# Mass Loss \& Rotational Spindown of Magnetic Massive Stars 

Stan Owocki<br>University of Delaware<br>Newark, Delaware USA

Collaborators

- Asif ud-Doula
- Rich Townsend


## Earth's Magnetosphere



## Solar Corona in EUV \& X-rays

Composite EUV image from EIT/SOHO

## Solar Corona in EUV \& X-rays

Composite EUV image from EIT/SOHO
X-ray Corona from SOHO

## Corona during Solar Eclipse

## Solar Activity: Coronal Mass Ejections

2001/04/01 00:18

## Hot, Luminous, Massive Stars

- Strong, radiatively driven stellar wind

$$
-\mathrm{M}_{\mathrm{dot}} \sim 10^{-9-10-5} \mathrm{M}_{\mathrm{O}} / \mathrm{yr} ; \mathrm{V}_{\infty}>1000 \mathrm{~km} / \mathrm{s} \gg \mathrm{~V}_{\text {sound }}
$$

## Hot, Luminous, Massive Stars

- Strong, radiatively driven stellar wind
$-\mathrm{M}_{\mathrm{dot}} \sim 10^{-9-10-5} \mathrm{M}_{\mathrm{O}} / \mathrm{yr} ; \mathrm{V}_{\infty}>1000 \mathrm{~km} / \mathrm{s} \gg \mathrm{V}_{\text {sound }}$
- Some have observed dipole field $\sim 10^{3}-10^{4} \mathrm{G}$ - stable; not from convective dynamo; fossil?
- Fast rotation with $\mathrm{V}_{\text {rot }} \sim 250 \mathrm{~km} / \mathrm{s} \sim \mathrm{V}_{\text {crit }} / 2$ $-\mathrm{P}_{\text {rot }} \sim$ few days


## Questions

## Questions

- How does a strong magnetic field affect radiatively driven wind outflow?
- wind channeling
- magnetically confined wind shocks
- wind-fed rotational magnetospheres


## Questions

- How does a strong magnetic field affect radiatively driven wind outflow?
- wind channeling
- magnetically confined wind shocks
- wind-fed rotational magnetospheres
- How does angular momentum loss \& spindown scale with B*, Mdot, n-pole order, etc.?
- can we explain slow rotators w/ magnetic spindown?
- what are implications for stellar evolution


## Wind Magnetic Confinement

## Ratio of magnetic to kinetic energy density:

$$
\eta(r) \equiv \frac{B^{2} / 8 \pi}{\rho v^{2} / 2}
$$

## Wind Magnetic Confinement

## Ratio of magnetic to kinetic energy density:

$$
\eta(r) \equiv \frac{B^{2} / 8 \pi}{\rho v^{2} / 2}=\frac{B_{*}^{2} R_{*}^{2}}{\dot{M} \mathrm{v}_{\infty}} \frac{\left(r / R_{*}\right)^{-2 n}}{\left(1-R_{*} / r\right)^{\beta}}
$$

## Wind Magnetic Confinement

## Ratio of magnetic to kinetic energy density:

$$
\eta(r) \equiv \frac{B^{2} / 8 \pi}{\rho v^{2} / 2}=\frac{B_{*}^{2} R_{*}^{2}}{\dot{M} \mathrm{v}_{\infty}} \frac{\left(r / R_{*}\right)^{-2 \sqrt{n}}}{\left(1-R_{*} / r\right)^{\beta}} \quad \begin{gathered}
\text { for n-pole } \\
\mathrm{B}(\mathrm{r}) \sim 1 / \mathrm{r} \mathrm{n}+1
\end{gathered}
$$

## Wind Magnetic Confinement

## Ratio of magnetic to kinetic energy density:



## Wind Magnetic Confinement

## Ratio of magnetic to kinetic energy density:



Note also $\eta=\frac{\mathrm{V}_{\mathrm{A}}^{2}}{\mathrm{v}^{2}}$ so Alfven Radius, where $\mathrm{v}=\mathrm{V}_{\mathrm{A}}$, has $\eta\left(\mathbf{R}_{\mathrm{A}}\right) \equiv \mathbf{1}$

## Wind Magnetic Confinement

## Ratio of magnetic to kinetic energy density:



Note also $\eta=\frac{\mathrm{V}_{\mathrm{A}}^{2}}{\mathrm{v}^{2}}$ so Alfven Radius, where $\mathrm{v}=\mathrm{V}_{\mathrm{A}}$, has $\eta\left(\mathbf{R}_{\mathrm{A}}\right) \equiv \mathbf{1}$
For $\beta=0: R_{A}=\eta_{*}^{1 / 2 n} R_{*}$

## Wind Magnetic Confinement

## Ratio of magnetic to kinetic energy density:



Note also $\eta=\frac{\mathrm{V}_{\mathrm{A}}^{2}}{\mathrm{v}^{2}}$ so Alfven Radius, where $\mathrm{v}=\mathrm{V}_{\mathrm{A}}$, has $\eta\left(\mathbf{R}_{\mathrm{A}}\right) \equiv \mathbf{1}$
For $\beta=0: \quad R_{A}=\eta_{*}^{1 / 2 n} R_{*}$

$$
\text { e.g., for dipole, } \mathbf{n}=2: \mathbf{R}_{\mathrm{A}}=\eta_{*}^{1 / 4} \mathbf{R}_{*}
$$

## Magnetic confinement vs. Wind + Rotation

Wind mag. confinement

$$
\eta_{*} \equiv \frac{B_{*}^{2} R_{*}^{2}}{\dot{M} V_{\infty}}
$$

Rotation vs. critical

$$
W \equiv \frac{V_{r o t}}{\sqrt{G M / R_{*}}}
$$

Alfven radius for n-pole

$$
\begin{aligned}
R_{A} & =\eta_{*}^{1 / 2 n} R_{*} \\
& =\eta_{*}^{1 / 4} R_{*} \text { for n=2 dipole }
\end{aligned}
$$

