Memorial lecture for Prof. Hayashi
Discovery of Hayashi Phase
and
his way of thinking

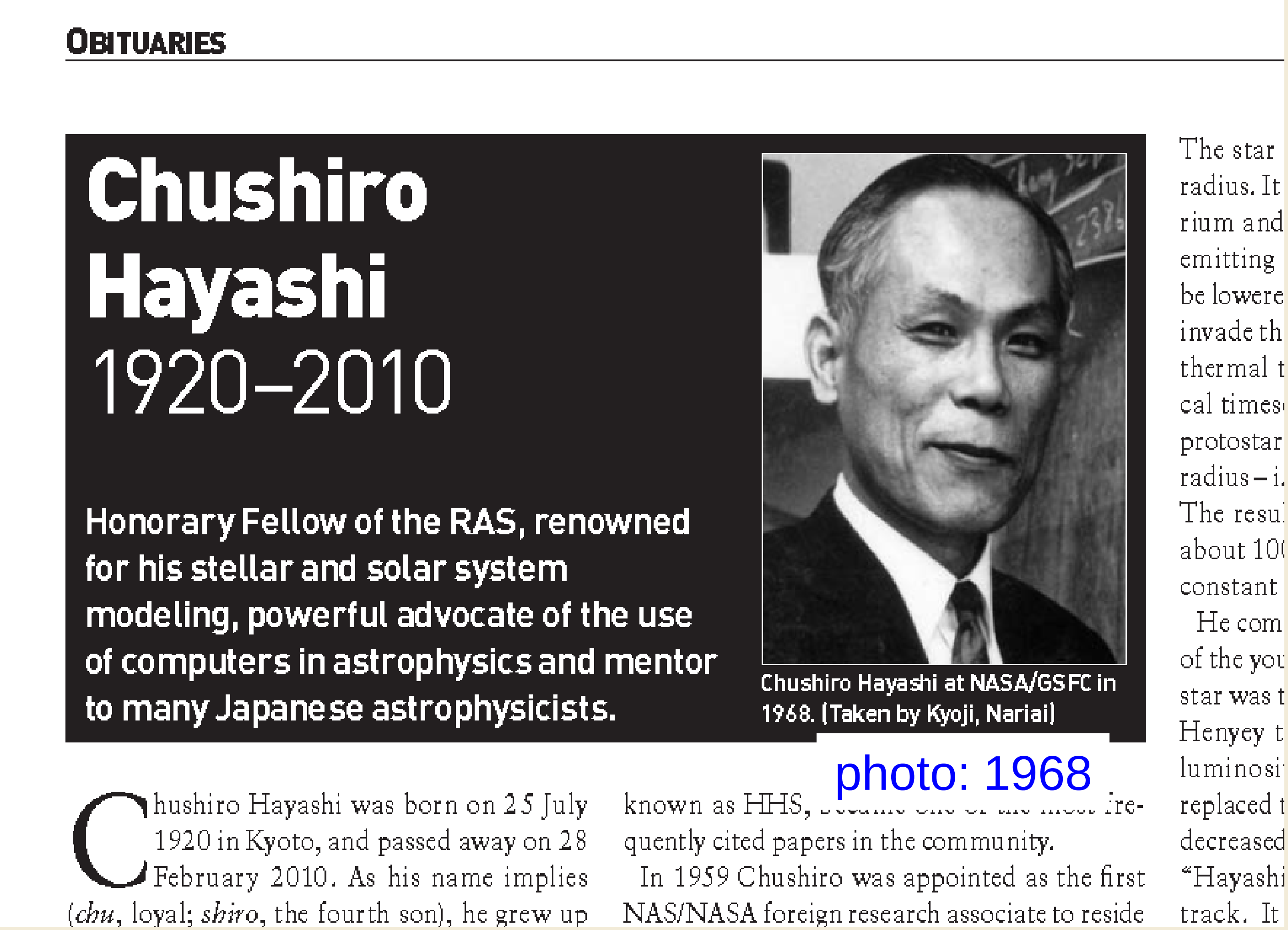
@Kobe planetary school

Daiichiro Sugimoto

former affiliations: Univ. Tokyo

& Open University of Japan

Jan 15, 2011



OBITUARIES

Chushiro Hayashi

1920–2010

Honorary Fellow of the RAS, renowned for his stellar and solar system modeling, powerful advocate of the use of computers in astrophysics and mentor to many Japanese astrophysicists.

Chushiro Hayashi at NASA/GSFC in 1968. (Taken by Kyoji, Nariai)

Chushiro Hayashi was born on 25 July 1920 in Kyoto, and passed away on 28 February 2010. As his name implies (*chu*, loyal; *shiro*, the fourth son), he grew up

known as HHS, became one of the most frequently cited papers in the community.

In 1959 Chushiro was appointed as the first NAS/NASA foreign research associate to reside

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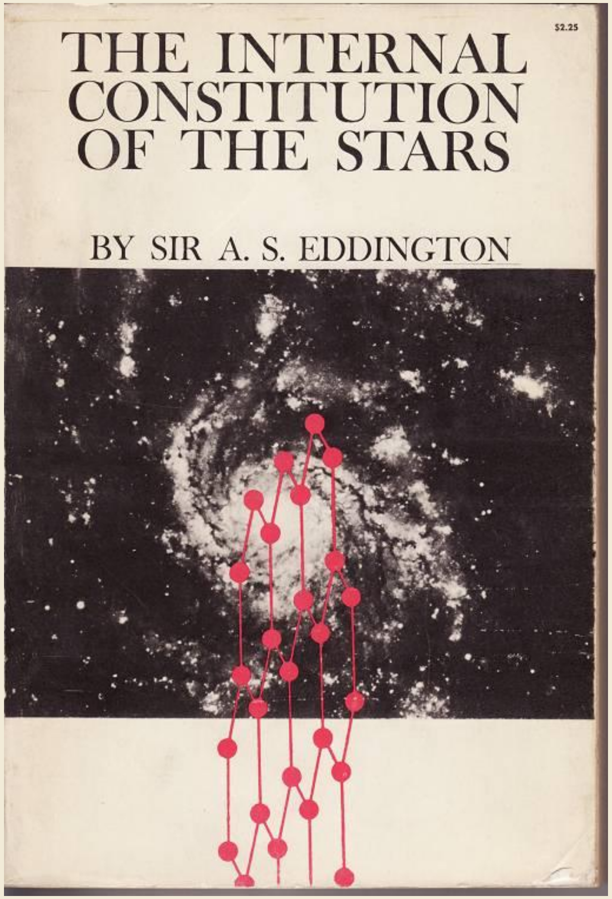
photo: 1968

Astronomy & Geophysics, 51: 3.36.
24 MAY 2010, by D.Sugimoto

Fundamental Textbooks

A.S. Eddington 1926

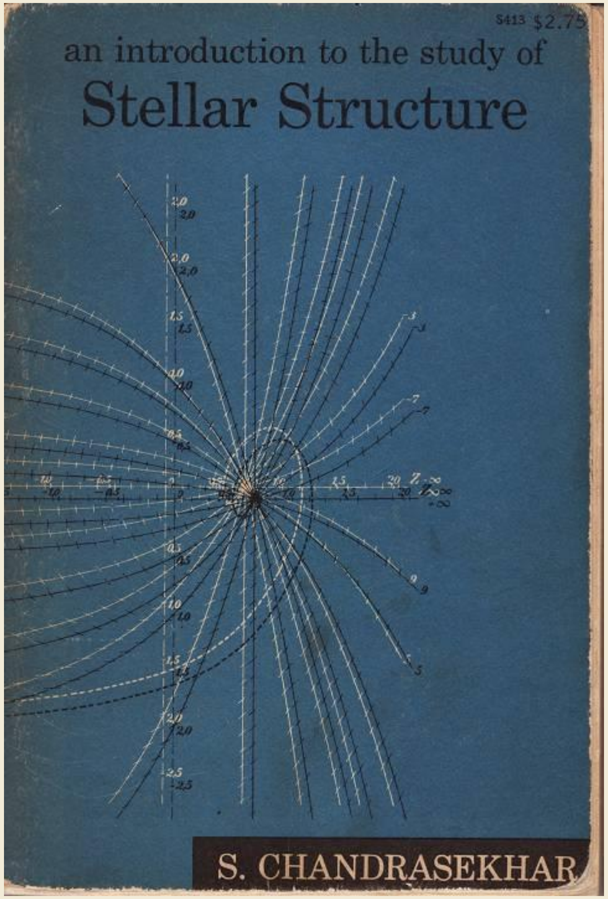
S. Chandrasekhar 1939



$2.25

THE INTERNAL CONSTITUTION OF THE STARS

BY SIR A. S. EDDINGTON

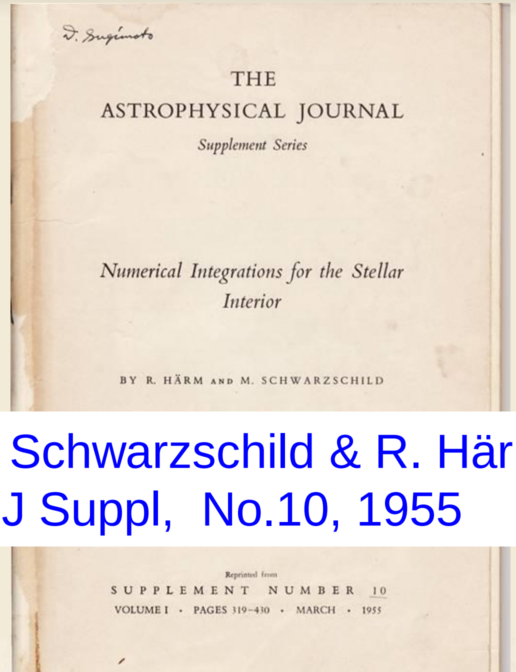


S413 $2.75

an introduction to the study of

Stellar Structure

S. CHANDRASEKHAR



D. Sugimoto

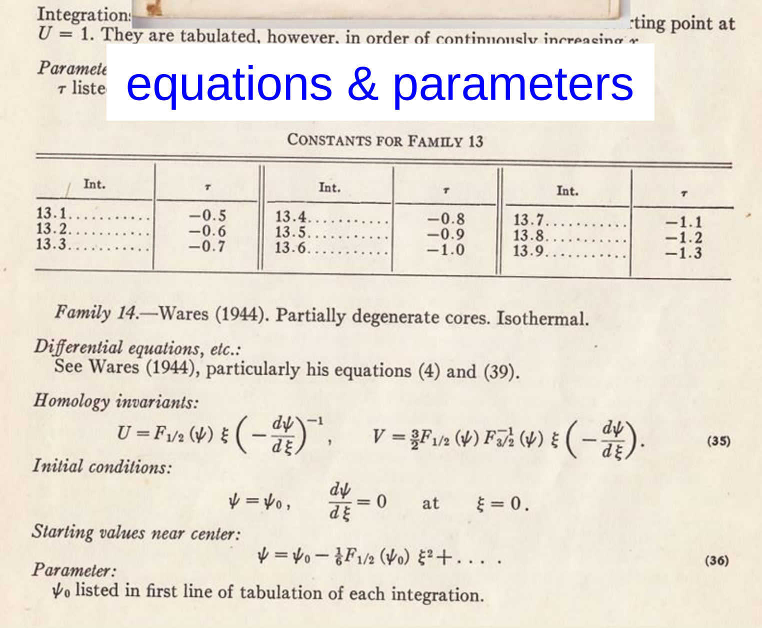
THE
ASTROPHYSICAL JOURNAL
Supplement Series

Numerical Integrations for the Stellar Interior

BY R. HÄRM AND M. SCHWARZSCHILD

Reprinted from
SUPPLEMENT NUMBER 10
VOLUME I · PAGES 319–430 · MARCH · 1955

M. Schwarzschild & R. Härm,
ApJ Suppl, No.10, 1955



Integration … rting point at
$U = 1$. They are tabulated, however, in order of continuously increasing

Paramete
τ liste

equations & parameters

CONSTANTS FOR FAMILY 13

Int.	τ	Int.	τ	Int.	τ
13.1	−0.5	13.4	−0.8	13.7	−1.1
13.2	−0.6	13.5	−0.9	13.8	−1.2
13.3	−0.7	13.6	−1.0	13.9	−1.3

Family 14.—Wares (1944). Partially degenerate cores. Isothermal.

Differential equations, etc.:
See Wares (1944), particularly his equations (4) and (39).

Homology invariants:

$$U = F_{1/2}(\psi)\,\xi\left(-\frac{d\psi}{d\xi}\right)^{-1}, \qquad V = \tfrac{3}{2}F_{1/2}(\psi)\,F_{3/2}^{-1}(\psi)\,\xi\left(-\frac{d\psi}{d\xi}\right). \tag{35}$$

Initial conditions:

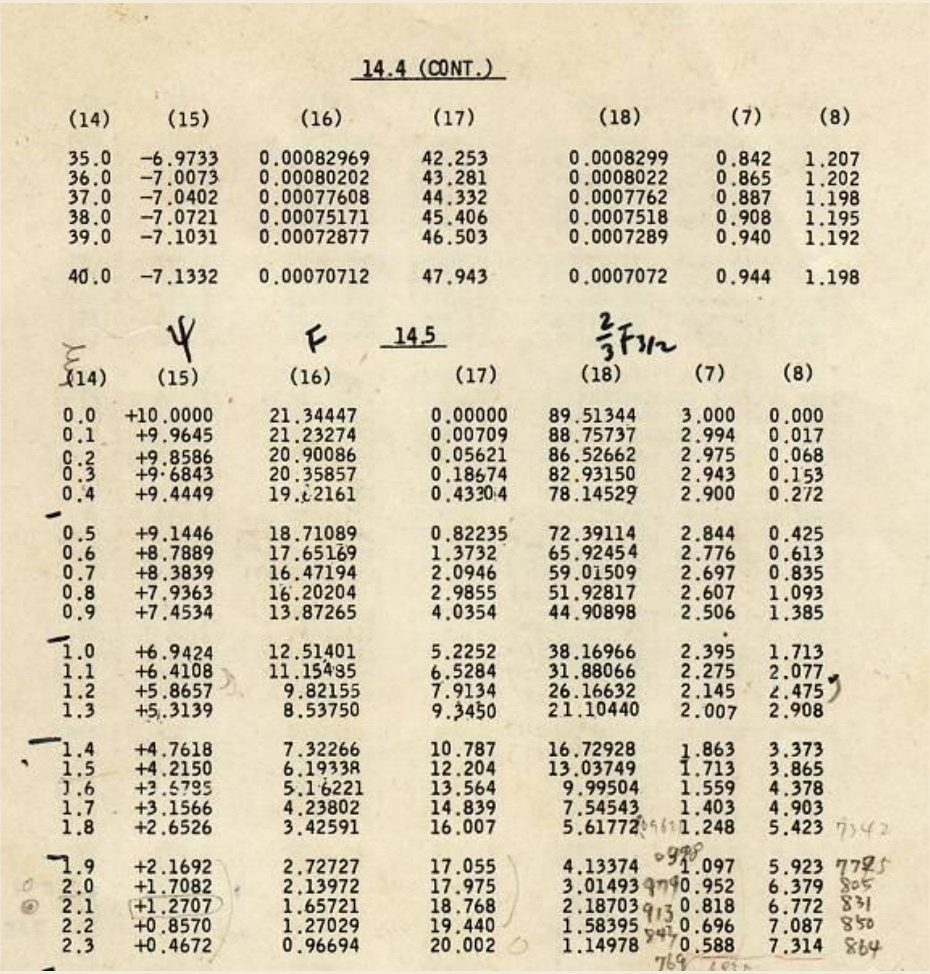
$$\psi = \psi_0, \qquad \frac{d\psi}{d\xi} = 0 \quad \text{at} \quad \xi = 0.$$

Starting values near center:

$$\psi = \psi_0 - \tfrac{1}{6}F_{1/2}(\psi_0)\,\xi^2 + \dots . \tag{36}$$

Parameter:
ψ_0 listed in first line of tabulation of each integration.

Numerical Table



14.4 (CONT.)

