

**Workshop on Magneto-Rotational
Instability in Protoplanetary Disks**

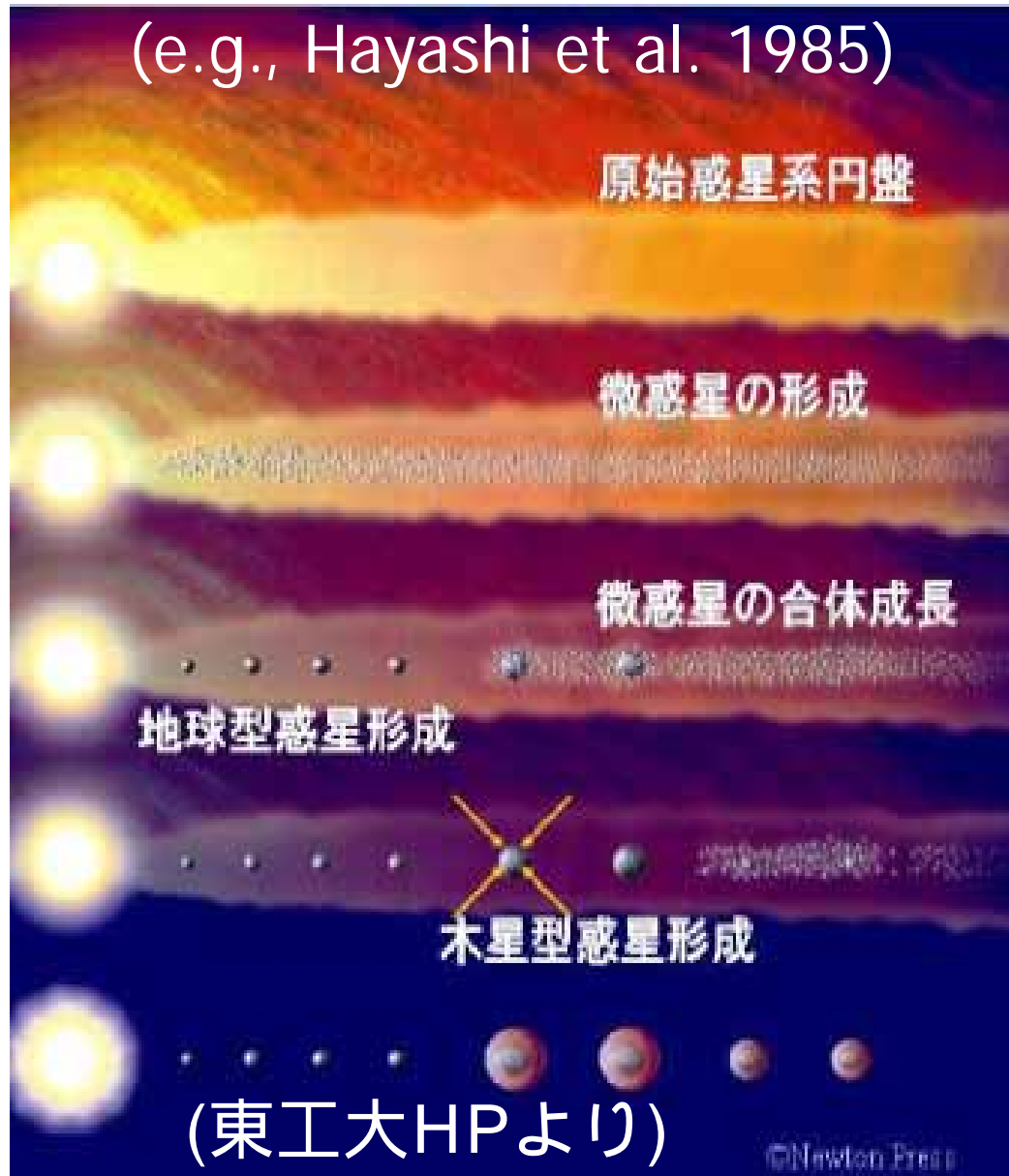
**Dust evolution in
protoplanetary disks:
Effect on observations
of dust emission**

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§ 1 Introduction

From protoplanetary disk to planets

(e.g., Hayashi et al. 1985)