## MiMeS

## Magnetism in Massive Stars

P.I.: Gregg A. Wade, Royal Military College

50+ Co-Is, 2008-2012, CFHT Allocation: 640 hours
http://www.physics.queensu.ca/~wade/mimes/
MiMeS Magnetism in Massive Stars.html


## Magnetically Confined Wind-Shocks

## Babel \& Montmerle 1997

## Magnetic $\mathrm{A}_{\mathrm{p}}-\mathrm{B}_{\mathrm{p}}$ stars



Rigid Field - Hydro Model

## Rigid Field - Hydro Model

Log Number Density (cm ${ }^{-3}$ )
Log Temperature (K)


## Isothermal

No Rotation
Confinement
parameter

$$
\eta_{*}=1 / 3
$$

## A. ud Doula

PhD thesis 2002

## MHID Simulation of Wind Channeling

## Isothermal

No Rotation
Confinement parameter

$$
\eta_{*}=1 / 3
$$

A. ud Doula PhD thesis 2002


## Isothermal <br> No Rotation

Confinement
parameter

$$
\eta_{*}=1
$$

## MHID Simulation of Wind Channeling

## Isothermal No Rotation <br> Confinement parameter

$$
\eta_{*}=1
$$



## Isothermal <br> No Rotation

Confinement
parameter

$$
\eta_{*}=3
$$

## MHID Simulation of Wind Channeling

## Isothermal No Rotation <br> <br> \section*{Confinement <br> <br> \section*{Confinement parameter}} parameter

}$$
\eta_{*}=3
$$

## MHID Simulation of Wind Channeling

## Isothermal

No Rotation
Confinement
parameter

$$
\eta_{*}=10
$$

## MHID Simulation of Wind Channeling

## Isothermal

No Rotation

## Confinement

 parameter$$
\eta_{*}=10
$$

$\zeta$ Pup, $\mathrm{B}_{0}=930 \mathrm{G}$ (Pole), $\left.\Delta=15 \mathrm{ksec}, \angle \hat{(\mathrm{min}}\right)=0.3^{0}$ isothermal, $B_{6}$ and $V_{6}$ independertly $u$ pdated, $7 / 4 / 01$

## Field-aligned rotation

$$
\begin{aligned}
& \eta_{*}=100 \\
& \mathbf{R}_{\mathrm{A}}=3.2 \mathbf{R}_{*} \\
& \mathrm{~W}=1 / 2 \\
& \mathbf{R}_{\mathrm{K}}=1.6 \mathrm{R}^{2}
\end{aligned}
$$


$\mathbf{R}_{\mathrm{K}} \quad \mathbf{R}_{\mathrm{A}}$

## Field-aligned rotation

$$
\begin{aligned}
& \eta_{*}=100 \\
& \mathbf{R}_{\mathrm{A}}=3.2 \mathbf{R}_{*} \\
& \mathrm{~W}=1 / 2 \\
& \mathbf{R}_{\mathrm{K}}=1.6 \mathrm{R}_{*}
\end{aligned}
$$



## Strong Field + Rapid rotation $\eta_{*}=100$ <br> W=1/2



## Strong Field + Rapid rotation $\eta_{*}=100$ <br> W=1/2



## Radial Mass Distribution



$$
\frac{d m_{e}(r, t)}{d r} \equiv 2 \pi r^{2} \int_{\pi / 2-\Delta \theta / 2}^{\pi / 2+\Delta \theta / 2} \rho(r, \theta, t) \sin \theta d \theta
$$

Time evolution of Radial distribution of equatorial disk mass

$$
\eta_{*}=100 \quad \& \quad \text { Vrot } / \text { Vcrit }=1 / 2
$$



Ud-Doula et al. 2008, MNRAS, 385, 97

# Temporal evolution of radial distribution of equatorial disk mass 


$\mathrm{t}=0-3 \mathrm{Msec}$
Stronger Magnetic Confinement
Ud-Doula et al. 2008, MNRAS, 385, 97

Strongest MHD sim

$$
\begin{aligned}
& \eta_{*}=1000 \\
& \mathrm{~W}=1 / 2
\end{aligned}
$$



## Magnetic Bp Stars

## Magnetic Bp Stars

- o Ori E (B2p V)
- $\mathrm{P}_{\text {rot }}=1.2$ days $\Rightarrow \mathrm{v}_{\text {rot }} / \mathrm{v}_{\text {crit }} \sim 1 / 2$
$-\mathrm{B}_{\text {obs }} \sim 10^{4} \mathrm{G} \Rightarrow \eta_{\approx} \sim 10^{7}$ !
$-\Rightarrow V_{\text {Alviven }}$ very large $=>$ Courant time very small
$-\Rightarrow$ Direct MHD impractical


## Magnetic Bp Stars

- o Ori E (B2p V)
- $\mathrm{P}_{\text {rot }}=1.2$ days $\Rightarrow \mathrm{v}_{\text {rot }} / \mathrm{v}_{\text {crit }} \sim 1 / 2$
$-\mathrm{B}_{\text {obs }} \sim 10^{4} \mathrm{G} \Rightarrow \eta_{\approx} \sim 10^{7}$ !
$-\Rightarrow V_{\text {Alviven }}$ very large $=>$ Courant time very small
$-\Rightarrow$ Direct MHD impractical


## Magnetic Bp Stars

- $\quad$ o Ori E (B2p V)
- $\mathrm{P}_{\text {rot }}=1.2$ days $\Rightarrow \mathrm{v}_{\text {rot }} / \mathrm{v}_{\text {crit }} \sim 1 / 2$
$-\mathrm{B}_{\text {obs }} \sim 10^{4} \mathrm{G} \Rightarrow \eta_{*} \sim 10^{7}$ !
- $=>V_{\text {Alfven }}$ very large $=>$ Courant time very small
- => Direct MHD impractical
- Instead treat fields lines as Rigid guides - Torque up wind outflow
- Hold down disk material vs. centrifugal force