(14)	(15)	(16)	(17)	(18)	(7)	(8)
35.0	−6.9733	0.00082969	42.253	0.0008299	0.842	1.207
36.0	−7.0073	0.00080202	43.281	0.0008022	0.865	1.202
37.0	−7.0402	0.00077608	44.332	0.0007762	0.887	1.198
38.0	−7.0721	0.00075171	45.406	0.0007518	0.908	1.195
39.0	−7.1031	0.00072877	46.503	0.0007289	0.940	1.192
40.0	−7.1332	0.00070712	47.943	0.0007072	0.944	1.198

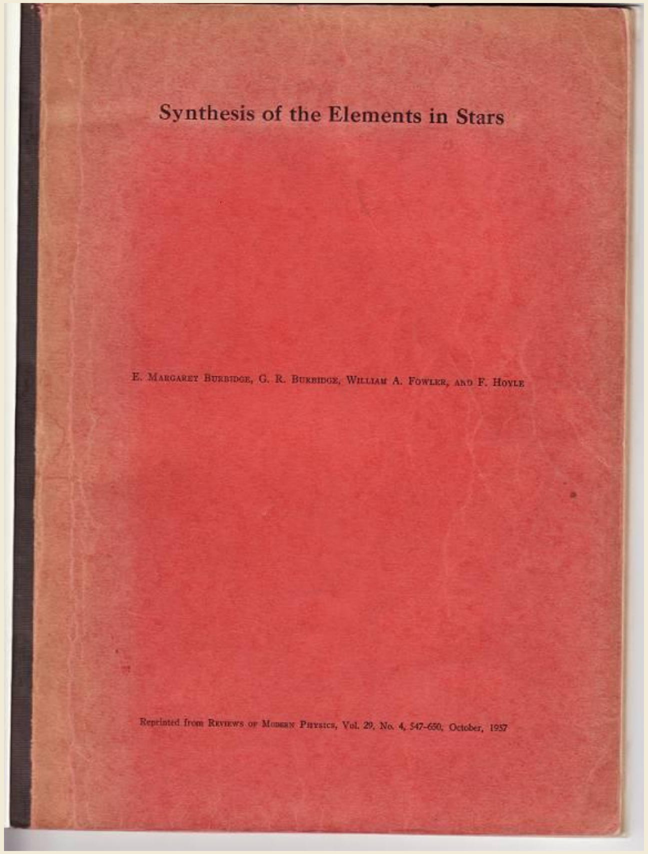
14.5

ξ	ψ	F		$\frac{2}{3}F_{3/2}$				
(14)	(15)	(16)	(17)	(18)		(7)	(8)	
0.0	+10.0000	21.34447	0.00000	89.51344		3.000	0.000	
0.1	+9.9645	21.23274	0.00709	88.75737		2.994	0.017	
0.2	+9.8586	20.90086	0.05621	86.52662		2.975	0.068	
0.3	+9.6843	20.35857	0.18674	82.93150		2.943	0.153	
0.4	+9.4449	19.22161	0.43304	78.14529		2.900	0.272	
0.5	+9.1446	18.71089	0.82235	72.39114		2.844	0.425	
0.6	+8.7889	17.65169	1.3732	65.92454		2.776	0.613	
0.7	+8.3839	16.47194	2.0946	59.01509		2.697	0.835	
0.8	+7.9363	16.20204	2.9855	51.92817		2.607	1.093	
0.9	+7.4534	13.87265	4.0354	44.90898		2.506	1.385	
1.0	+6.9424	12.51401	5.2252	38.16966		2.395	1.713	
1.1	+6.4108	11.15485	6.5284	31.88066		2.275	2.077	
1.2	+5.8657	9.82155	7.9134	26.16632		2.145	2.475	
1.3	+5.3139	8.53750	9.3450	21.10440		2.007	2.908	
1.4	+4.7618	7.32266	10.787	16.72928		1.863	3.373	
1.5	+4.2150	6.19338	12.204	13.03749		1.713	3.865	
1.6	+3.6795	5.16221	13.564	9.99504		1.559	4.378	
1.7	+3.1566	4.23802	14.839	7.54543		1.403	4.903	
1.8	+2.6526	3.42591	16.007	5.61772		1.248	5.423	7342
1.9	+2.1692	2.72727	17.055	4.13374	0998	1.097	5.923	7775
2.0	+1.7082	2.13972	17.975	3.01493	9790	0.952	6.379	805
2.1	+1.2707	1.65721	18.768	2.18703	913	0.818	6.772	831
2.2	+0.8570	1.27029	19.440	1.58395	947	0.696	7.087	850
2.3	+0.4672	0.96694	20.002	1.14978		0.588	7.314	864

769

B^2FH for Nucleosynthesis

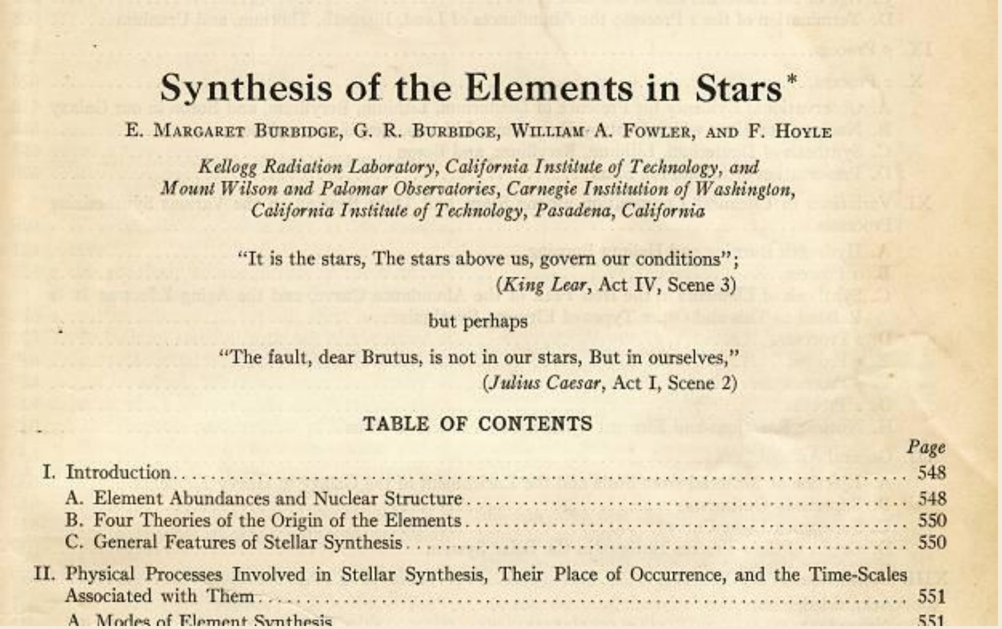
E.M. Burbidge, G.R. Burbidge,
W.A. Fowler & F. Hoyle,
Rev. Mod. Phys., 1957



Synthesis of the Elements in Stars

E. Margaret Burbidge, G. R. Burbidge, William A. Fowler, and F. Hoyle

Reprinted from Reviews of Modern Physics, Vol. 29, No. 4, 547-650, October, 1957



Synthesis of the Elements in Stars*

E. Margaret Burbidge, G. R. Burbidge, William A. Fowler, and F. Hoyle

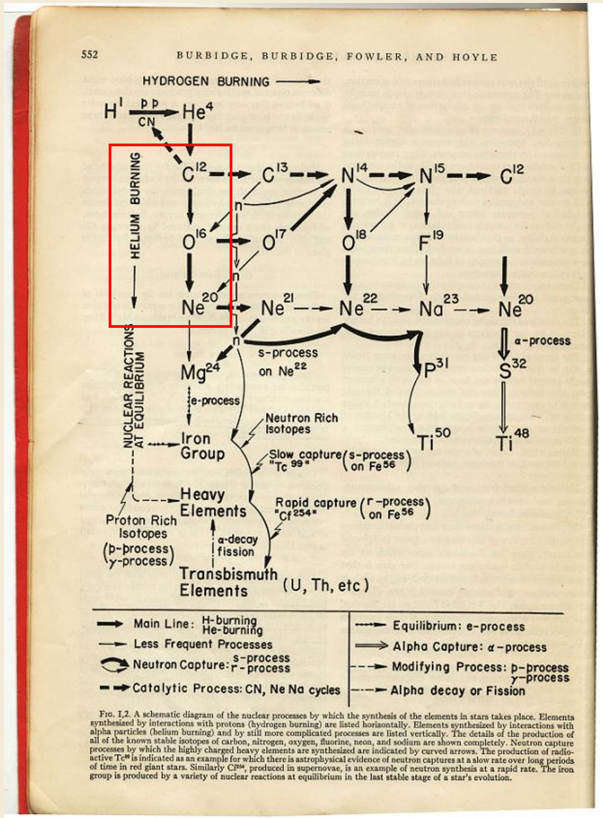
Kellogg Radiation Laboratory, California Institute of Technology, and
Mount Wilson and Palomar Observatories, Carnegie Institution of Washington,
California Institute of Technology, Pasadena, California

> "It is the stars, The stars above us, govern our conditions";
> (*King Lear*, Act IV, Scene 3)
>
> but perhaps
>
> "The fault, dear Brutus, is not in our stars, But in ourselves,"
> (*Julius Caesar*, Act I, Scene 2)

TABLE OF CONTENTS

Page
I. Introduction 548
  A. Element Abundances and Nuclear Structure 548
  B. Four Theories of the Origin of the Elements 550
  C. General Features of Stellar Synthesis 550
II. Physical Processes Involved in Stellar Synthesis, Their Place of Occurrence, and the Time-Scales Associated with Them 551
  A. Modes of Element Synthesis 551

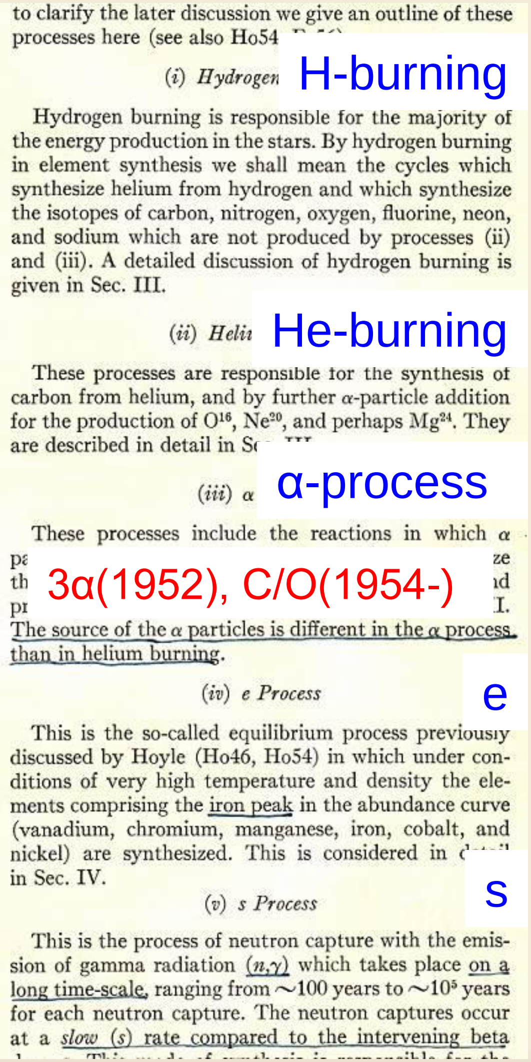
Classification of processes for nucleosynthesis,
Rev. Mod. Phys. 1957



552 BURBIDGE, BURBIDGE, FOWLER, AND HOYLE

Fig. I,2. A schematic diagram of the nuclear processes by which the synthesis of the elements in stars takes place. Elements synthesized by interactions with protons (hydrogen burning) are listed horizontally. Elements synthesized by interactions with alpha particles (helium burning) and by still more complicated processes are listed vertically. The details of the production of all of the known stable isotopes of carbon, nitrogen, oxygen, fluorine, neon, and sodium are shown completely. Neutron capture processes by which the highly charged heavy elements are synthesized are indicated by curved arrows. The production of radioactive Tc^{99} is indicated as an example for which there is astrophysical evidence of neutron captures at a slow rate over long periods of time in red giant stars. Similarly Cf^{254}, produced in supernovae, is an example of neutron synthesis at a rapid rate. The iron group is produced by a variety of nuclear reactions at equilibrium in the last stable stage of a star's evolution.

almost the same figure
in Fowler, B & B 1955



to clarify the later discussion we give an outline of these processes here (see also Ho54

(i) *Hydrogen*

H-burning

Hydrogen burning is responsible for the majority of the energy production in the stars. By hydrogen burning in element synthesis we shall mean the cycles which synthesize helium from hydrogen and which synthesize the isotopes of carbon, nitrogen, oxygen, fluorine, neon, and sodium which are not produced by processes (ii) and (iii). A detailed discussion of hydrogen burning is given in Sec. III.

(ii) *Heli*

He-burning

These processes are responsible for the synthesis of carbon from helium, and by further α-particle addition for the production of O^{16}, Ne^{20}, and perhaps Mg^{24}. They are described in detail in Sc

(iii) α

α-process

These processes include the reactions in which α
pa ze
th nd
pr I.

3α(1952), C/O(1954-)

The source of the α particles is different in the α process than in helium burning.

(iv) *e Process*

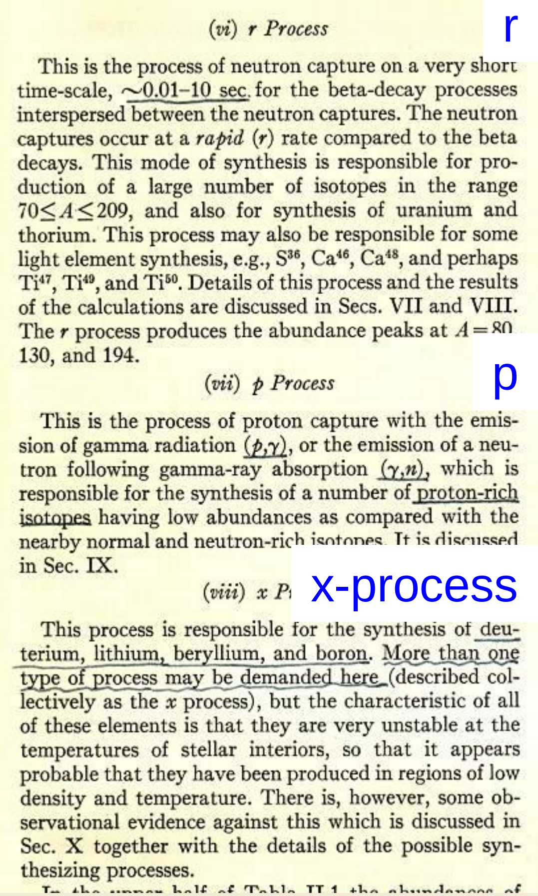
e

This is the so-called equilibrium process previously discussed by Hoyle (Ho46, Ho54) in which under conditions of very high temperature and density the elements comprising the iron peak in the abundance curve (vanadium, chromium, manganese, iron, cobalt, and nickel) are synthesized. This is considered in d in Sec. IV.

(v) *s Process*

s

This is the process of neutron capture with the emission of gamma radiation (n,γ) which takes place on a long time-scale, ranging from ∼100 years to $\sim 10^5$ years for each neutron capture. The neutron captures occur at a *slow* (*s*) rate compared to the intervening beta



(vi) *r Process*

r

This is the process of neutron capture on a very short time-scale, ∼0.01–10 sec. for the beta-decay processes interspersed between the neutron captures. The neutron captures occur at a *rapid* (*r*) rate compared to the beta decays. This mode of synthesis is responsible for production of a large number of isotopes in the range $70 \leq A \leq 209$, and also for synthesis of uranium and thorium. This process may also be responsible for some light element synthesis, e.g., S^{36}, Ca^{46}, Ca^{48}, and perhaps Ti^{47}, Ti^{49}, and Ti^{50}. Details of this process and the results of the calculations are discussed in Secs. VII and VIII. The *r* process produces the abundance peaks at $A=80$, 130, and 194.

(vii) *p Process*

p

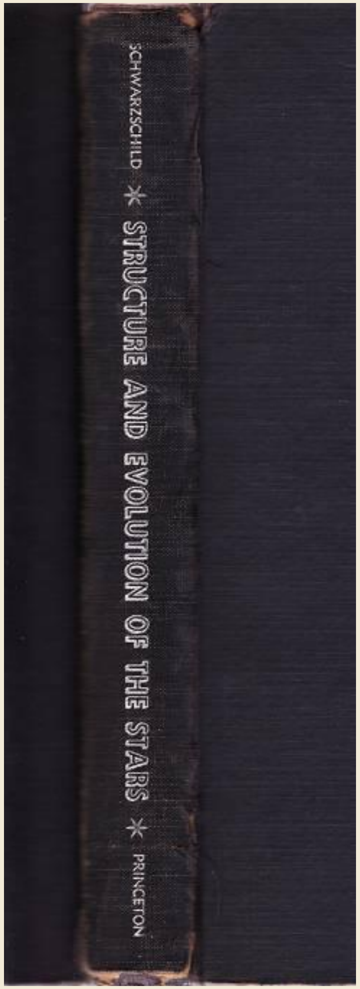
This is the process of proton capture with the emission of gamma radiation (p,γ), or the emission of a neutron following gamma-ray absorption (γ,n), which is responsible for the synthesis of a number of proton-rich isotopes having low abundances as compared with the nearby normal and neutron-rich isotopes. It is discussed in Sec. IX.

