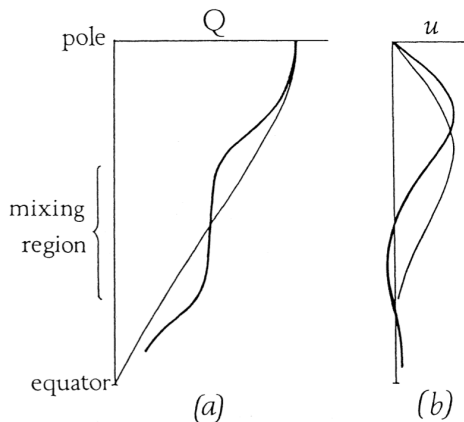


A general theorem on angular-momentum changes due to potential vorticity mixing

Jet sharpening as a result of inhomogeneous potential vorticity (PV) mixing plays an important role in jetstream formation in planetary atmospheres.

As an example consider this schematic from McIntyre 1982



Here most of the mixing is on the equatorward flank of an idealized stratospheric polar-night jet, in a broad midlatitude surf zone due to the breaking of Rossby waves arriving from below. The PV profile Q can be thought of as giving a somewhat blurred, zonally-averaged picture. The light and heavy curves are for conditions before and after the mixing event, where after means after the wave has largely decayed.

The correspondingly sharpened zonal velocity u is displayed for comparison.

It is well known that in simple examples of such a

scenario, the change in angular momentum is negative. This is to be expected, the missing angular momentum is supplied by the Rossby wave.

Such mixing scenarios involve an irreversible rearrangement of PV. A simple tool to model such irreversible rearrangements is to use an integral kernel or Green's function $R(y', y)$ in the standard way¹. Considering quasigeostrophic shallow-water on a β -plane, start from an initially zonally symmetric PV profile $q_i(y)$. The zonally averaged (but not necessarily zonally symmetric) PV distribution at a later time $\bar{q}_\ell(y)$ can be written

$$\bar{q}_\ell(y) = \int_{-L}^L dy' q_i(y') R(y', y) \quad (1)$$

¹e.g., Pasquill and Smith 1983; Fiedler 1984; Stull 1994; Plumb and McConalogue 1988; Shnirelman 1993; Thuburn and McIntyre 1997; Esler 2008

where $\int_{-L}^L dy R(y', y) = 1$ for all $y' \in [-L, L]$, (2)

$$\int_{-L}^L dy' R(y', y) = 1 \quad \text{for all } y \in [-L, L] , \quad (3)$$

and $R(y', y) \geq 0$ for all $y, y' \in [-L, L]$. (4)

The central result of this poster is that the resulting change in angular momentum ΔM can be written

$$\Delta M = \rho_0 H \int_{-L}^L dy q_\ell(y) y \quad (5)$$

$$= -\rho_0 H \int_{-L}^L dy' \frac{\partial q_i(y')}{\partial y'} \mathfrak{H}(y') , \quad (6)$$

where

$$\mathfrak{H}(y'; R(\cdot)) \geq 0 . \quad (7)$$

Furthermore, $\mathfrak{H}(y'; R(\cdot))$ is only equal to 0 in the trivial case of the PV being left unchanged. (see Wood and McIntyre 2009 on arXiv.org for the proof).

Equations (6)–(7) show that the change in momentum is always negative when an initially zonally symmetric PV profile has positive gradient, calling into

question recent research (e.g. Dunkerton and Scott 2008).

This result can easily be extended to include 2D vortex dynamics on a sphere as well as to a stratified quasigeostrophic model. However, an extension beyond quasigeostrophic models on to more accurate methods of balance presents a significant challenge.

Note that this result can be used to prove an extension to the Rayleigh-Kuo stability theorem where mixing is initially allowed. Unfortunately, it appears very likely that Arnol'd's theorems cannot be extended in the same way.

As illustrated in the figure, breaking Rossby waves in the flanks of a jet can cause the jet core to sharpen.

To see how general this is, we can once again use a quasigeostrophic shallow-water model with the integral kernel R . This time, R has the further restriction

$$R(y', y; y_0) = 0 \quad \text{if } y < y_0 < y' \quad \text{or} \quad y' < y_0 < y. \quad (8)$$

Hence mixing does not occur across the latitude y_0 , which represents the latitude of the jet core.

In this thought experiment, the net change $\Delta \bar{u}(y_0)$

in the zonal-mean zonal flow can be written

$$\Delta \bar{u}(y_0) = \int_{-L}^L dy' \frac{\partial q_i(y')}{\partial y'} \widehat{\mathbb{U}}(y', y_0) , \quad (9)$$

where in a similar way to before, but this time only for finite Rossby deformation length L_D ,

$$\widehat{\mathbb{U}}(y', y_0; R(\cdot)) \geq 0 . \quad (10)$$

(see Wood and McIntyre 2009 on arXiv.org for the proof). Hence in this model, the jet-core is always accelerated by mixing in the flanks of the jet.

This result has an important caveat: the jet must be zonally symmetric both before and after, although not during, the mixing event.