## Effective Gravitational+Centrifugal Potential



## Effective Gravitational+Centrifugal Potential



## Rigidly Rotating Magnetosphere



Townsend \& Owocki (2005)

## Rigidly Rotating Magnetosphere



Townsend \& Owocki (2005)

## Rigidly Rotating Magnetosphere



Townsend \& Owocki (2005)

## Rigidly Rotating Magnetosphere ${ }_{-1.3}^{\square} \square-0.4$



Accumulation Surfaces
observed from $\mathbf{i}=\mathbf{6 0}^{\circ}$


RRM model for $\sigma$ Ori $E$

## $\mathrm{B}_{*} \sim 10^{4} \mathrm{G}$

$$
\eta_{*} \sim 10^{6}!
$$

$$
\text { tilt } \sim 55^{\circ}
$$



# RRM model 

```
\[
B_{*} \sim 10^{4} \mathrm{G}
\]
\[
\Rightarrow \eta_{\imath} \sim 10^{6}!
\]
\[
\text { tilt } \sim 55^{\circ}
\]
```


## RRM model for $\sigma$ Ori E

$$
\begin{aligned}
& \mathrm{B}_{*} \sim 10^{4} \mathrm{G} \\
& \Rightarrow \eta_{*} \sim 10^{6}!
\end{aligned}
$$

tilt $\sim 55^{\circ}$

EM +B-field


$\mathrm{H} \alpha$
photometry


polarimetry

## RRM Model



# $\sigma$ Ori E 

Townsend et al. 2005, ApJ, 630, 81

## RRM Model

H $\alpha$ Emission

## H $\alpha$ Observations

H $\alpha$ Emission
$-0.1 \lcm{\square}-15$



## Angular Momentum Loss \& Spindown Weber and Davis (1967)

Monopole field at solar surface


$$
\dot{J}=\frac{2}{3} \dot{M} \Omega R_{A}^{2}
$$

## Weber \& Davis 1967

## Spindown for $\mathrm{n}=1$ monopole field

Total equatorial Ang. mom/mass

$$
j=V_{\phi} r-\frac{\begin{array}{c}
\text { gas } \\
B_{\phi} B_{r} r \\
\rho V_{r}
\end{array}}{\text { fied }}
$$

## Weber \& Davis 1967

## Spindown for $\mathrm{n}=1$ monopole field

Total equatorial Ang. mom/mass

$$
j=V_{\phi} r-\frac{B_{\phi} B_{r} r}{\rho V_{r}} \quad \& \quad \frac{B_{\phi}}{B_{r}}=\frac{\Omega r-V_{\phi}}{V_{r}}
$$

## Weber \& Davis 1967

## Spindown for $\mathrm{n}=1$ monopole field

Total equatorial Ang. mom/mass

$$
\begin{aligned}
& \begin{array}{l}
\text { gas field } \\
j= \\
V_{\phi} r
\end{array} \\
& \Rightarrow \quad \frac{B_{\phi} B_{r} r}{\rho V_{r}} \quad \& \quad \frac{B_{\phi}}{B_{r}}=\frac{\Omega r-V_{\phi}}{V_{r}} \\
& \Rightarrow \quad j_{g a s} \equiv V_{\phi} r=\frac{j M_{A}^{2}-\Omega r^{2}}{M_{A}^{2}-1}
\end{aligned}
$$

## Weber \& Davis 1967

Spindown for $\mathrm{n}=1$ monopole field

Total equatorial Ang. mom/mass

\[

\]

Frozen flux

## Spindown

$$
\begin{aligned}
& \dot{J}=\frac{2}{3} \dot{M} \Omega R_{A}^{2} \quad \text { contribution from both matter \& field } \\
& \tau_{\text {spin }} \equiv \frac{J}{\dot{J}} \approx \frac{\frac{3}{2} I}{M R^{2}} \frac{M}{\dot{M}} \frac{1}{\eta_{*}^{1 / n}}=\tau_{\text {mass }} \frac{\frac{3}{2} k}{\eta_{*}^{1 / n}} \\
& \text { For dipole: } \quad \frac{\tau_{\text {spin }}}{\tau_{\text {mass }}} \approx \frac{0.15}{\sqrt{\eta_{*}}}
\end{aligned}
$$

# Dynamical simulations of magnetically channelled line-driven stellar winds - III. Angular momentum loss and rotational spin-down 

Asif ud-Doula, ${ }^{1 \star}$ Stanley P. Owocki ${ }^{2}$ and Richard H. D. Townsend ${ }^{3}$<br>${ }^{1}$ Department of Physics, Morrisville State College, Morrinille, NY 13408, USA<br>${ }^{2}$ Bartol Research Instiate, University of Delaware, Newark, DE I97I6, USA<br>${ }^{3}$ Department of Astronony, Univerrity of Wisconsin-Madison, 5534 Sterling Hall, 475 N Charter Street, Madison, WI 53706, USA

Accepted 2008 Ottober 23. Received 2008 September 25 ; in original form 2008 August 4