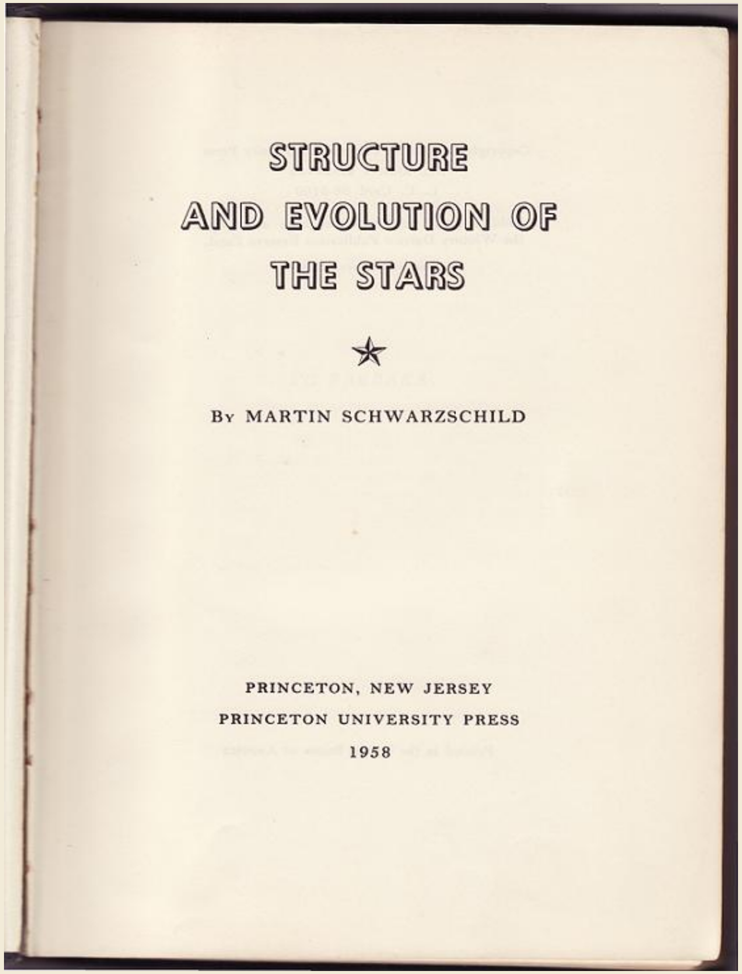
(viii) *x P*

x-process

This process is responsible for the synthesis of deuterium, lithium, beryllium, and boron. More than one type of process may be demanded here (described collectively as the *x* process), but the characteristic of all of these elements is that they are very unstable at the temperatures of stellar interiors, so that it appears probable that they have been produced in regions of low density and temperature. There is, however, some observational evidence against this which is discussed in Sec. X together with the details of the possible synthesizing processes.

Standard text book “Stellar Structure & Evolution of the Stars
M. Schwarzschild, 1958





STRUCTURE
AND EVOLUTION OF
THE STARS

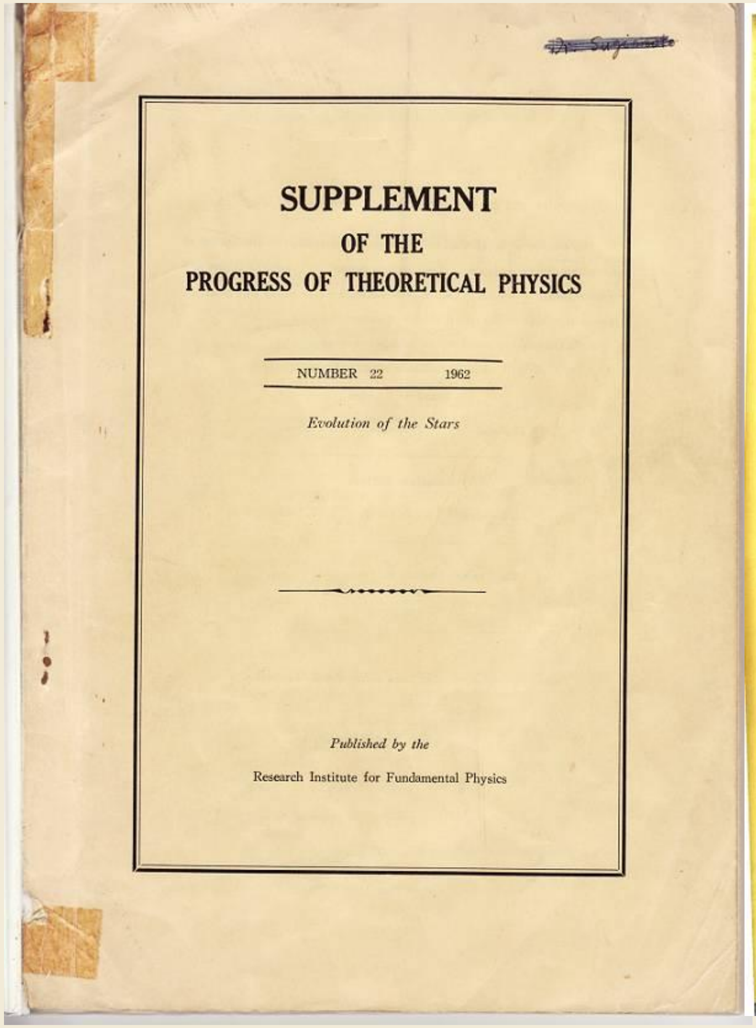
★

By MARTIN SCHWARZSCHILD

PRINCETON, NEW JERSEY
PRINCETON UNIVERSITY PRESS
1958

HHS for Evolution of the Stars

C. Hayashi, R.Hoshi & Sugimoto 1962



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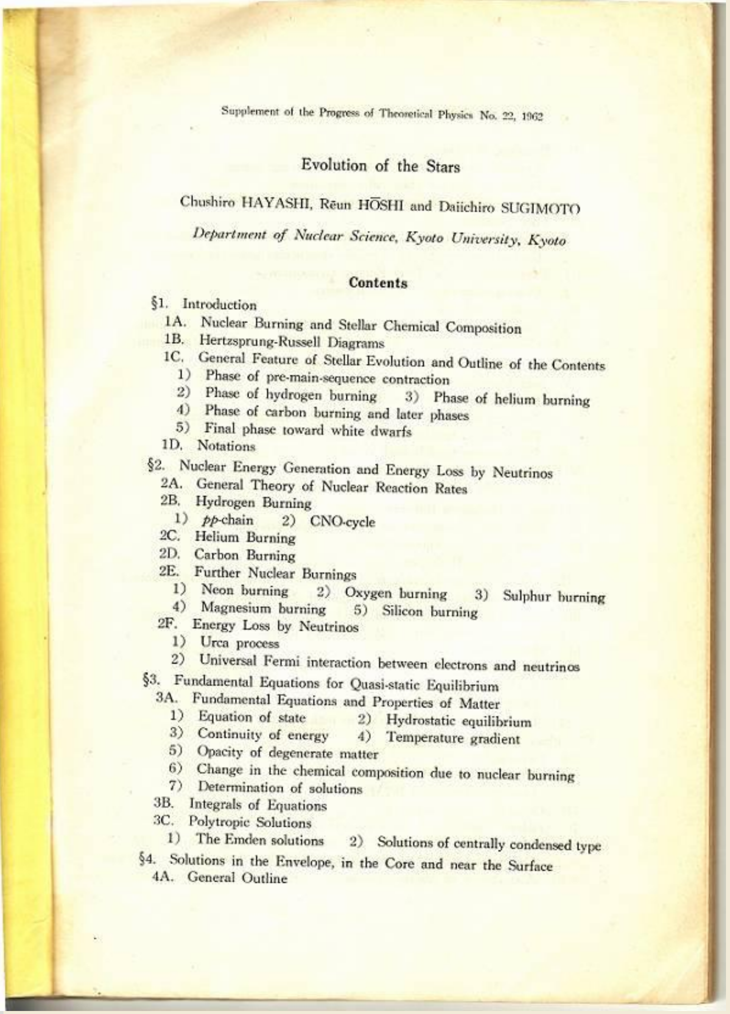
SUPPLEMENT
OF THE
PROGRESS OF THEORETICAL PHYSICS

NUMBER 22 1962

Evolution of the Stars

Published by the
Research Institute for Fundamental Physics



Supplement of the Progress of Theoretical Physics No. 22, 1962

Evolution of the Stars

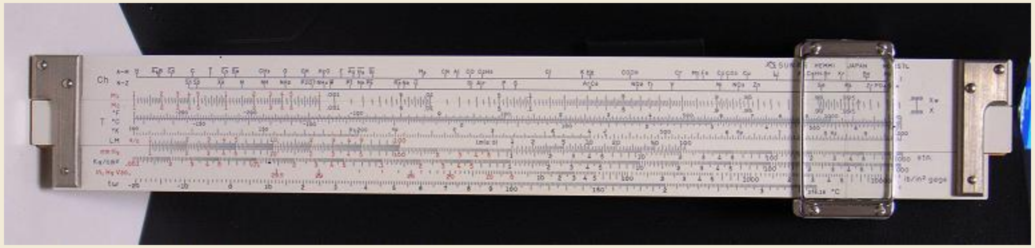
Chushiro HAYASHI, Rēun HŌSHI and Daiichiro SUGIMOTO

Department of Nuclear Science, Kyoto University, Kyoto

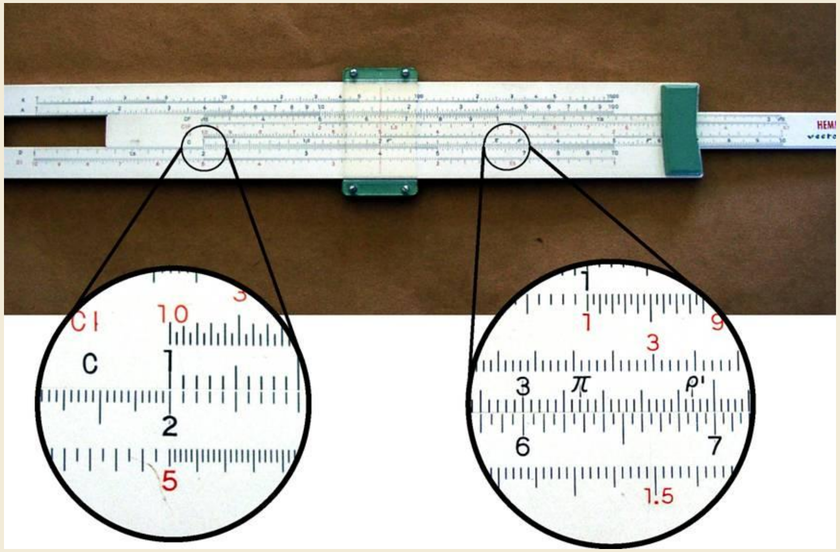
Contents

§1. Introduction
  1A. Nuclear Burning and Stellar Chemical Composition
  1B. Hertzsprung-Russell Diagrams
  1C. General Feature of Stellar Evolution and Outline of the Contents
    1) Phase of pre-main-sequence contraction
    2) Phase of hydrogen burning 3) Phase of helium burning
    4) Phase of carbon burning and later phases
    5) Final phase toward white dwarfs
  1D. Notations
§2. Nuclear Energy Generation and Energy Loss by Neutrinos
  2A. General Theory of Nuclear Reaction Rates
  2B. Hydrogen Burning
    1) *pp*-chain 2) CNO-cycle
  2C. Helium Burning
  2D. Carbon Burning
  2E. Further Nuclear Burnings
    1) Neon burning 2) Oxygen burning 3) Sulphur burning
    4) Magnesium burning 5) Silicon burning
  2F. Energy Loss by Neutrinos
    1) Urca process
    2) Universal Fermi interaction between electrons and neutrinos
§3. Fundamental Equations for Quasi-static Equilibrium
  3A. Fundamental Equations and Properties of Matter
    1) Equation of state 2) Hydrostatic equilibrium
    3) Continuity of energy 4) Temperature gradient
    5) Opacity of degenerate matter
    6) Change in the chemical composition due to nuclear burning
    7) Determination of solutions
  3B. Integrals of Equations
  3C. Polytropic Solutions
    1) The Emden solutions 2) Solutions of centrally condensed type
§4. Solutions in the Envelope, in the Core and near the Surface
  4A. General Outline

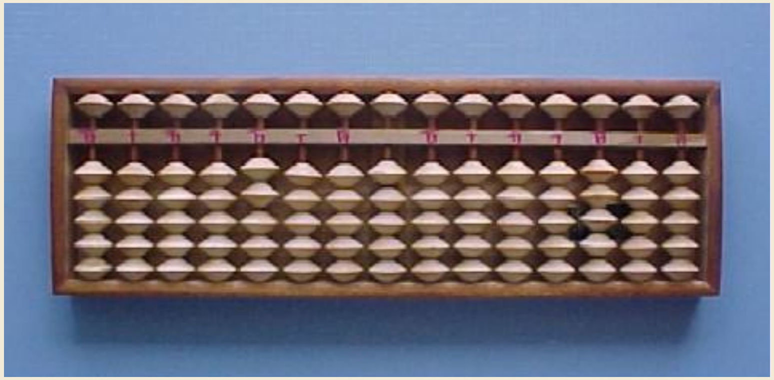
TOOLS



slide rule

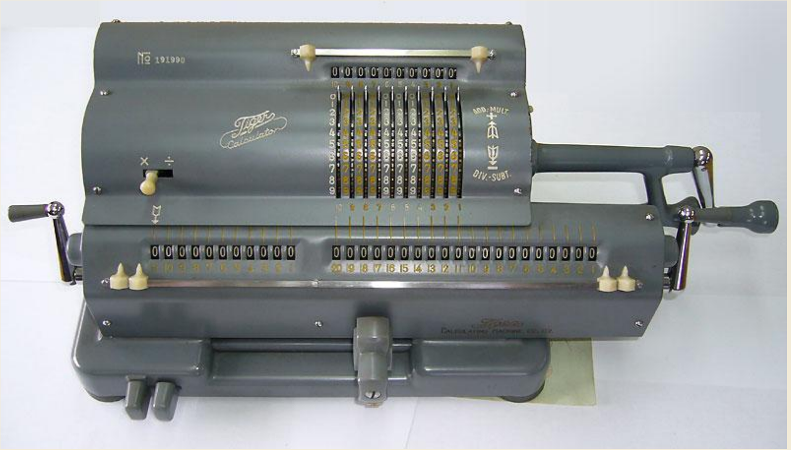


2 x 3 = 6



abacus

TOOLS-2
Machine (gear) calculators



TIGER
by hand

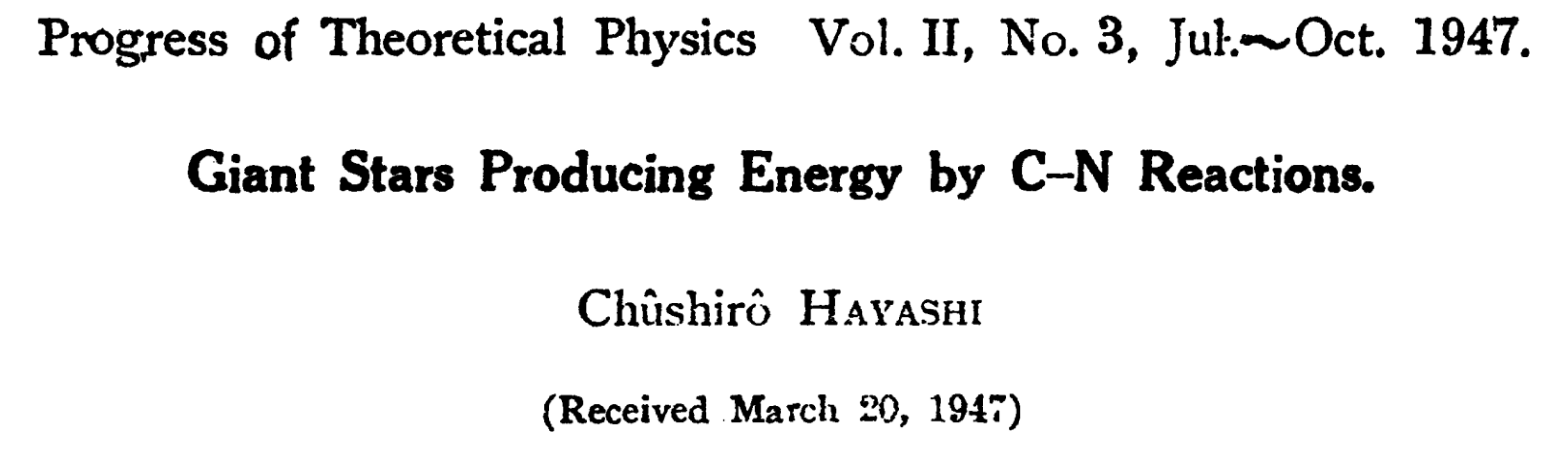
MONROE
electrically driven

Hayashi’s sayings (aphorisms)
林 語録

- Extend the problem as wide as possible
- Then, concentrate to the central problem
- Avoid unclear assumptions
- Return to physics (esp. to elementary processes)
- Construct a system from elementary processes