**Dust size growth
& settling**

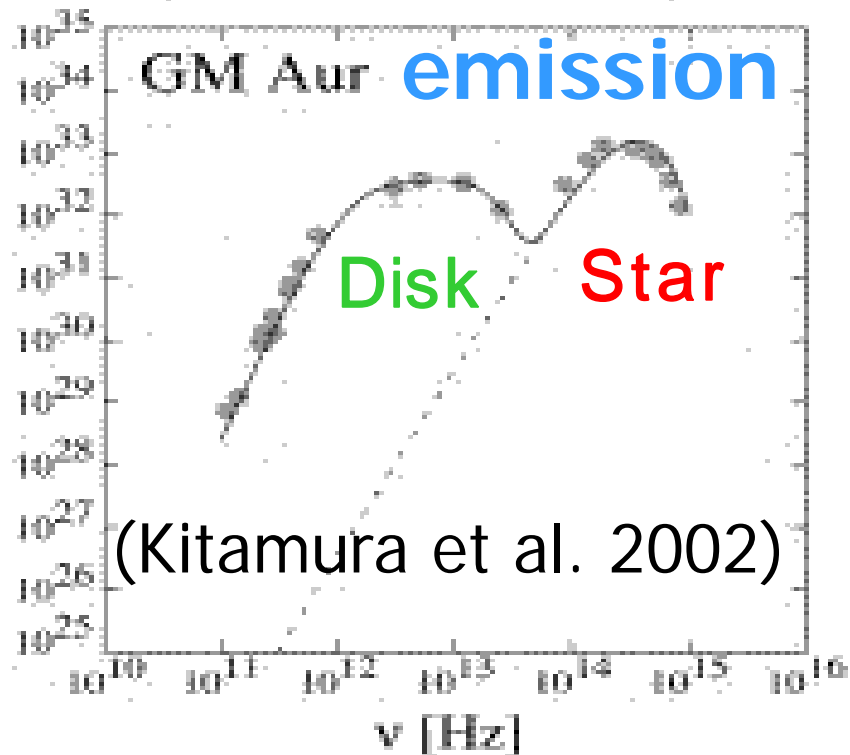
**Planetesimal
formation**

**Collisional growth,
Planet formation**

**Gas dispersal
Planetary
system formation**

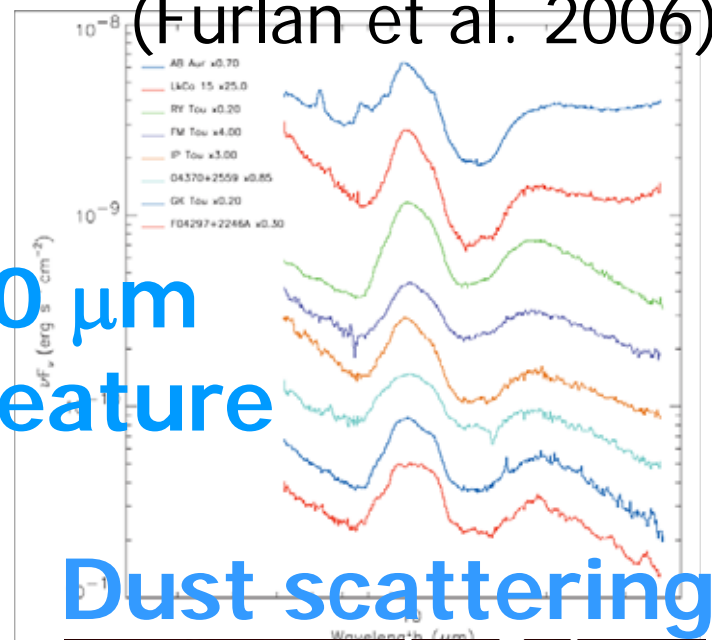
Obs. of Dust Emission from PPDs