## ABSTRACT

We examine the angular momentum loss and associated rotational spin-down for magnetic hot stars with a line-driven stellar wind and a rotation-aligned dipole magnetic field. Our analysis here is based on our previous two-dimensional numerical magnetohydrodynamics simulation study that examines the interplay among wind, field and rotation as a function of two dimensionless parameters: one characterizing the wind magnetic confinement ( $\eta_{*} \equiv$ $B_{\text {eq }}^{2} R_{*}^{2} / \dot{M} v_{\infty}$ ) and the other the ratio ( $W \equiv V_{\text {rot }} / V_{\text {ct }}$ ) of stellar rotation to critical (orbital) speed. We compare and contrast the two-dimensional, time-variable angular momentum loss of this dipole model of a hot-star wind with the classical one-dimensional steady-state analysis by Weber and Davis (WD), who used an idealized monopole field to model the angular momentum loss in the solar wind. Despite the differences, we find that the total angular momentum loss $\bar{J}$ averaged over both solid angle and time closely follows the general WD scaling $\dot{J}=(2 / 3) \dot{M} \Omega R_{\mathrm{A}}^{2}$, where $\dot{M}$ is the mass-loss rate, $\Omega$ is the stellar angular velocity and $R_{\mathrm{A}}$ is a characteristic Alfvén radius. However, a key distinction here is that for a dipole field, this Alfvén radius has a strong-field scaling $R_{\mathrm{A}} / R_{*} \approx \eta_{*}^{1 / 4}$, instead of the scaling $R_{\mathrm{A}} / R_{*} \sim \sqrt{\eta_{*}}$ for a monopole field. This leads to a slower stellar spin-down time that in the dipole case scales as $\tau_{\text {spin }}=\tau_{\text {mass }} 1.5 \mathrm{k} / \sqrt{\eta_{*}}$, where $\tau_{\text {mass }}=M / \dot{M}$ is the characteristic mass loss time and $k$ is the dimensionless factor for stellar moment of inertia. The full numerical scaling relation that we cite gives typical spin-down times of the order of 1 Myr for several known magnetic massive stars.

Key words: MHD-stars: early-type - stars: magnetic fields - stars: mass loss - stars: rotation - stars: winds, outflows.

## Time variation of total

## Angular Momentum Loss

## Gas <br> Field <br> Total



Ud-Doula et al. 2009, MNRAS, 392, 1022

Angular Momentum Loss vs.

## latitude \& time



Outer Boundary

Field


Gas+Field


Inner Boundary


Field


Ud-Doula et al. 2009, MNRAS, 392, 1022

## Spindown Time

## W=1/2

$$
\mathrm{W}=1 / 4
$$

Ud-Doula et al. 2009, MNRAS, 392, 1022


Magnetic confinement parameter $=>$

## Dipole spindown times

Table 1. Estimated spin-down time for selected known magnetic stars.

| Star $^{a}$ | $M / \mathrm{M}_{\odot}$ | $R_{*} / \mathrm{R}_{\odot}$ | $\mathrm{P}(\mathrm{d})$ | $k$ | $\dot{M}\left(10^{-9} \mathrm{M}_{\odot} \mathrm{yr}^{-1}\right)$ | $v_{\infty}\left(1000 \mathrm{~km} \mathrm{~s}^{-1}\right)$ | $B_{\mathrm{p}}(\mathrm{kG})$ | $\eta_{*}$ | $\tau_{\text {spin }}(\mathrm{Myr})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\theta^{1} \mathrm{OriC}^{1}$ | 40 | 8 | 15.4 | 0.28 | 400 | 2.5 | 1.1 | 15.7 | -8 |
| ${\mathrm{HDl} 191612^{2}}^{2}$ | 40 | 18 | 538 | 0.17 | 6100 | 2.5 | 1.6 | 7.6 | 0.4 |
| $\zeta \mathrm{Cas}^{3}$ | 8 | 5.9 | 5.37 | 0.1 | 0.3 | 0.8 | 0.34 | 3200 | 65.2 |
| $\sigma$ Ori $^{4}$ | 8.9 | 5.3 | 1.2 | 0.1 | 2.4 | 1.46 | 9.6 | $1.4 \times 10^{5}$ | $\boxed{1.4}$ |
| $\rho$ Leo $^{5}$ | 22 | 35 | 7.47 | 0.12 | 630 | 1.1 | 0.24 | 20 | 1.1 |

Ud-Doula et al. 2009, MNRAS, 392, 1022

$$
\begin{aligned}
\tau_{\text {spin }} & \approx \tau_{\text {mass }} \frac{\frac{3}{2} k}{\sqrt{\eta_{*}}} \\
& \approx 11 M y r \frac{k_{-1}}{B_{k G}} \frac{M_{*}}{R_{*}} \sqrt{\frac{V_{8}}{\dot{M_{-9}}}}
\end{aligned}
$$

## DISCOVERY OF ROTATIONAL BRAKING IN THE MAGNETIC HELIUM-STRONG STAR SIGMA ORIONIS E

R. H. D. Townsend ${ }^{1}$, M. E. Oksala ${ }^{2}$, D. H. Cohen ${ }^{3}$, S. P. Owocki ${ }^{2}$, and A. ud-Doula ${ }^{4}$<br>${ }^{1}$ Department of Astrenomy, University of Wisconsin-Madison, Sterling Hall, 475 N. Charter Street, Madison, WI 53706, USA; townsende astro,wisc-edu<br>${ }^{2}$ Bartol Research Institute, Department of Physics and Astronomy, University of Delaware, Newark, DE 19716, USA<br>${ }^{3}$ Department of Physics and Astronomy, Swarthmore College, Swarthmore, PA 19081, USA<br>${ }^{4}$ Penn State Worthington Scranton, 120 Ridge View Drive, Dunmore, PA 18512, USA<br>Received 2010 March 12; accepted 2010 April 14; published 2010 April 26

## ABSTRACT

We present new $U$-band photometry of the magnetic helium-strong star $\sigma$ Ori E, obtained over 2004-2009 using the SMARTS 0.9 m telescope at Cerro Tololo Inter-American Observatory. When combined with historical measurements, these data constrain the evolution of the star's 1.19 day rotation period over the past three decades. We are able to rule out a constant period at the $p_{\text {null }}=0.05 \%$ level, and instead find that the data are well described ( $p_{\text {mall }}=$ $99.3 \%$ ) by a period increasing linearly at a rate of 77 ms per year. This corresponds to a characteristic spin-down time of 1.34 Myr , in good agreement with theoretical predictions based on magnetohydrodynamical simulations of angular momentum loss from magnetic massive stars. We therefore conclude that the observations are consistent with $\sigma$ Ori E undergoing rotational braking due to its magnetized line-driven wind.