  (René Descartes: Discours de la méthode pour bien conduire sa raison, et chercher la vérité dans les sciences 方法序説)
- Construct a whole story (of evolution)

Electron deg core leads to red giant (1947)

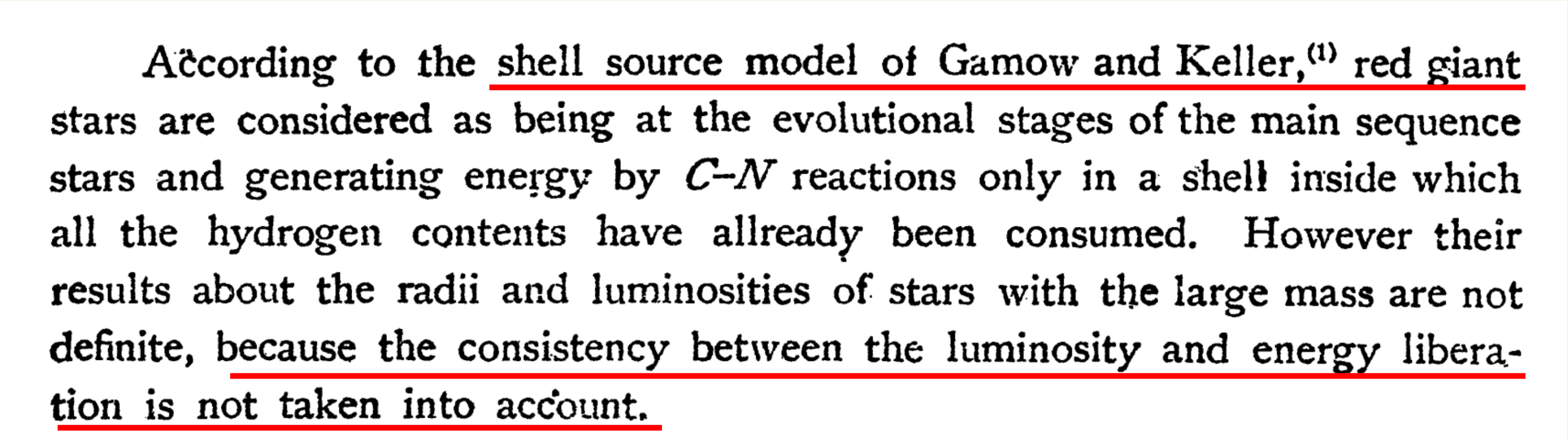


Progress of Theoretical Physics Vol. II, No. 3, Jul.∼Oct. 1947.

Giant Stars Producing Energy by C–N Reactions.

Chûshirô HAYASHI

(Received March 20, 1947)



According to the shell source model of Gamow and Keller,(1) red giant stars are considered as being at the evolutional stages of the main sequence stars and generating energy by *C–N* reactions only in a shell inside which all the hydrogen contents have allready been consumed. However their results about the radii and luminosities of stars with the large mass are not definite, because the consistency between the luminosity and energy liberation is not taken into account.

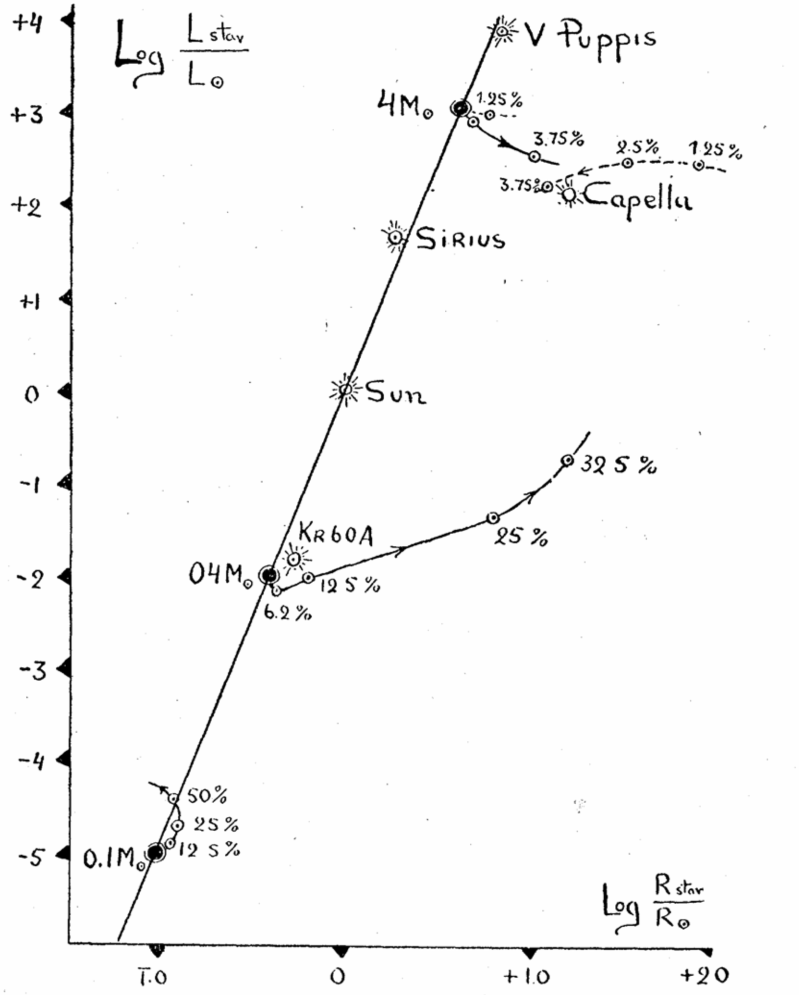
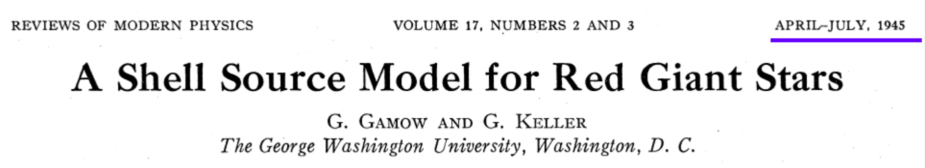


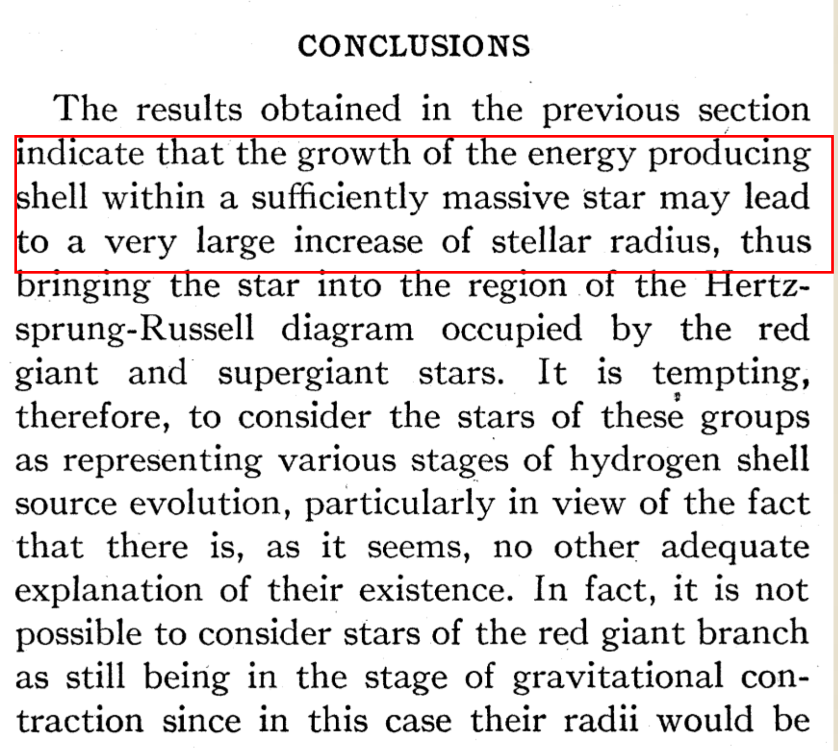
FIG. 8. Evolutionary tracks calculated for the stars of 0.1, 0.4, and 4 $M\odot$. The values of the radii of the stars on the 4 $M\odot$ curve are very uncertain and could be actually much larger than indicated.


REVIEWS OF MODERN PHYSICS VOLUME 17, NUMBERS 2 AND 3 APRIL–JULY, 1945

A Shell Source Model for Red Giant Stars

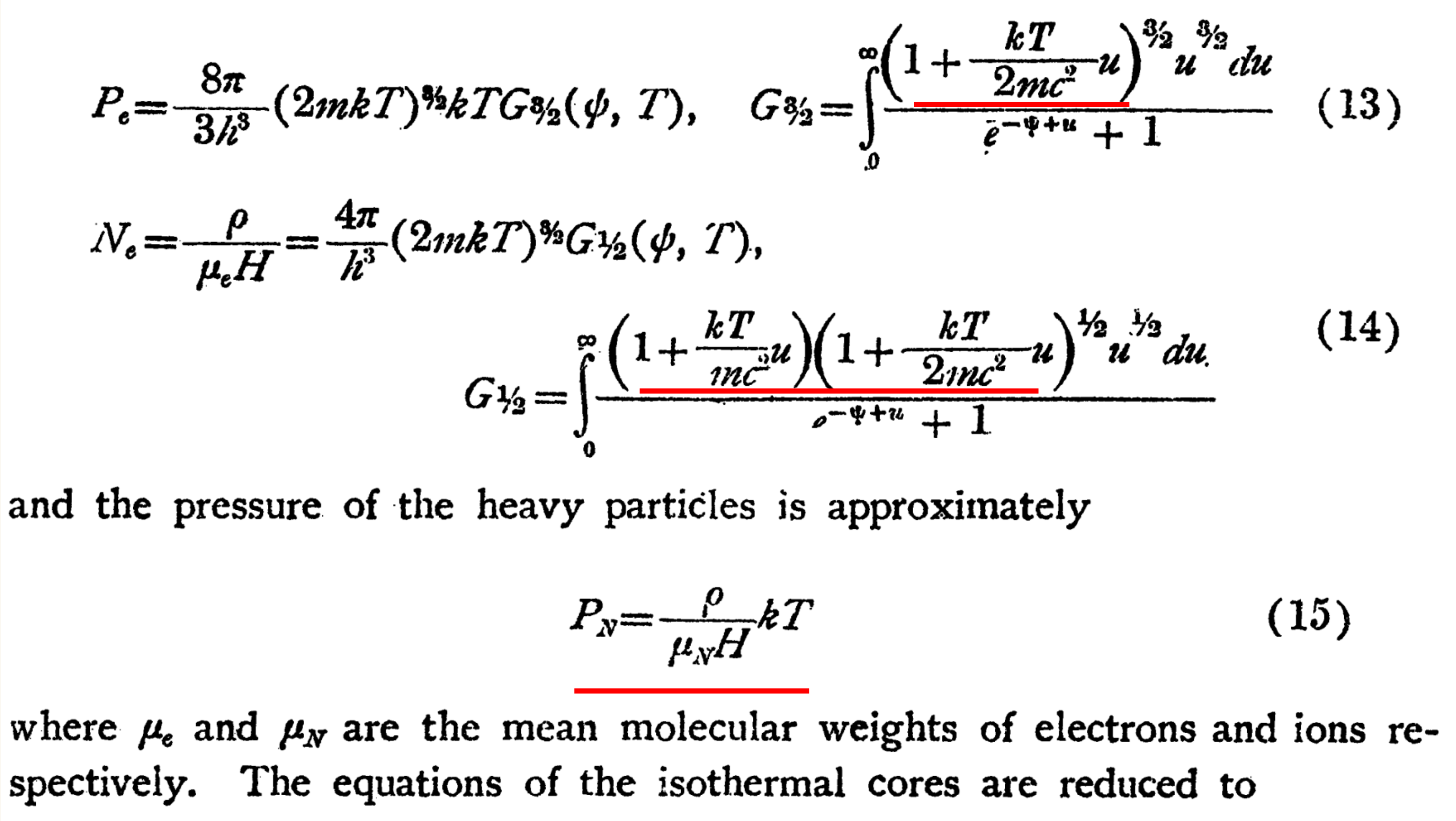
G. Gamow and G. Keller
The George Washington University, Washington, D. C.

1945: fixed $T^* = 2 \times 10^7\mathrm{K}$


CONCLUSIONS

The results obtained in the previous section indicate that the growth of the energy producing shell within a sufficiently massive star may lead to a very large increase of stellar radius, thus bringing the star into the region of the Hertzsprung-Russell diagram occupied by the red giant and supergiant stars. It is tempting, therefore, to consider the stars of these groups as representing various stages of hydrogen shell source evolution, particularly in view of the fact that there is, as it seems, no other adequate explanation of their existence. In fact, it is not possible to consider stars of the red giant branch as still being in the stage of gravitational contraction since in this case their radii would be

Included: (REL)
Degenerate electrons + ions
radiation pressure & pressure ionization



$$P_e = \frac{8\pi}{3h^3}(2mkT)^{3/2} kT G_{3/2}(\psi, T), \quad G_{3/2} = \int_0^\infty \frac{\underline{\left(1+\frac{kT}{2mc^2}u\right)}^{3/2} u^{3/2}\,du}{e^{-\psi+u}+1} \qquad (13)$$

$$N_e = \frac{\rho}{\mu_e H} = \frac{4\pi}{h^3}(2mkT)^{3/2} G_{1/2}(\psi, T),$$

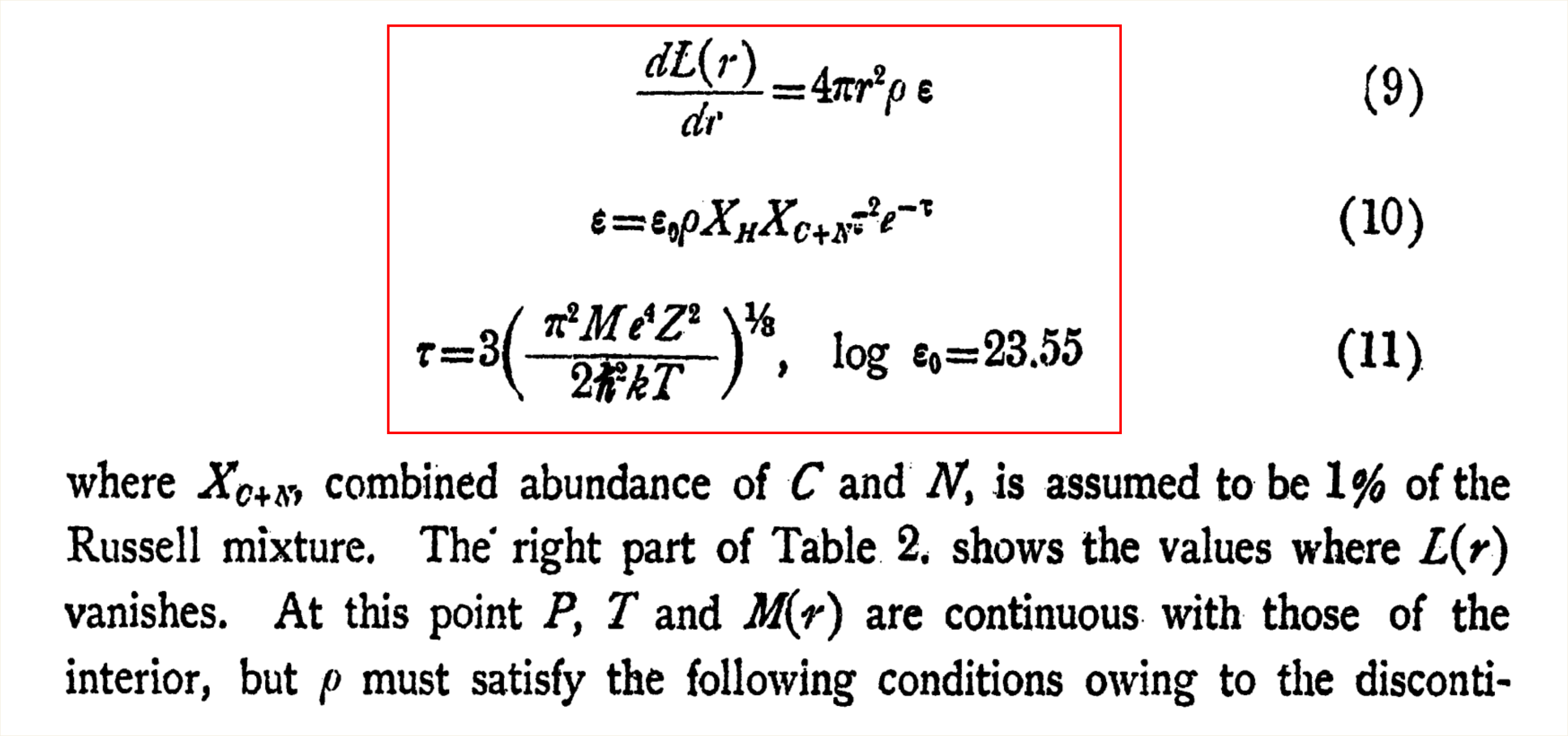
$$G_{1/2} = \int_0^\infty \frac{\underline{\left(1+\frac{kT}{mc^2}u\right)\left(1+\frac{kT}{2mc^2}u\right)}^{1/2} u^{1/2}\,du}{e^{-\psi+u}+1} \qquad (14)$$

and the pressure of the heavy particles is approximately

$$\underline{P_N = \frac{\rho}{\mu_N H} kT} \qquad (15)$$

where μ_e and μ_N are the mean molecular weights of electrons and ions respectively. The equations of the isothermal cores are reduced to