Thermal dust



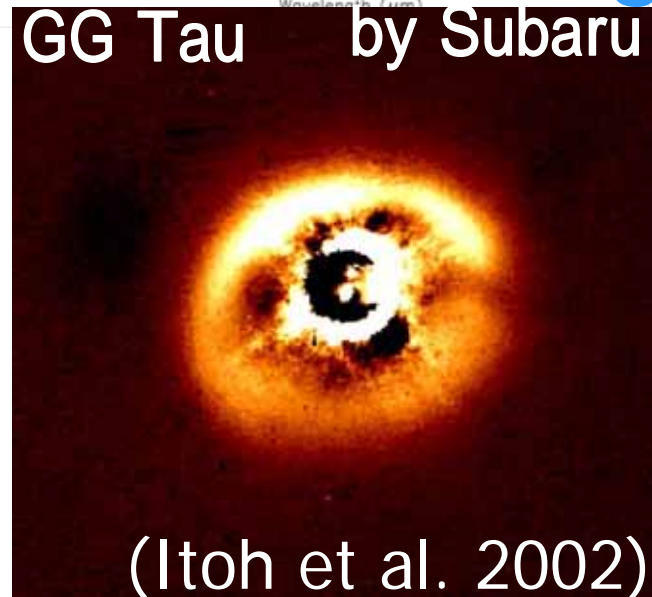
(Furlan et al. 2006)

10 μm
Si feature

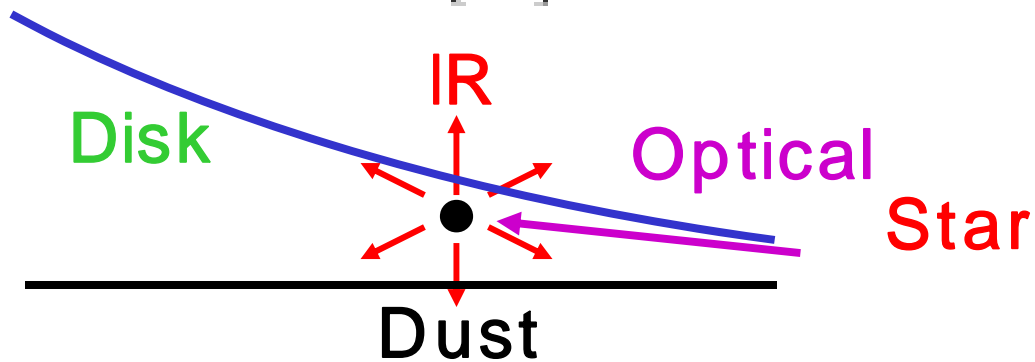


Dust scattering

GG Tau by Subaru

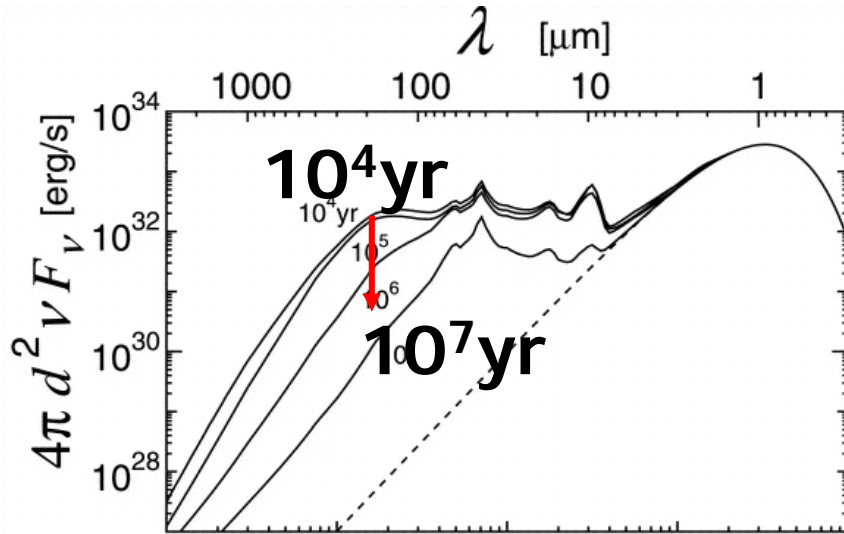


(Itoh et al. 2002)