## Spindown age

$$
P=P_{o} e^{t / \tau_{\text {pin }}}
$$

$$
P_{o} \approx f P_{c}
$$

## Spindown age

$$
\begin{aligned}
P=P_{o} e^{t / \tau_{\text {pin }}} & P_{o} \approx f P_{c} \\
P_{c}=0.21 d R \sqrt{R / M} & \Rightarrow P_{o} \sim d a y
\end{aligned}
$$

## Spindown age

$$
\begin{gathered}
P=P_{o} e^{t / \tau_{\text {vipin }}} \quad P_{o} \approx f P_{c} \\
P_{c}=0.21 d R \sqrt{R / M} \quad \Rightarrow P_{o} \sim d a y \\
\tau_{a g e}=2.3\left(\log P_{d a y}-\log P_{o, d a y}\right) \tau_{\text {spin }}
\end{gathered}
$$

## Spindown age

$$
\begin{gathered}
P=P_{o} e^{t / \tau_{\text {vipin }}} \quad P_{o} \approx f P_{c} \\
P_{c}=0.21 d R \sqrt{R / M} \quad \Rightarrow P_{o} \sim d a y \\
\tau_{a g e}=2.3\left(\log P_{d a y}-\log P_{o, d a y}\right) \tau_{\text {spin }}
\end{gathered}
$$

e.g. HD191612, with $\mathrm{P}_{\mathrm{o}}=0.5$ to 1 day $=>$ now $\mathrm{P}=630$ day:

$$
\tau_{\text {age }} \approx 6.3 \rightarrow 6.9 \tau_{\text {spin }} \approx 2.5 \rightarrow 2.9 \mathrm{Myr}
$$



## Extrapolated spindown law for higher order multipoles?

$\frac{\tau_{\text {spin }}}{\tau_{\text {mass }}}=\frac{\frac{3}{2} k}{\eta_{*}^{1 / n}}$
$\mathrm{n}=1$ monopole
$=2$ dipole
=3 quadrapole
... etc.

## Extrapolated spindown law for higher order multipoles?

$\tau_{\text {spin }}=\frac{\frac{3}{2} k}{\boldsymbol{\eta}_{*}^{1 / n}} \quad \begin{array}{r}\mathrm{n}=1 \text { monopo } \\ =2 \text { dipole } \\ =3 \text { quadra } \\ \tau_{\text {mass }}\end{array}$
$=>$ Spindown weaker for more complex fields?
If so, hard to explain tau Sco by spindown??

## Extrapolated spindown law for higher order multipoles?

$\frac{\tau_{\text {spin }}}{\tau_{\text {mass }}}=\frac{\frac{3}{2} k}{\eta_{*}^{1 / n}} \quad \begin{aligned} & \text { n=1 monopole } \\ & =2 \text { dipole } \\ & =3 \text { quadrapole } \\ & \ldots \text { etc. }\end{aligned}$
=> Spindown weaker for more complex fields?
If so, hard to explain tau Sco by spindown??
Need 3D MHD sims to test this!


## Summary

- Wind feeding of magnetosphere
- balanced by inner \& outer "leakage"?
- observations should estimate $\mathrm{M}_{\mathrm{tot}}$
- breakout analysis predicts $\mathrm{M}_{\mathrm{tot}}$ indep of $\mathrm{M}_{\mathrm{dot}}$ !
- Wind Magnetic Spindown
$-\mathrm{t}_{\text {spin }} \sim \mathrm{t}_{\text {mass }}$ /Sqrt[eta*] for aligned dipole
- complex field => slower spindown?
- need 3D sims to confirm!


## Mass Loss \& Rotational Spindown of Magnetic Massive Stars

Stan Owocki<br>University of Delaware<br>Newark, Delaware USA

Collaborators

- Asif ud-Doula
- Rich Townsend


## Earth's Magnetosphere



## Solar Corona in EUV \& X-rays

Composite EUV image from EIT/SOHO
X-ray Corona from SOHO

## Corona during Solar Eclipse

## Solar Activity: Coronal Mass Ejections

## Hot, Luminous, Massive Stars

- Strong, radiatively driven stellar wind
$-\mathrm{M}_{\mathrm{dot}} \sim 10^{-9}-10^{-5} \mathrm{M}_{\mathrm{o}} / \mathrm{yr} ; \mathrm{V}_{\square}>1000 \mathrm{~km} / \mathrm{s} \gg \mathrm{V}_{\text {sound }}$
- Some have observed dipole field $\sim 10^{3}-10^{4} \mathrm{G}$
- stable; not from convective dynamo; fossil?
- Fast rotation with $\mathrm{V}_{\text {rot }} \sim 250 \mathrm{~km} / \mathrm{s} \sim \mathrm{V}_{\text {crit }} / 2$ $-\mathrm{P}_{\text {rot }} \sim$ few days


## Questions

- How does a strong magnetic field affect radiatively driven wind outflow?
- wind channeling
- magnetically confined wind shocks
- wind-fed rotational magnetospheres
- How does angular momentum loss \& spindown scale with B*, Mdot, n-pole order, etc.?
- can we explain slow rotators w/ magnetic spindown?
- what are implications for stellar evolution