Integrated envelope solution inwards, subtracting nuclear energy generation until L(r)=0 to obtain the core-mass



$$\frac{dL(r)}{dr} = 4\pi r^2 \rho\, \varepsilon \qquad (9)$$

$$\varepsilon = \varepsilon_0 \rho X_H X_{C+N} \tau^2 e^{-\tau} \qquad (10)$$

$$\tau = 3\left(\frac{\pi^2 M e^4 Z^2}{2\hbar^2 kT}\right)^{1/3}, \quad \log\ \varepsilon_0 = 23.55 \qquad (11)$$

where X_{C+N}, combined abundance of C and N, is assumed to be 1% of the Russell mixture. The right part of Table 2. shows the values where $L(r)$ vanishes. At this point P, T and $M(r)$ are continuous with those of the interior, but ρ must satisfy the following conditions owing to the disconti-

mass fraction of the core M^*/M

Capella	0.047
Zeta Aur.	0.028

smaller than S-C limit (1942)

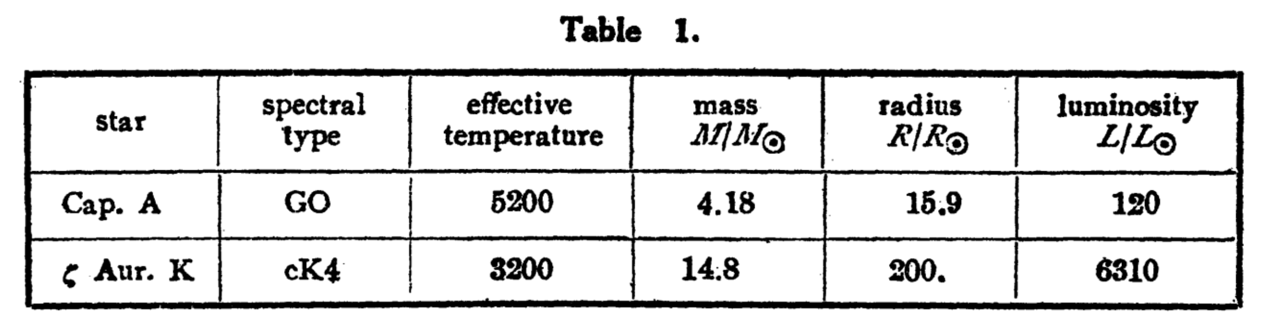


Table 1.

star	spectral type	effective temperature	mass $M/M_\odot$	radius $R/R_\odot$	luminosity $L/L_\odot$
Cap. A	GO	5200	4.18	15.9	120
ζ Aur. K	cK4	3200	14.8	200.	6310

logU*	log V*
-2.03	-1.28
-2.85	-1.19

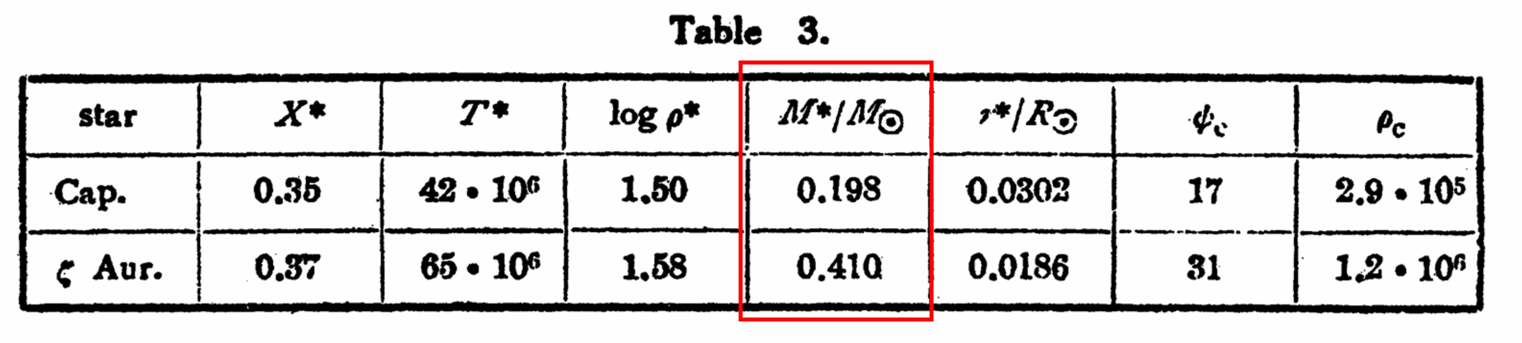
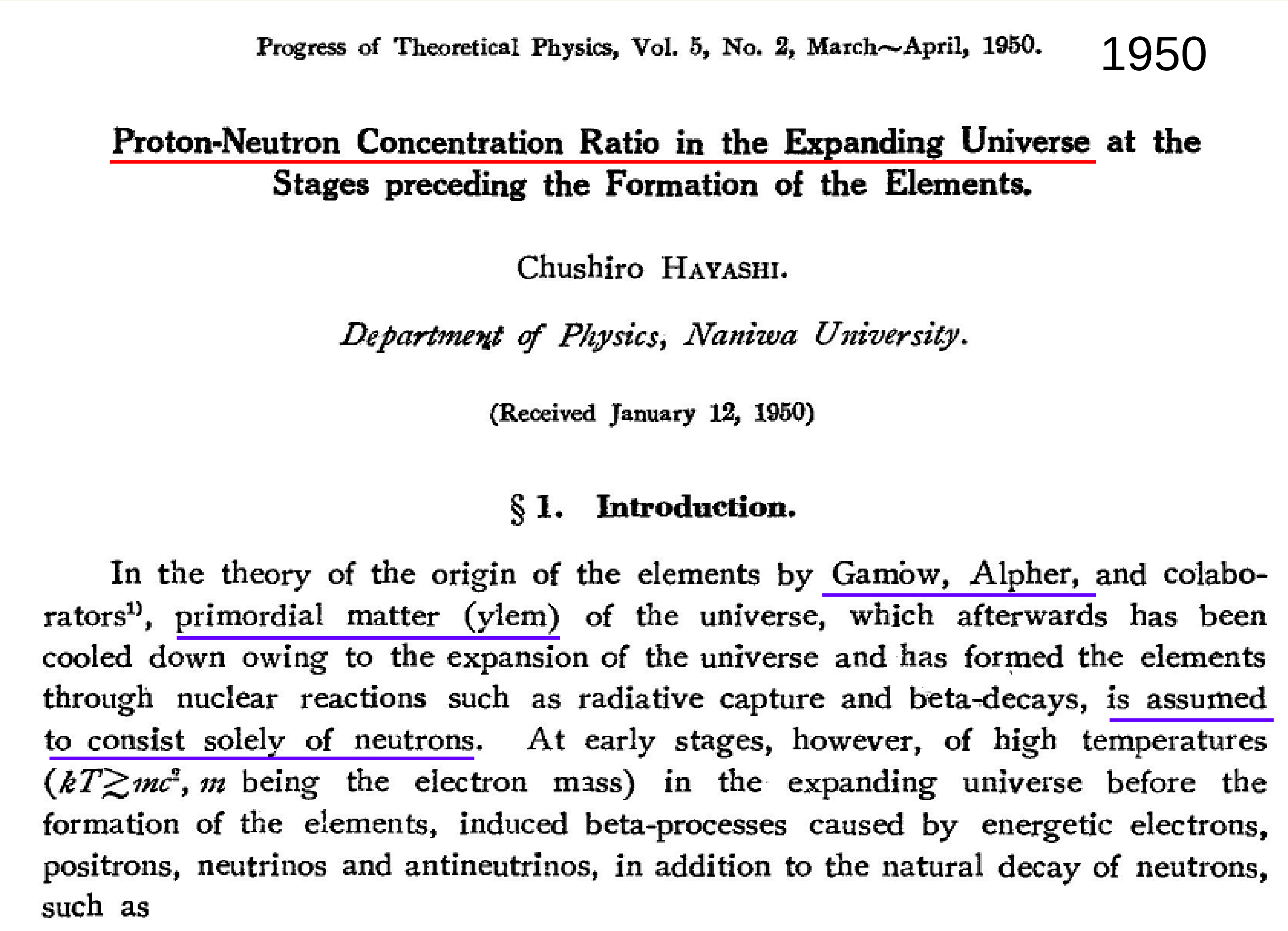


Table 3.

star	X^*	T^*	$\log \rho^*$	$M^*/M_\odot$	$r^*/R_\odot$	ψ_c	ρ_c
Cap.	0.35	$42 \cdot 10^6$	1.50	0.198	0.0302	17	$2.9 \cdot 10^5$
ζ Aur.	0.37	$65 \cdot 10^6$	1.58	0.410	0.0186	31	$1.2 \cdot 10^6$

M*/M
0.047
0.028

Hayashi C.,1947,PThPh,2,127



Progress of Theoretical Physics, Vol. 5, No. 2, March~April, 1950.

1950

Proton-Neutron Concentration Ratio in the Expanding Universe at the Stages preceding the Formation of the Elements.

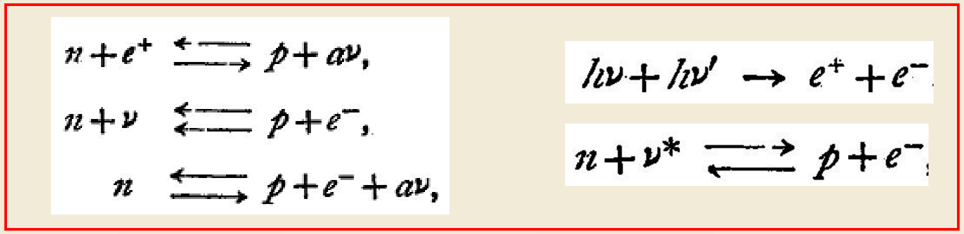
Chushiro HAYASHI.

Department of Physics, Naniwa University.

(Received January 12, 1950)

§ 1. Introduction.

In the theory of the origin of the elements by Gamow, Alpher, and colaborators1), primordial matter (ylem) of the universe, which afterwards has been cooled down owing to the expansion of the universe and has formed the elements through nuclear reactions such as radiative capture and beta-decays, is assumed to consist solely of neutrons. At early stages, however, of high temperatures ($kT \gtrsim mc^2$, m being the electron mass) in the expanding universe before the formation of the elements, induced beta-processes caused by energetic electrons, positrons, neutrinos and antineutrinos, in addition to the natural decay of neutrons, such as



$$n + e^+ \rightleftarrows p + a\nu,$$
$$n + \nu \rightleftarrows p + e^-,$$
$$n \rightleftarrows p + e^- + a\nu,$$

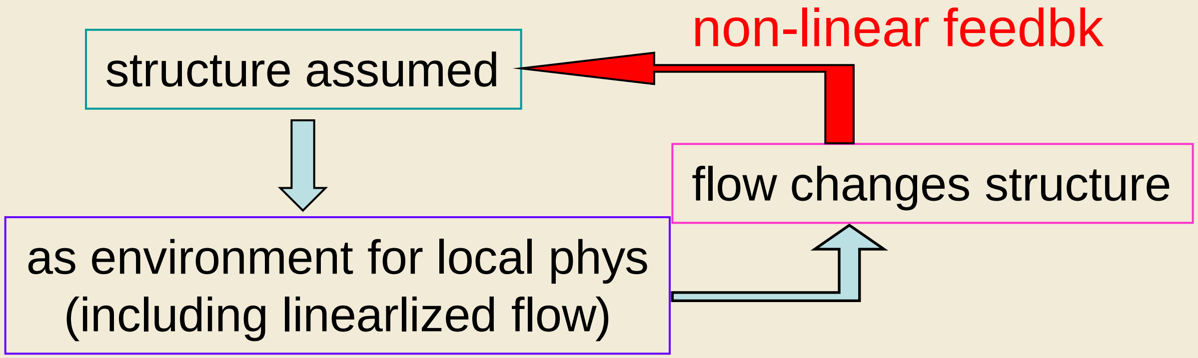
$$h\nu + h\nu' \rightarrow e^+ + e^-$$
$$n + \nu^* \rightleftarrows p + e^-$$

Three types of Physics

Type A) Local Physics
- micro processes under given environment (Descartes)
- e.g., p/n-ratio in early Universe, origin of the elements
  though the environment changes in time as specified by other principle

Type B) Physics including (spatial) structure
- characteristic of Astronomy
- e.g., stellar spectra formed in stratified layers
  < structure given by other principle (incl perturbation method)

Type C) Global Physics for structure & its formation
- should be solved as a whole system (beyond Descartes)
  > behavior as a whole appears beyond the sum of local physics



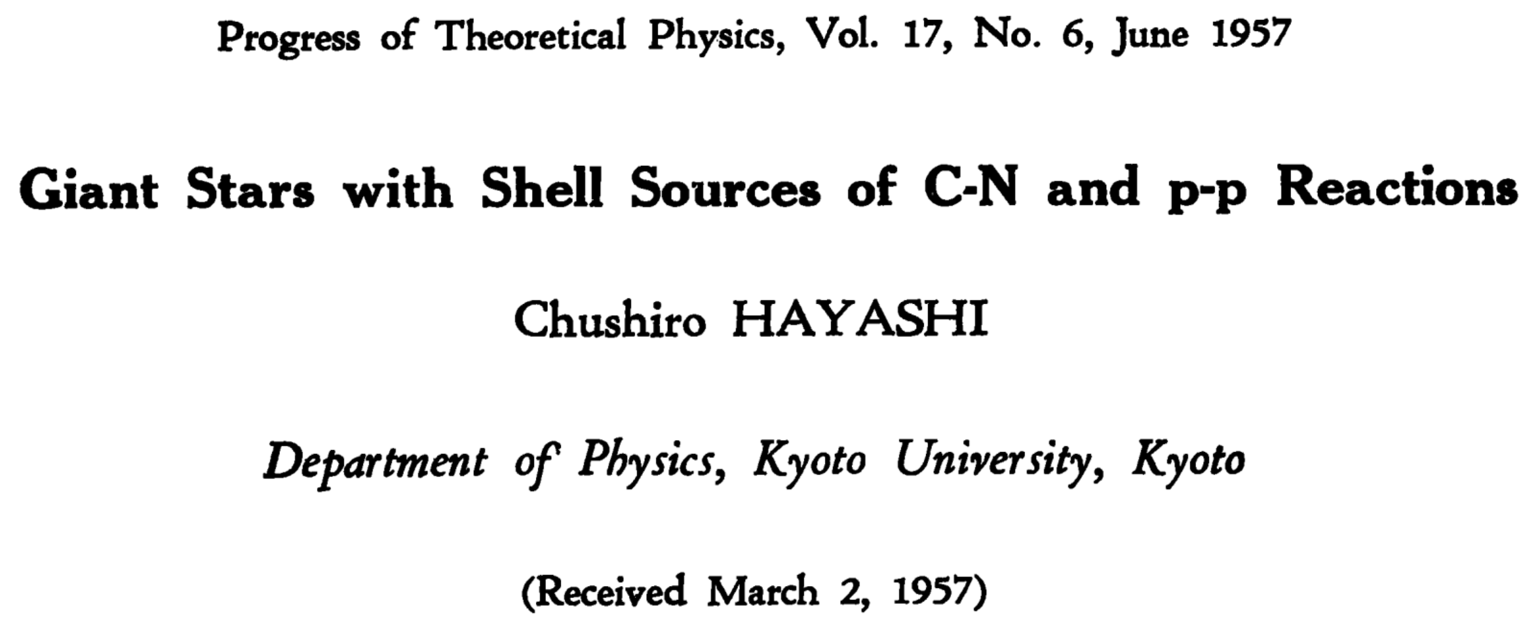
non-linear feedbk

structure assumed

flow changes structure

as environment for local phys
(including linearlized flow)

el-deg core; effect of non-deg ion (1957)



