\cdot\mathbf{g} + Q_{rad}$$

Numerical Scheme consists of two steps : (i) An alternating direction implicit (ADI) with large time steps & first order accuracy and (ii) an explicit method (ADE) with second order accuracy. When the flow reaches statistical relaxation, simulation is switched to the explicit schemes incorporating the second order predictor-corrector time integration.

Domain is set to be a plain-parallel, closed box with stress-free top & bottom and periodic sides. Computational domain extends $3^2 \times 2$ Mm covering several granules and 11~14 pressure scale heights with the resolution of $102^2 \times 200$ staggered mesh grids (Chan & Wolff 1982). 3-D Snapshots have been accumulated during 900min in real time scale, which covers sufficiently the typical convective turn-over time.

Radiative Transfer In our RHD code, radiation routines have been constructed using the generalized 3-D Eddington approximation as anisotropic diffusion in the upper region (Unno & Spiegel, 1966).





Snapshots A vertical slice **(top)** and a horizontal slice **(bottom)** of the 3-D thermodynamic structure are presented. After a relaxation of numerical fluid, the 3-D snapshots have been collected with the second order accuracy during the several convective turn-over times.

Discussion

Through a series of studies on a high resolution spectroscopic analysis, evolutionary computation, and the 3-D hydrodynamical simulation, the physical dimensions of μ Cas binary system have been determined. From our elemental analysis, it is known that μ Cas has $\alpha-$enhanced chemical composition. With the well-defined mixture, we find the mass ratio (0.8 $M_{\odot}$, 0.2 $M_{\odot}$) and the evolutionary status (Age ~ 10 Gyr) of the system. From the calibrated parameters, the p-mode frequency spectrum of the primary has been estimated. In addition, the 3-D numerical simulation for the surface convection region including radiative transfer has been investigated. The consistent stellar modelling will provide deeper insight of physical processes inside stars.

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