## Wind Magnetic Confinement

Ratio of magnetic to kinetic energy density:
$\eta(r) \equiv \frac{B^{2} / 8 \pi}{\rho v^{2} / 2}=\begin{array}{ll}\left.\right|_{*} ^{2} R_{*}^{2} \\ \frac{\dot{M} \mathrm{v}_{\infty}}{} & \frac{\left(r / R_{*}\right)^{-2 \pi}}{\left(1-R_{*} / r\right)^{\beta}}\end{array} \quad \begin{gathered}\text { for n-pole } \\ \mathrm{B}(\mathrm{r}) \sim 1 / \mathrm{r}^{\mathrm{n}+1}\end{gathered}$
Note also $\eta=\frac{\mathrm{V}_{\mathrm{A}}^{2}}{\mathrm{v}^{2}}$ so Alfven Radius, where $\mathrm{v}=\mathrm{V}_{\mathrm{A}}$, has $\mid\left(\mathbf{R}_{\mathrm{A}}\right) \propto \mathbf{1}$
For $R=0$ : $\quad \mathrm{R}_{\mathrm{A}}=\left.\right|_{*} ^{1 / 2 \mathrm{n}} \mathrm{R}_{*}$
e.g., for dipole, $\mathbf{n}=2$ : $\mathbf{R}_{\mathrm{A}}=\mid{ }_{*}{ }^{1 / 4} \mathbf{R}_{*}$

## Magnetic confinement vs. Wind + Rotation

Wind mag. confinement

$$
\eta_{*} \equiv \frac{B_{*}^{2} R_{*}^{2}}{\dot{M} V_{\infty}}
$$

Rotation vs. critical

$$
W \equiv \frac{V_{r o t}}{\sqrt{G M / R_{*}}}
$$

Alfven radius for n-pole

$$
\begin{aligned}
R_{A} & =\eta_{*}^{1 / 2 n} R_{*} \quad R_{K}=W^{-2 / 3} R_{*} \\
& =\eta_{*}^{1 / 4} R_{*} \text { for } \mathrm{n}=2 \text { dipole }
\end{aligned}
$$

Kepler radius

## MiMeS

## Magnetism in Massive Stars

## P.I.: Gregg A. Wade, Royal Military College

50+ Co-Is, 2008-2012, CFHT Allocation: 640 hours
http://www.physics.queensu.ca/~wade/mimes/MiMeS Magnetism Massive Stars.html


## Magnetically Confined Wind-Shocks

## Babel \& Montmerle 1997

## Magnetic $\mathrm{A}_{\mathrm{p}}-\mathrm{B}_{\mathrm{p}}$ stars



## Rigid Field - Hydro Model

## MHD Simulation of Wind Channeling

## Isothermal

No Rotation
Confinement
parameter

$$
\left.\right|_{*}=1 / 3
$$

A. ud Doula

PhD thesis 2002

## MHD Simulation of Wind Channeling

## Isothermal

No Rotation
Confinement
parameter
$\left.\right|_{*}=1$

## MHD Simulation of Wind Channeling

## Isothermal

No Rotation
Confinement
parameter

$$
\mid *=3
$$

## MHD Simulation of Wind Channeling

## Isothermal

No Rotation
Confinement
parameter

$$
\left.\right|_{*}=10
$$

## Field-aligned rotation

$$
\begin{aligned}
& r_{*}=100 \\
& \mathbf{R}_{\mathrm{A}}=3.2 \mathbf{R}_{*} \\
& \mathbf{W}=1 / 2 \\
& \mathbf{R}_{\mathrm{K}}=1.6 \mathbf{R}^{2}
\end{aligned}
$$



## Strong Field + Rapid rotation *=100 W=1/2



## Radial Mass Distribution



$$
\frac{d m_{e}(r, t)}{d r} \equiv 2 \pi r^{2} \int_{\pi / 2-\Delta \theta / 2}^{\pi / 2+\Delta \theta / 2} \rho(r, \theta, t) \sin \theta d \theta
$$

Time evolution of Radial distribution of equatorial disk mass

$$
\left.\right|_{*}=100 \quad \& \text { Vrot } / \text { Vcrit }=1 / 2
$$



## Temporal evolution of radial distribution of equatorial disk mass <br> $\mathrm{r}=1-5$




Ud-Doula et al. 2008, MNRAS, 385, 97

## Strongest MHD sim

$$
\begin{aligned}
& *=1000 \\
& W=1 / 2
\end{aligned}
$$



## Magnetic Bp Stars

- $\int$ Ori E (B2p V)
- $\mathrm{P}_{\text {rot }}=1.2$ days $=>\mathrm{v}_{\text {rot }} / \mathrm{v}_{\text {crit }} \sim 1 / 2$
$-\mathrm{B}_{\mathrm{obs}} \sim 10^{4} \mathrm{G}=>\left.\right|_{*} \sim 10^{7}$ !
- => $\mathrm{V}_{\text {Alfven }}$ very large => Courant time very small
- => Direct MHD impractical
- Instead treat fields lines as Rigid guides
- Torque up wind outflow
- Hold down disk material vs. centrifugal force


## Effective Gravitational+Centrifugal Potential



## Rigidly Rotating Magnetosphere



Townsend \& Owocki (2005)

## Accumulation Surfaces

 observed from $\mathbf{i}=60^{\circ}$

RRM model
for $\int$ Ori E
$\mathrm{B}_{*} \sim 10^{4} \mathrm{G}$
*~10 ${ }^{6}$ !
tilt $\sim 55^{\circ}$

# RRM model for C Ori E 

$\mathrm{B}_{*} \sim 10^{4} \mathrm{G}$
$\Rightarrow \mid * \sim 10^{6}$ !
tilt $\sim 55^{\circ}$

## COri E

RRM Model
H $\alpha$ Emission

$-0.1$| $\square \square$ | 0.15 |
| :---: | :---: |



## H〈 Observations

H $\alpha$ Emission
$- 0 . 1 \longdiv { \square } 0 . 1 5$


## Angular Momentum Loss \& Spindown

Weber and Davis (1967)