Progress of Theoretical Physics, Vol. 17, No. 6, June 1957

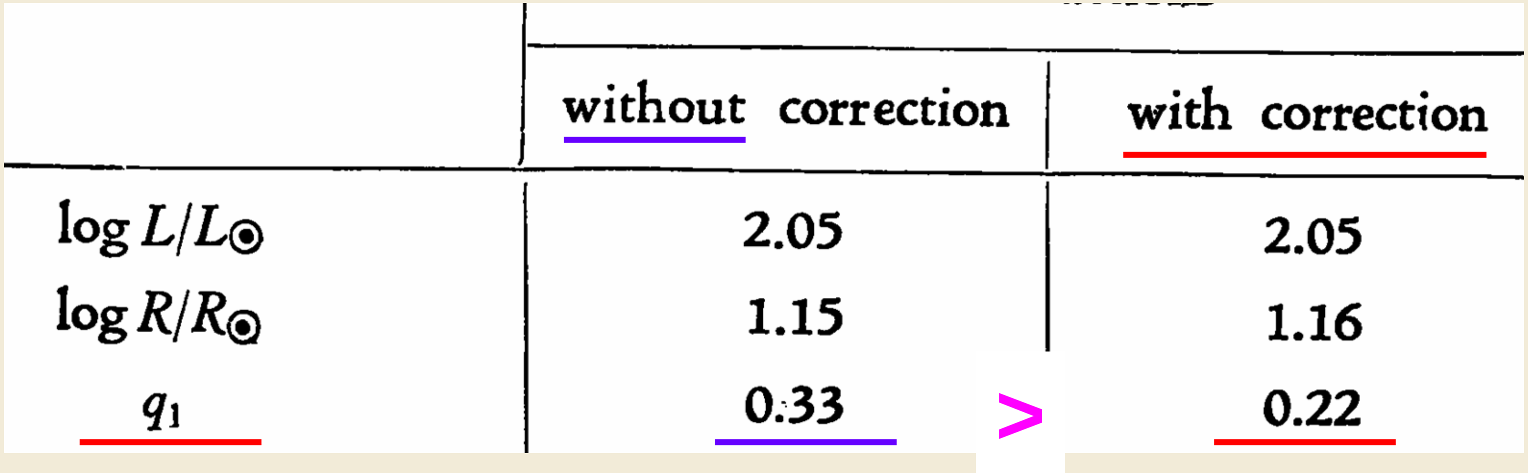
Giant Stars with Shell Sources of C-N and p-p Reactions

Chushiro HAYASHI

Department of Physics, Kyoto University, Kyoto

(Received March 2, 1957)

Ion pressure makes the core mass smaller?



	without correction	with correction
$\log L/L_\odot$	2.05	2.05
$\log R/R_\odot$	1.15	1.16
q_1	0.33 >	0.22

Hayashi C.,1947,PThPh,2,127

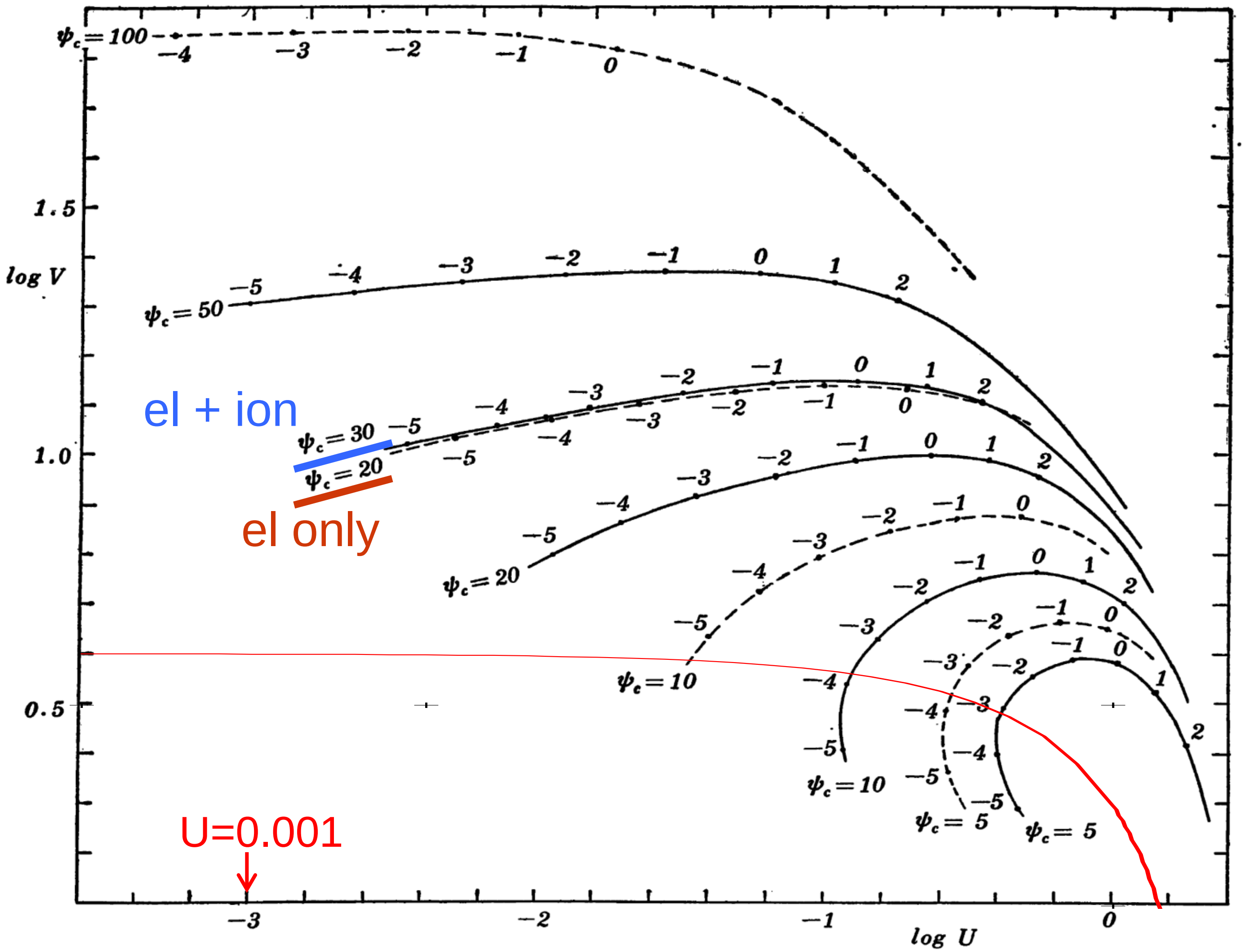
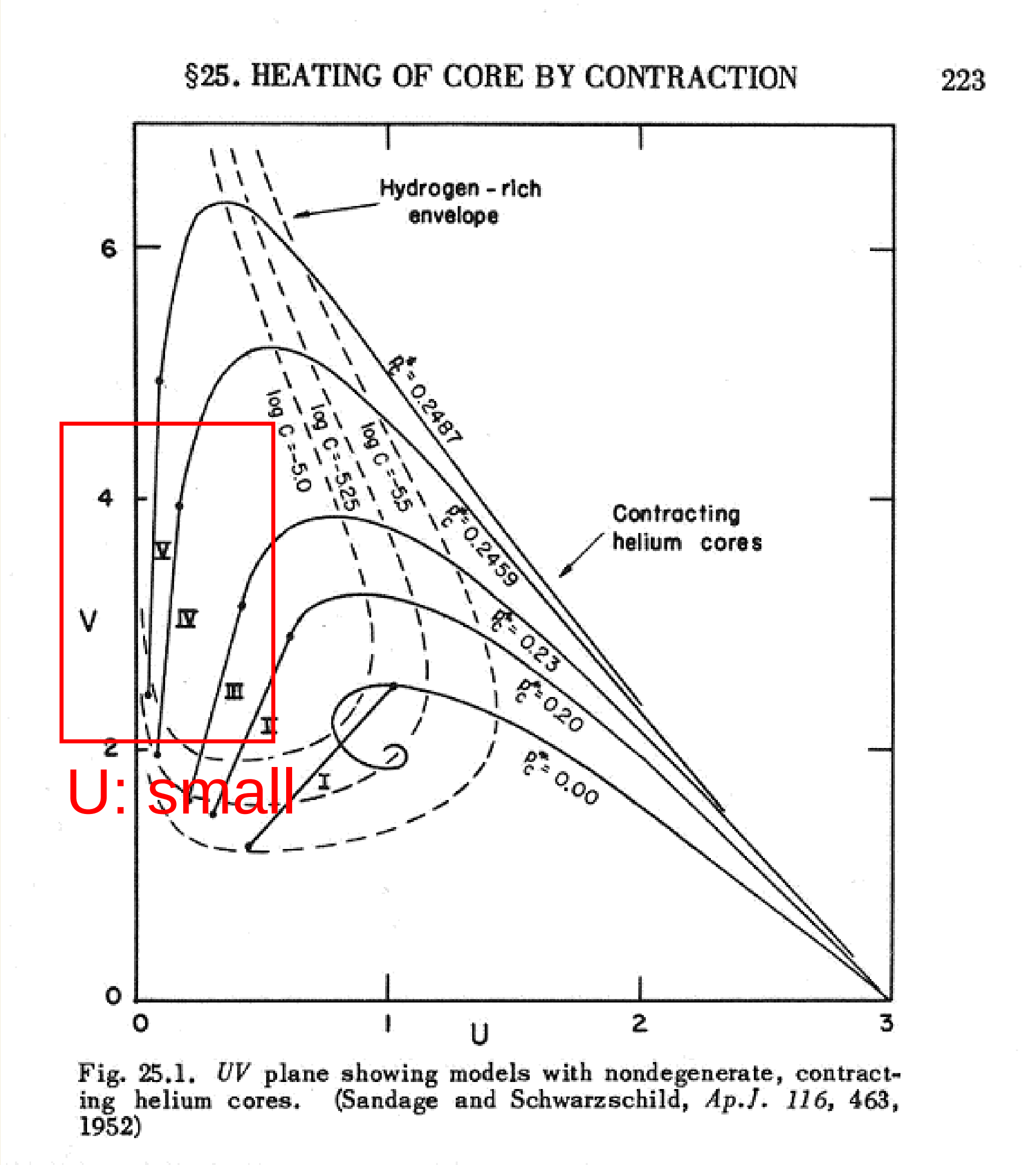


Fig. 1. U–V curves of the solutions of the partially degenerate isothermal cores. Full and dotted curves show the cases $\mu_n/\mu_e = 2$ and $\mu_n/\mu_e = \infty$, respectively. Values of ψ are shown on the points on these curves.

log U
vs
log V
plane

admired
by Martin
as
Hayashi's
invention

Hayachi C.,1957:PThPh,17,727



§25. HEATING OF CORE BY CONTRACTION 223

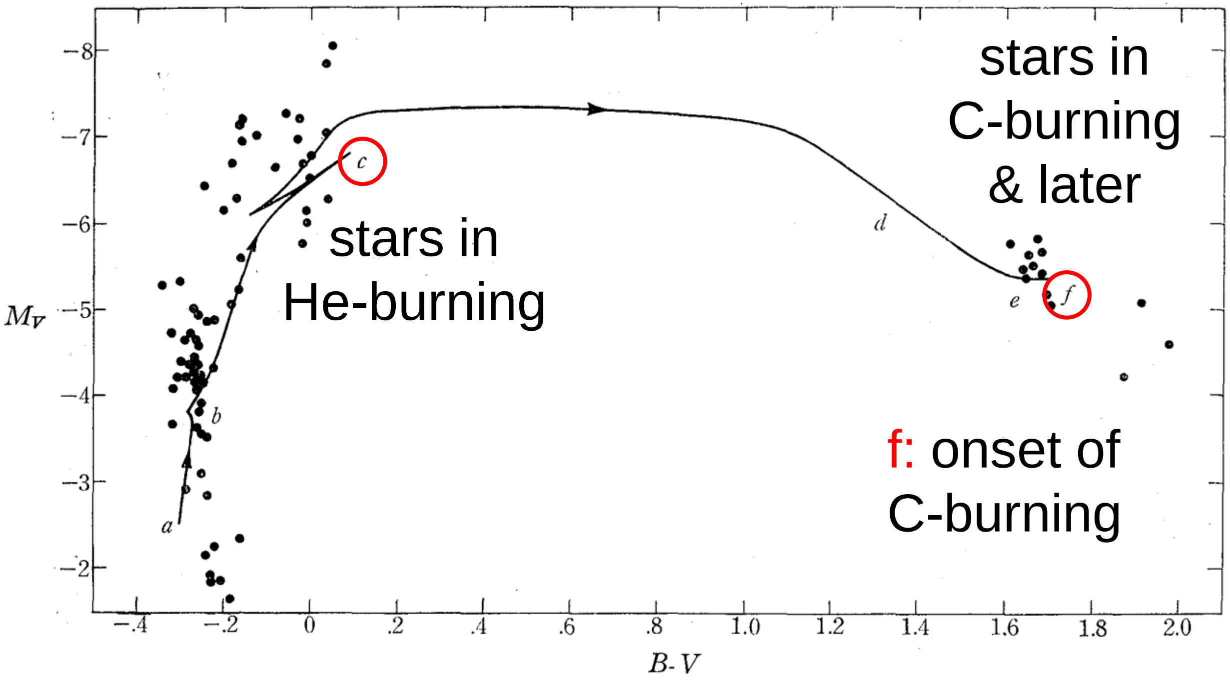
Fig. 25.1. *UV* plane showing models with nondegenerate, contracting helium cores. (Sandage and Schwarzschild, *Ap.J.* *116*, 463, 1952)

contracting He core
by Sandage
& Schwarzschild 1952

similar figure
for el-deg He core
(no ion-pressure)
by Schwarzschild
Rabinowitz & Härm1953

models with H-He
intermediate zone
(stability criterion)
by Härm
& Schwarzschild 1955