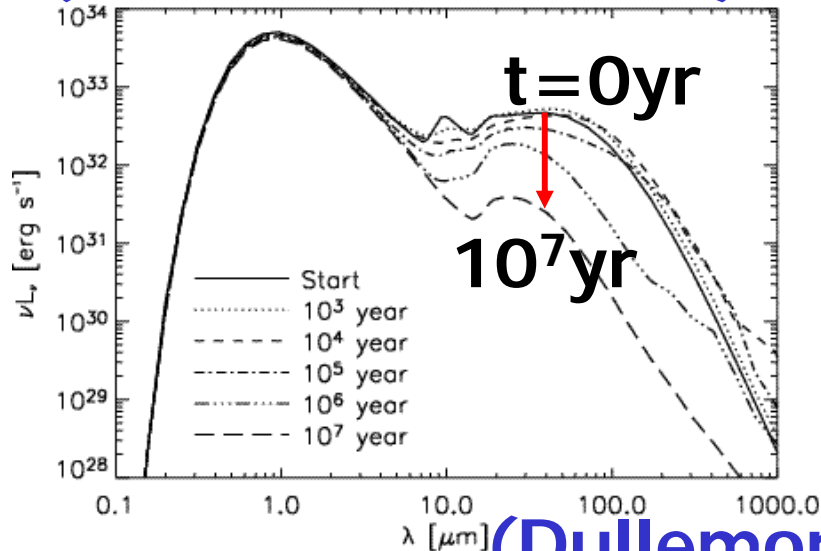
Dust Evolution & SED

Quiescent disk



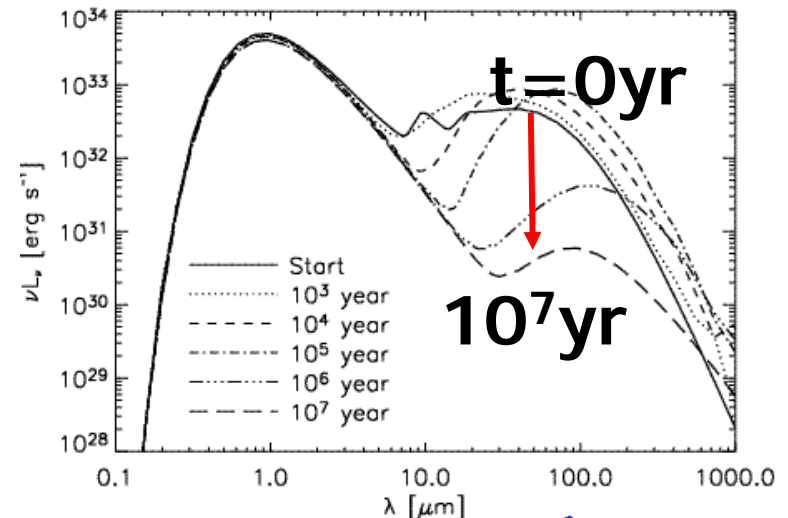
(Tanaka et al. 2005)

Dust evolution in disks
SED model calculation
Unable to reproduce
observations especially
in turbulent disks



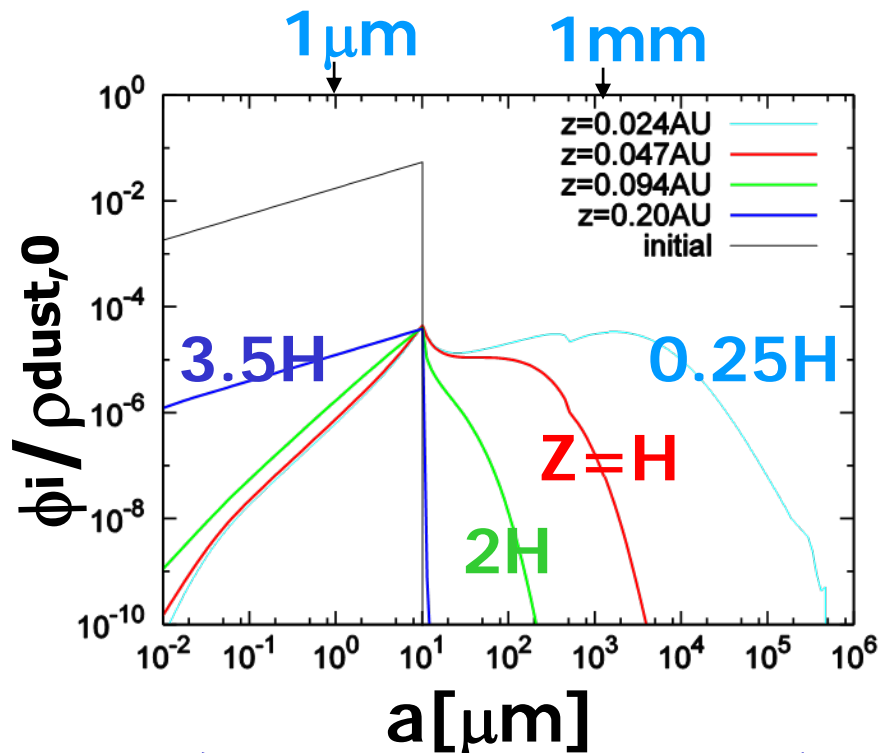
(Dullemond & Dominik 2005)