## Weber \& Davis 1967

Spindown for n=1 monopole field

Total equatorial Ang. mom/mass
Frozen flux

$$
\begin{aligned}
& \text { gas field } \\
& j=V_{\phi} r-\frac{B_{\phi} B_{r} r}{\rho V_{r}} \quad \& \quad \frac{B_{\phi}}{B_{r}}=\frac{\Omega r-V_{\phi}}{V_{r}} \\
& =>j_{g a s} \equiv V_{\phi} r=\frac{j M_{A}^{2}-\Omega r^{2}}{M_{A}^{2}-1} \\
& \begin{array}{l}
\text { Atr= } \mathrm{R}_{\mathrm{A}}, \\
\mathrm{M}_{\mathrm{A}}=1 \text { implies }
\end{array}
\end{aligned}
$$

## Spindown

## $\stackrel{\mathrm{g}}{\mathrm{J}}=\frac{2}{3} \stackrel{\mathrm{~g}}{M} \Omega R_{A}^{2} \quad$ contribution from both mater \& field

$\tau_{\text {spin }} \equiv \frac{J}{J} \approx \frac{\frac{3}{2} I}{M R^{2}} \frac{M}{M} \frac{1}{\eta_{*}^{1 / n}}=\tau_{\text {mass }} \frac{\frac{3}{2} k}{\eta_{*}^{1 / n}}$

For dipole:

$$
\frac{\tau_{\text {spin }}}{\tau_{\text {mass }}} \approx \frac{0.15}{\sqrt{\eta_{*}}}
$$

# Dynamical simulations of magnetically channelled line-driven stellar winds - III. Angular momentum loss and rotational spin-down 

Asif ud-Doula, ${ }^{1 \star}$ Stanley P. Owocki ${ }^{2}$ and Richard H. D. Townsend ${ }^{3}$

${ }^{1}$ Department of Physics, Morris ville State College, Morrisville, NY 13408, USA
${ }^{2}$ Bartol Research Institute, University of Delaware, Newark, DE 19716, USA
${ }^{3}$ Department of Astronomy, University of Wisconsin-Madison, 5534 Sterling Hall, 475 N Charter Street, Madison, WI 53706, USA

Accepted 2008 October 23. Received 2008 September 25; in original form 2008 August 4


#### Abstract

We examine the angular momentum loss and associated rotational spin-down for magnetic hot stars with a line-driven stellar wind and a rotation-aligned dipole magnetic field. Our analysis here is based on our previous two-dimensional numerical magnetohydrodynamics simulation study that examines the interplay among wind, field and rotation as a function of two dimensionless parameters: one characterizing the wind magnetic confinement $\left(\eta_{*} \equiv\right.$ $B_{\text {eq }}^{2} R_{*}^{2} / \dot{M} v_{\infty}$ ) and the other the ratio ( $W \equiv V_{\text {rot }} / V_{\text {orb }}$ ) of stellar rotation to critical (orbital) speed. We compare and contrast the two-dimensional, time-variable angular momentum loss of this dipole model of a hot-star wind with the classical one-dimensional steady-state analysis by Weber and Davis (WD), who used an idealized monopole field to model the angular momentum loss in the solar wind. Despite the differences, we find that the total angular momentum loss $\boldsymbol{J}$ averaged over both solid angle and time closely follows the general WD scaling $\dot{J}=(2 / 3) \dot{M} \Omega R_{\mathrm{A}}^{2}$, where $\dot{M}$ is the mass-loss rate, $\Omega$ is the stellar angular velocity and $R_{\mathrm{A}}$ is a characteristic Alfvén radius. However, a key distinction here is that for a dipole field, this Alfvén radius has a strong-field scaling $R_{\mathrm{A}} / R_{*} \approx \eta_{*}^{1 / 4}$, instead of the scaling $R_{\mathrm{A}} / R_{*} \sim \sqrt{\eta_{*}}$ for a monopole field. This leads to a slower stellar spin-down time that in the dipole case scales as $\tau_{\text {spin }}=\tau_{\text {mass }} 1.5 k / \sqrt{\eta_{*}}$, where $\tau_{\text {mass }} \equiv M / \dot{M}$ is the characteristic mass loss time and $k$ is the dimensionless factor for stellar moment of inertia. The full numerical scaling relation that we cite gives typical spin-down times of the order of 1 Myr for several known magnetic massive stars.


Key words: MHD-stars: early-type - stars: magnetic fields - stars: mass loss - stars: rotation - stars: winds, outflows.

## Time variation of total Angular Momentum Loss

Gas


Field
Total


Ud-Doula et al. 2009, MNRAS, 392, 1022

## Angular lVomentum Loss vs.

latitude \& time


Inner Boundary


Field


Gas+Field


## Spindown Time

W=1/2 W=1/4

Ud-Doula et al. 2009, MNRAS, 392, 1022


## Dipole spindown times

Table 1. Estimated spin-down time for selected known magnetic stars.

| Star ${ }^{\text {a }}$ | $M / \mathrm{M}_{\odot}$ | $R_{*} / \mathrm{R}_{\odot}$ | P (d) | $k$ | $\dot{M}\left(10^{-9} \mathrm{M}_{\odot} \mathrm{yr}^{-1}\right)$ | $v_{\infty}\left(1000 \mathrm{~km} \mathrm{~s}^{-1}\right)$ | $B_{\mathrm{p}}(\mathrm{kG})$ | $\eta_{*}$ | $\tau_{\text {spin }}(\mathrm{Myr})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\theta^{1}$ Ori $\mathrm{C}^{1}$ | 40 | 8 | 15.4 | 0.28 | 400 | 2.5 | 1.1 | 15.7 | 8 |
| HD191612 ${ }^{2}$ | 40 | 18 | 538 | 0.17 | 6100 | 2.5 | 1.6 | 7.6 | 0.4 |
| $\zeta \mathrm{Cas}^{3}$ | 8 | 5.9 | 5.37 | 0.1 | 0.3 | 0.8 | 0.34 | 3200 | 65.2 |
| $\sigma$ Ori ${ }^{4}$ | 8.9 | 5.3 | 1.2 | 0.1 | 2.4 | 1.46 | 9.6 | $1.4 \times 10^{5}$ | 1.4 |
| $\rho$ Leo $^{5}$ | 22 | 35 | 7-47 | 0.12 | 630 | 1.1 | 0.24 | 20 | 1.1 |