Hayashi & Cameron
15.6 $M_\odot$, 1962

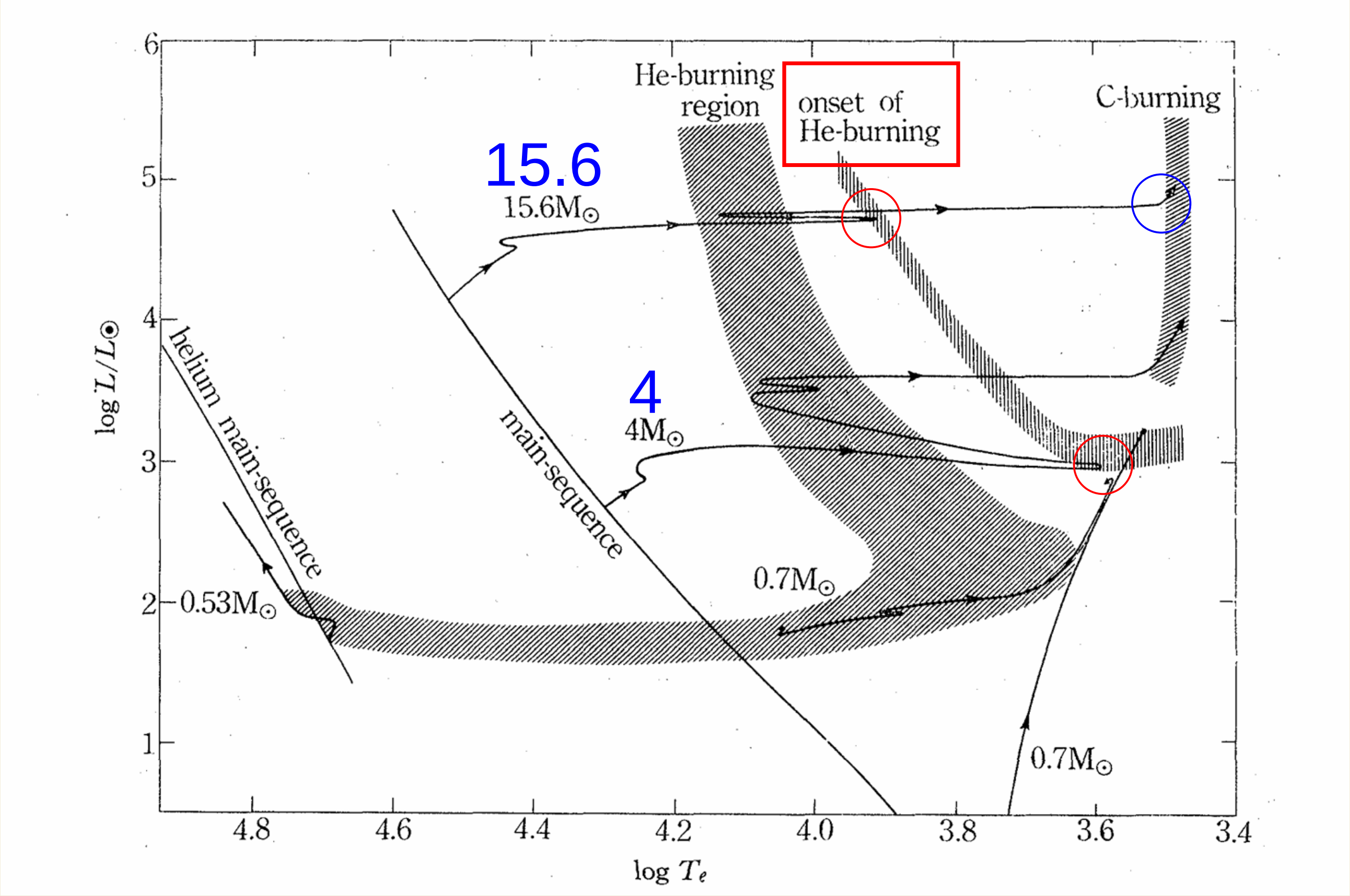


stars in He-burning

stars in C-burning & later

f: onset of C-burning

Fig. 7-6. Evolutionary track of a star of 15.6$M_\odot$ superposed on the color-magnitude diagrams of h and χ Persei. Segments of the track correspond to the phases: a–b, hydrogen depletion in the core; b–c, contracting helium core; c–d, helium depletion in the core; d–e, contracting carbon-oxygen core; and e–f carbon burning in the core.



HHS
(1962)

later comp → Red-giants of 15.6 $M_\odot$ contains stars in He burning
SN1978A → Star in the later phase could be a yellow giant,
if its envelope mass was lost in the preceding phase

To extend the evolution computation
through C-burning phase and beyond

it was necessary to formulate

the surface Boundary Conditions

including the effects of

convection

with finite efficiency of heat transport
co-existent with radiative transport,

incomplete ionization,

and opacity at low temperatures.

Boundary Condition fm photoshere outwards

photosphere: optical depth (↓with hydrostatic equil)

$$\frac{2}{3} = \int_{\mathrm{ph}}^{0} \mathrm{d}\tau = \int_{\mathrm{ph}}^{R} \kappa\rho \mathrm{d}r = \int_{0}^{\mathrm{ph}} \kappa \frac{r^2}{GM_r} \mathrm{d}P \simeq \frac{\langle\kappa\rangle R^2}{GM} P_{\mathrm{ph}}$$

homology variables: $P = p\frac{GM^2}{4\pi R^4}, \qquad \rho = f\frac{M}{4\pi R^3}$

effective polytropic index, and homology parameter:

$$N \quad \text{and} \quad Bp^N = f^{N+1}$$

eq of state only at the photosph: $P = \left(\frac{k}{\mu H}\right)\rho T$

luminosity: $L = 4\pi R^2 \sigma T_{\mathrm{eff}}^4$

HHS (1962)

$$\kappa = \kappa_0 P^{\alpha} T^{\gamma},$$

dimension ?

$$s = \frac{Xk}{H}\left\{(1+x)\left(\frac{5}{2}+\frac{\chi}{kT}\right)+2\ln\frac{x}{1-x} + \frac{5}{2}\delta + \delta\ln\frac{(8\pi H)^{3/2}(kT)^{5/2}(1+x+\delta)}{h^3 P\delta}\right\},$$

only

when $\Sigma_k \mu_k \mathrm{d}N_k = 0$

i.e., when in equil.

$$P_0 = (A/\kappa_0)^{1/(1+\alpha)},$$

result: P_0 almost const

dimension ?

↓

$$E = 4\pi K G^{3/2}(\mu H/k)^{5/2} M^{1/2} R^{3/2}.$$

$$K = \frac{P_d}{T_d^{5/2}}$$

E=45.46 ?

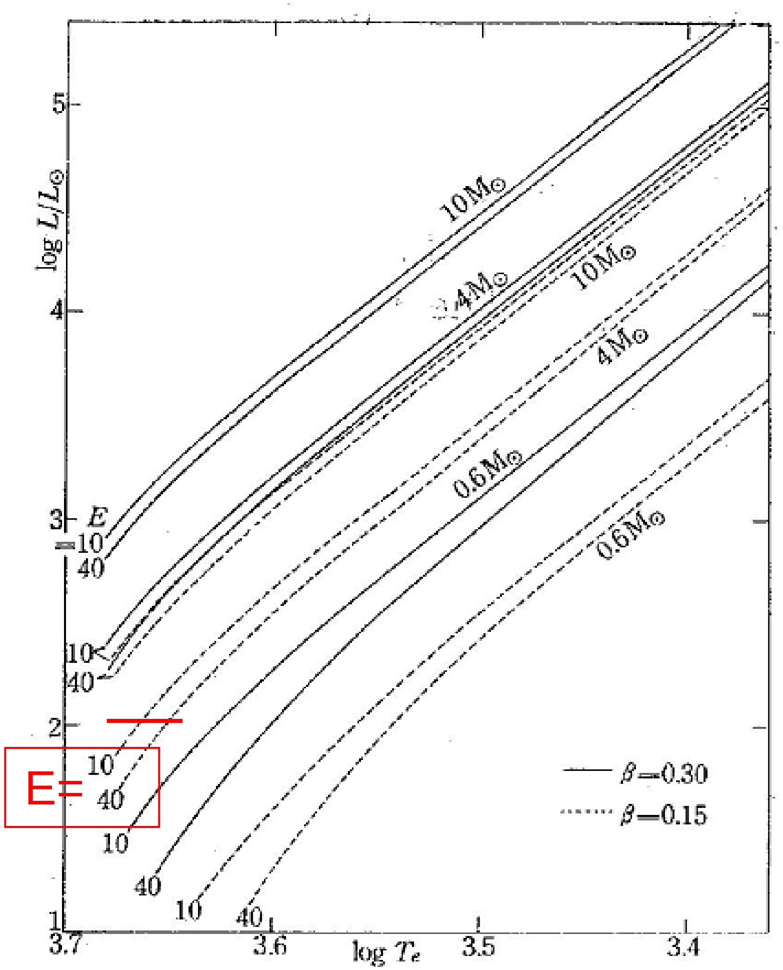


Fig. 4-13. Curves of $E=10$ and 40 in the HR diagram for given stellar masses (population I composition: $X=0.61$, $Y=0.37$, $Z=0.02$).

Success:
Hayashi lines
and Hayashi phase,
which opened
a new paradigm
for the origin
of the solar system

Hayashi lines
Hayashi & Hoshi (1961)

lower T_e
for larger E

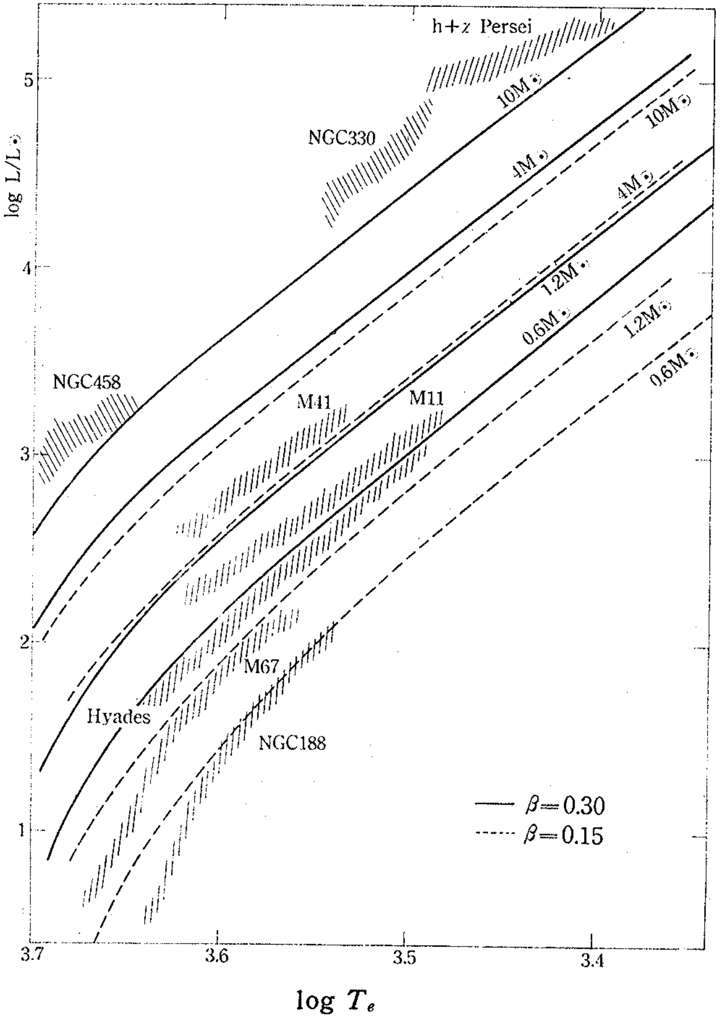
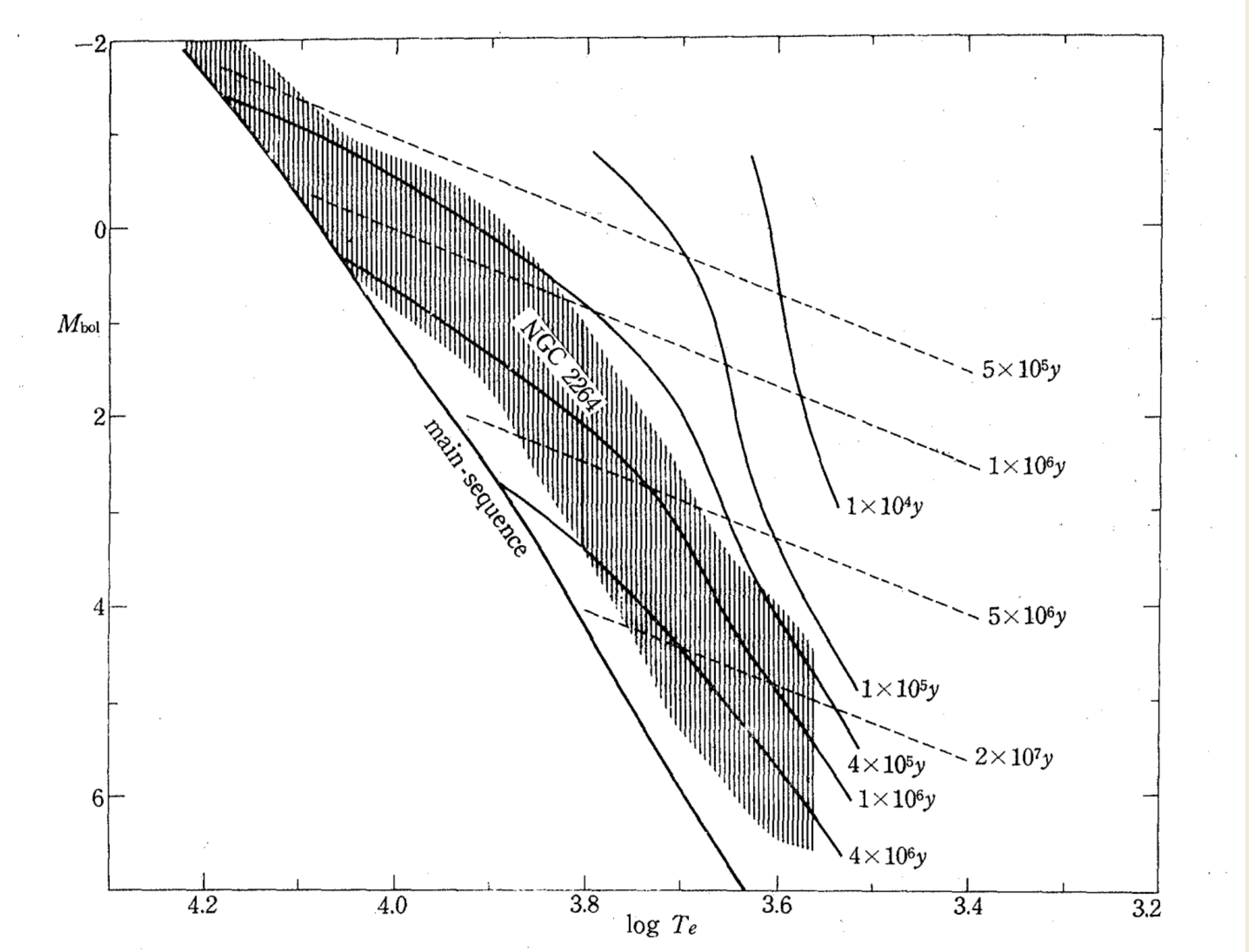


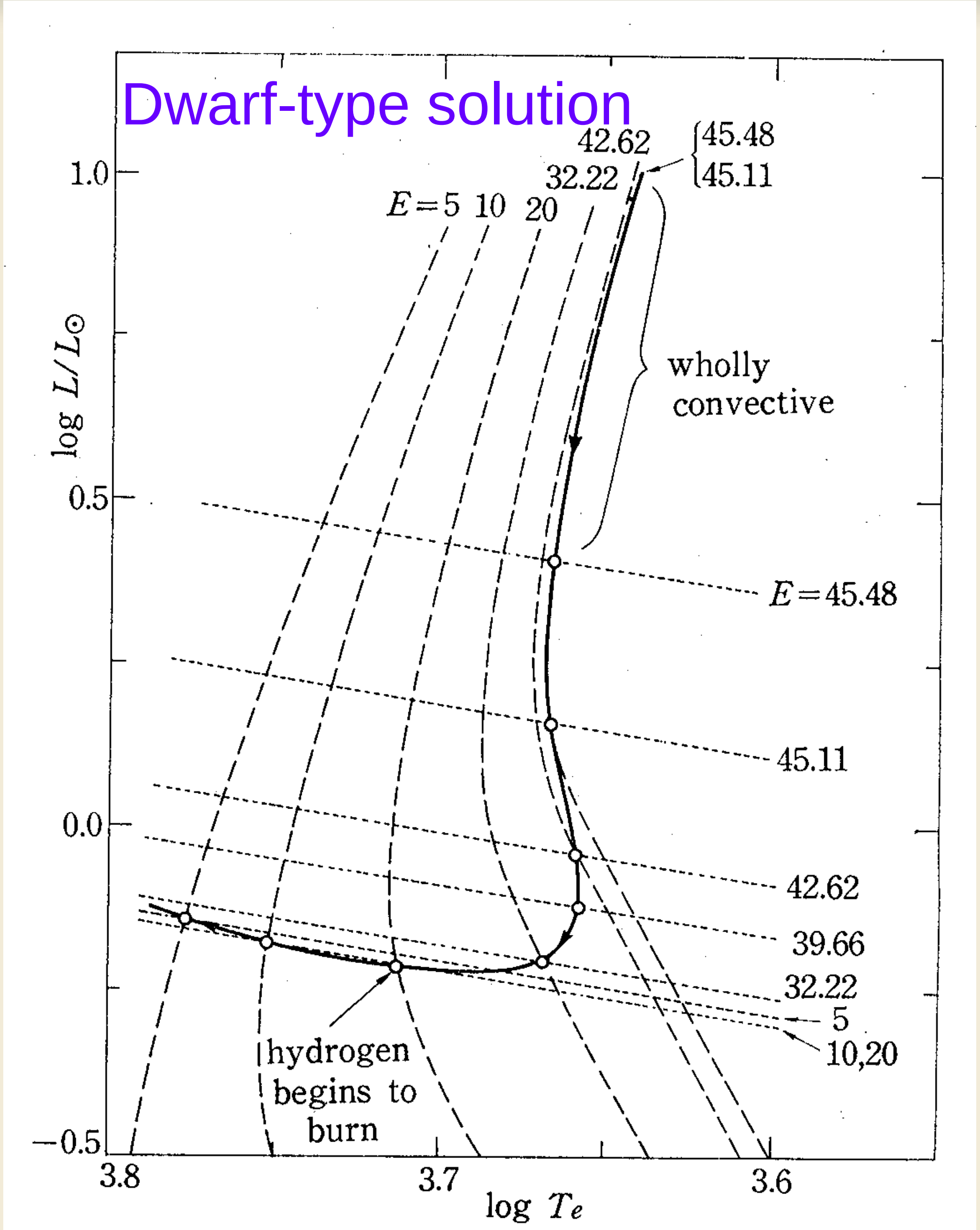
Fig. 4–16. Curves of $E=20$ for $\beta=0.15$ and 0.30 in the HR diagram as compared with the galactic clusters.

comparison with
observed giant branches
of some clusters
HHS (1962)

Comparison with young cluster NGC 2264
Hayashi, PASJ vol.13, pp. 450-452, 1961



Dwarf-type solution



Hayashi phase

pre-main-seq contr;
initially
wholly conv case

1 $M_\odot$

HHS (1962)

lower T_e
for
larger E

Hoyle & Schwarzschild 1955 for Globular Cluster

fitted to env sol

higher T_e for higher E

Described also in detail in Schwarzschild's book 1958

fitted also to core sol

(time sequence)

§24. GROWTH OF ISOTHERMAL CORE

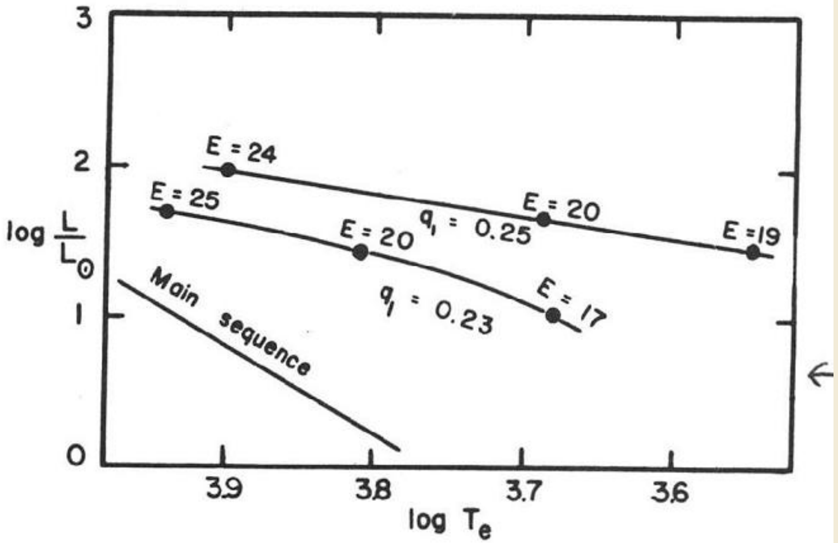


Fig. 24.2. Hertzsprung-Russell diagram representing two evolutionary phases with helium cores of moderate size, for various trial values of *E* (data from Table 24.1).

Giant-type env solution

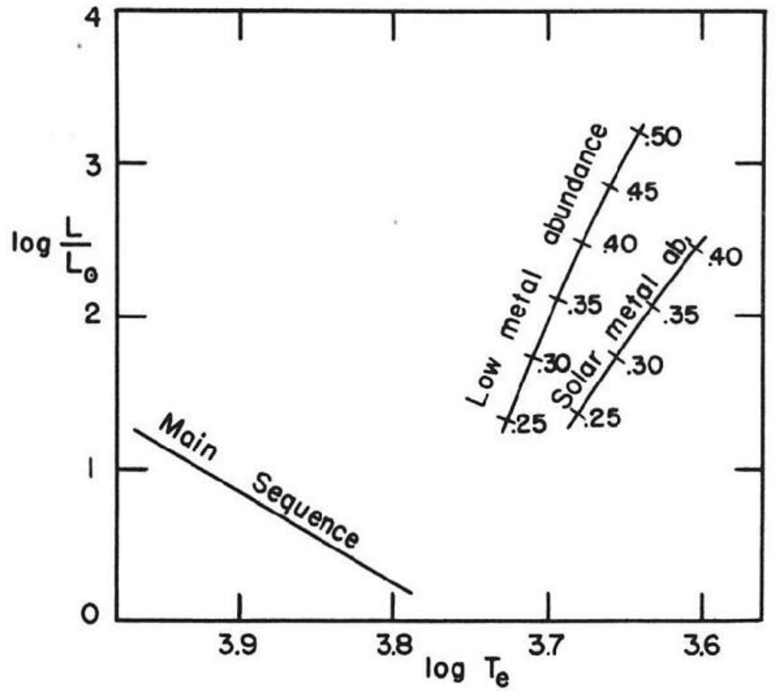


Fig. 24.3. Approximate evolutionary tracks in Hertzsprung-Russell diagram for stars of 1.2 $M_\odot$ in phases with large helium cores. The numbers give the mass fraction of the core, q_1, as it increases during the evolution. (Hoyle and Schwarzschild, *Ap.J.*, Supplement No. 13, 1955)

Hayashi lines for different $N_{\rm eff}$

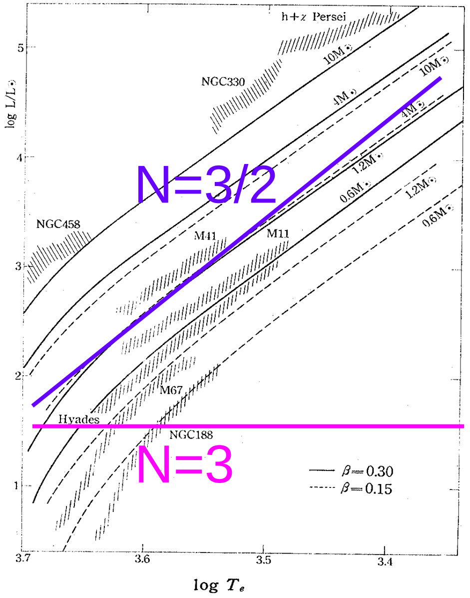


Fig. 4-16. Curves of $E=20$ for $\beta=0.15$ and 0.30 in the HR diagram as compared with the galactic clusters.

eliminate $R,\quad P_{\rm ph},\quad p_{\rm ph},\quad \rho_{\rm ph},\quad f_{\rm ph}$

result $L \sim T^{\boxed{\beta}}$:

$$\left(\frac{L}{4\pi\sigma}\right)^{(N-1)/2} = \left(\frac{4\pi}{B\langle\kappa\rangle}\right)\left(\frac{\mu H G}{k}\right)^{N+1} T_{\rm eff}^{N-3}$$

relation to homology parameter:

$N_{\rm eff}$:	13/4	3	2	3/2	1	0
β :	+2/9	0	-2	-6	∞	+6

$$\text{for}\quad N = 3/2\ :\ B = E$$

$$\text{for}\quad N = 3\quad\text{and}\quad \frac{1}{n+1} = \frac{3}{16\pi acG}\frac{P}{T^4}\frac{\kappa L_r}{M_r}\ :\ B = \frac{1}{4C_E}$$

Difference on E dependence comes from:

definition: $$E = 4\pi K G^{3/2} (\mu H/k)^{5/2} M^{1/2} R^{3/2}.$$ $$K = \frac{P_d}{T_d^{5/2}}$$

---

fitted to core: $$E = \exp(\sigma_c - \sigma_d) \underline{\left[\varphi_d^{1/2} \xi_d^{3/2} \left(\frac{M}{M_d} \right)^{1/2} \left(\frac{R}{R_d} \right)^{3/2} \right]}$$

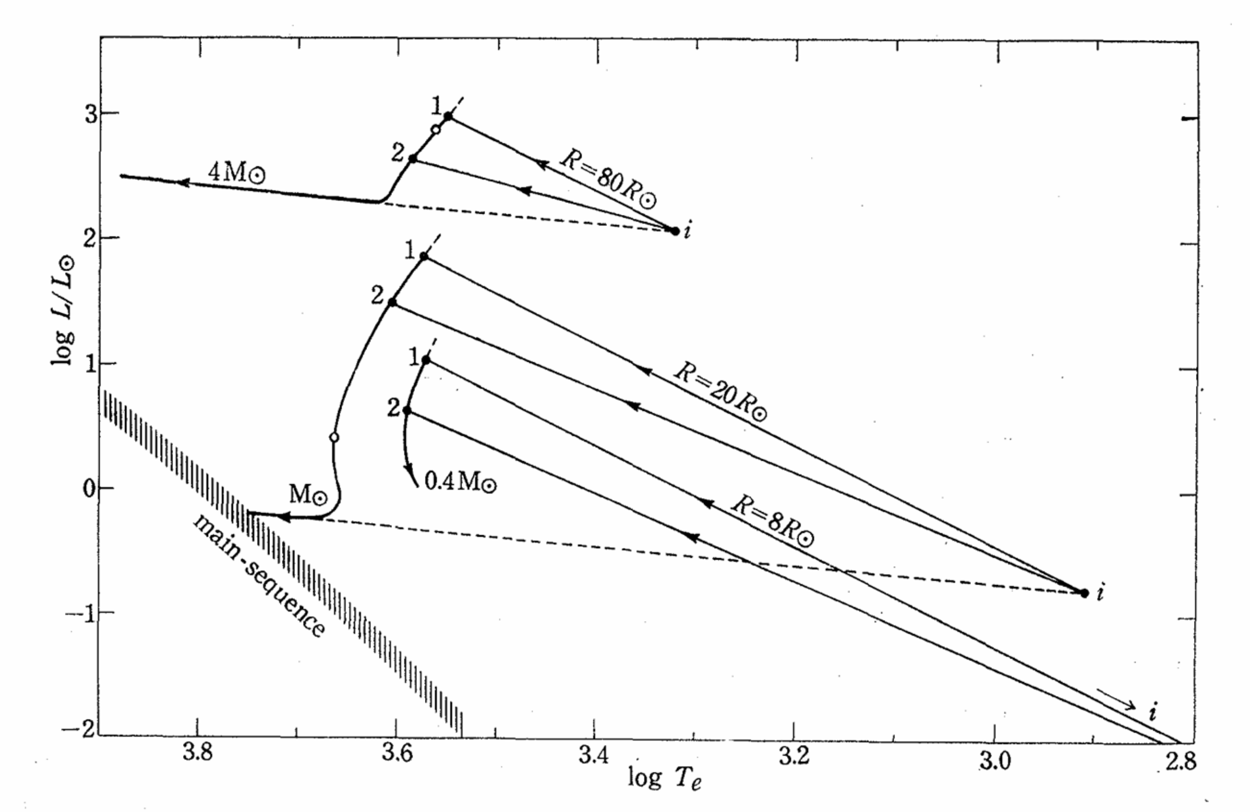
For dwarf-type sol: […] ~45. Radiative core results in smaller E
For giant-type sol: ξ very large but compensated by large diff inσ's

---

surface at pt *d* only: $$E = \frac{2(2\pi)^{1/2}}{(h/2\pi)^3} \left(\frac{\mu}{\mu^{(el)}} \right)^{3/2} \exp(5/2 - \sigma^{(el)}) M^{1/2} R^{3/2}$$
↑ phtosph cond

For R=const: smaller E→ largerσ→ higher $P_{\mathrm{rad}}/P_{\mathrm{gas}}$
→ more photons escape → higher L (Hayashi)

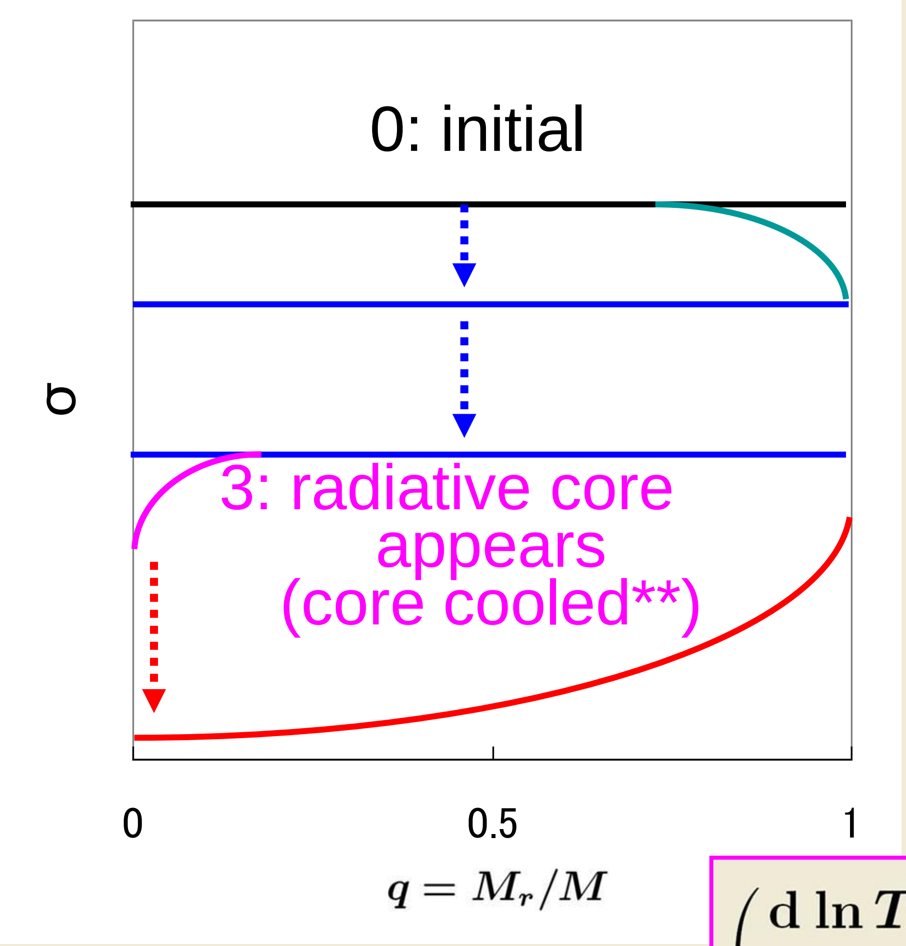
Evolution in the forbidden region ?



HHS (1962)

Not the dynamical evol, but thermal instability

Change of entropy distribution from birth through Hayashi phase



1: surface cooled*

2: convection
inwards
(Hayashi phase)

4: Henyey stage

---

* optically thin

** more transport
ideal gas & Kramers'

$$\left(\frac{\mathrm{d}\ln T}{\mathrm{d}\ln P}\right)_{\mathrm{rad}} \sim \frac{P}{T^4}\frac{\kappa L_r}{M_r} \sim \frac{\langle T\rangle}{T}\frac{1}{T^{5/2}}\Delta s$$

Why Prof. Hayashi so great

- Constructed not only the theory of stellar evolution and protostar formation, but also a systematic theory for the origin of the solar system, Kyoto model
- Awarded:

- for evolution of the stars (& protostar 1961-)

Eddington medal (1970); Imperial Prize of the Japanese Academy (1971)

- for origin of the Solar System 1970-

(promoted also cooperative research with geoscientists)

Order of Culture (1986); Order of Sacred Treasure, First Class (1994); Kyoto Prize (1995)

- for lifetime contribution, Bruce Medal (2004)

- Pioneering works (He began the fields before others can notice)

- Established the discipline of nuclear astrophysics in Japan

（Nurtured so many disciples）

References

- Astronomy & Geophysics, 51: 3.36. 24 MAY 2010, by D.Sugimoto
- Hayashi C.,Hoshi R.,SugimotoD.,1962:Evolution of the Stars,PThP,22,1 (HHS)
- Hayashi C.,1947:Giant Stars Producing Energy by C-N Reactions,PThPh,2,127
- Hayachi C.,1957:Giant Stars with Shell Sources of C=N and p-p Reactions,PThPh,17,727