$$
\tau_{\text {spin }} \approx \tau_{\text {mass }} \frac{\frac{3}{2} k}{\sqrt{\eta_{*}}}
$$

$$
\approx 11 \mathrm{Myr} \frac{\mathrm{k}_{-1}}{B_{k G}} \frac{M_{*}}{R_{*}} \sqrt{\frac{V_{8}}{\dot{M}_{-9}}}
$$

## DISCOVERY OF ROTATIONAL BRAKING IN THE MAGNETIC HELIUM-STRONG STAR SIGMA ORIONIS E

$$
\text { R. H. D. Townsend }{ }^{1} \text {, M. E. Oksala }{ }^{2} \text {, D. H. Cohen }{ }^{3} \text {, S. P. Owocki }{ }^{2} \text {, and A. ud-Doula }{ }^{4}
$$

${ }^{1}$ Department of Astronomy, University of Wisconsin-Madison, Sterling Hall, 475 N. Charter Street, Madison, WI 53706, USA; townsend @astro.wisc.edu
${ }^{2}$ Bartol Research Institute, Department of Physics and Astronomy, University of Delaware, Newark, DE 19716, USA
Department of Physics and Astronomy, Swarthmore College, Swarthmore, PA 19081, USA
${ }^{4}$ Penn State Worthington Scranton, 120 Ridge View Drive, Dunmore, PA 18512, USA
Received 2010 March 12; accepted 2010 April 14; published 2010 April 26

## ABSTRACT

We present new $U$-band photometry of the magnetic helium-strong star $\sigma$ Ori E, obtained over 2004-2009 using the SMARTS 0.9 m telescope at Cerro Tololo Inter-American Observatory. When combined with historical measurements, these data constrain the evolution of the star's 1.19 day rotation period over the past three decades. We are able to rule out a constant period at the $p_{\text {null }}=0.05 \%$ level, and instead find that the data are well described ( $p_{\text {null }}=$ $99.3 \%$ ) by a period increasing linearly at a rate of 77 ms per year. This corresponds to a characteristic spin-down time of 1.34 Myr , in good agreement with theoretical predictions based on magnetohydrodynamical simulations of angular momentum loss from magnetic massive stars. We therefore conclude that the observations are consistent with $\sigma$ Ori $E$ undergoing rotational braking due to its magnetized line-driven wind.


## Spindown age

$$
\begin{gathered}
P=P_{o} e^{t / \tau_{\text {spin }}} \quad P_{o} \approx f P_{c} \\
P_{c}=0.21 d R \sqrt{R / M} \quad \Rightarrow P_{o} \sim d a y \\
\tau_{\text {age }}=2.3\left(\log P_{d a y}-\log P_{o, d a y}\right) \tau_{\text {spin }}
\end{gathered}
$$

e.g. HD191612, with $\mathrm{P}_{0}=0.5$ to 1 day $=>$ now $\mathrm{P}=630$ day:

$$
\tau_{\text {age }} \approx 6.3 \rightarrow 6.9 \tau_{\text {spin }} \approx 2.5 \rightarrow 2.9 \mathrm{Myr}
$$



## Extrapolated spindown law for higher order multipoles?

$\frac{\tau_{\text {spin }}}{\tau_{\text {mass }}}=\frac{\frac{3}{2} k}{\eta_{*}^{1 / n}} \quad \begin{gathered}\text { n=1 monopole } \\ =2 \\ \text { =3 dipole } \\ \text { w.etc. }\end{gathered}$
=> Spindown weaker for more complex fields?
If so, hard to explain tau Sco by spindown??
Need 3D MHD sims to test this!

正

## Summary

- Wind feeding of magnetosphere
- balanced by inner \& outer "leakage"?
- observations should estimate M $_{\text {tot }}$
- breakout analysis predicts $\mathrm{M}_{\text {tot }}$ indep of $\mathrm{M}_{\text {dot }}$ !
- Wind Magnetic Spindown
- $\mathrm{t}_{\text {spin }} \sim \mathrm{t}_{\text {mass }}$ /Sqrt[eta*] for aligned dipole
- complex field => slower spindown?
- need 3D sims to confirm!


## References

- Babel, J. \& Montmerle, T. 1997, X-ray emission from Ap-Bp stars: a magnetically confined wind-shock model for IQ Aur., A\&A, 323, 121
- Ud-Doula, A. et al. 2008, Dynamical simulations of magnetically channelled line-driven stellar winds - II. The effects of field-aligned rotation, MNRAS, 385, 97
- Townsend, R.-H.-D. \& Owocki, S.-P. 2005, A rigidly rotating magnetosphere model for circumstellar emission from magnetic OB stars, MNRAS, 357, 251
- Townsend, R.-H.-D. et al. 2005, The Rigidly Rotating Magnetosphere of Ïf Orionis E, ApJ, 630, 81
- Weber, E.-J. \& Davis, L.-J. 1967, The Angular Momentum of the Solar Wind, ApJ, 148, 217
- Ud-Doula, A. et al. 2009, Dynamical simulations of magnetically channelled line-driven stellar winds - III. Angular momentum loss and rotational spin-down, MNRAS, 392, 1022
- Townsend, R.-H.-D. et al. 2010, Discovery of Rotational Braking in the Magnetic Helium-strong Star Sigma Orionis E, ApJ, 714, 318