Turbulent disk

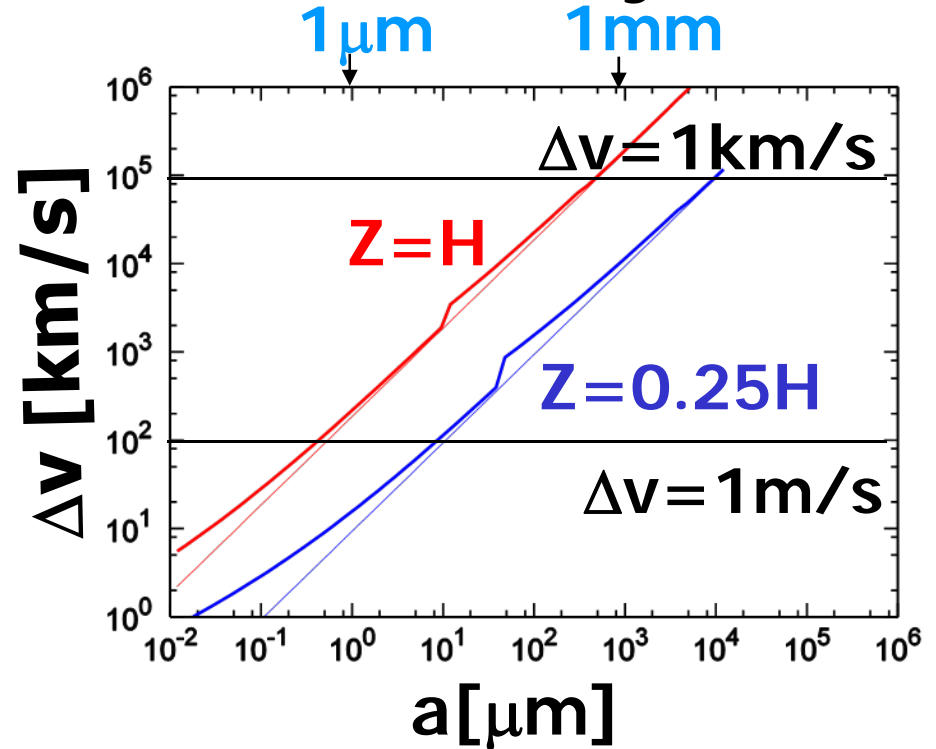


How to Supply Small Dust Grains?

Turbulent disk, $R=1\text{AU}$, $t=10^6\text{yr}$



(Nomura et al. 2007)



Fragmentation ?

Supply of small dust grains to inner disk

- Vertical: cloud disk midplane
- Radial: migrate with gas accretion flow

§ 2

**Size growth,
settling, & migration
of dust particles and
Disk model**

Dust size growth, settling, migration

Coagulation eq. for dust particles

$$\frac{\partial \varphi_i}{\partial t} + \frac{1}{R} \frac{\partial (R \varphi_i v_R)}{\partial R} + \frac{\partial (\varphi_i v_z)}{\partial z}$$

$$= \frac{1}{2} m_i \sum_{j=1}^{i-1} \beta_{i-j,j} \varphi_{i-j} \varphi_j - m_i \varphi_i \sum_{j=1}^N \beta_{i,j} \varphi_j$$

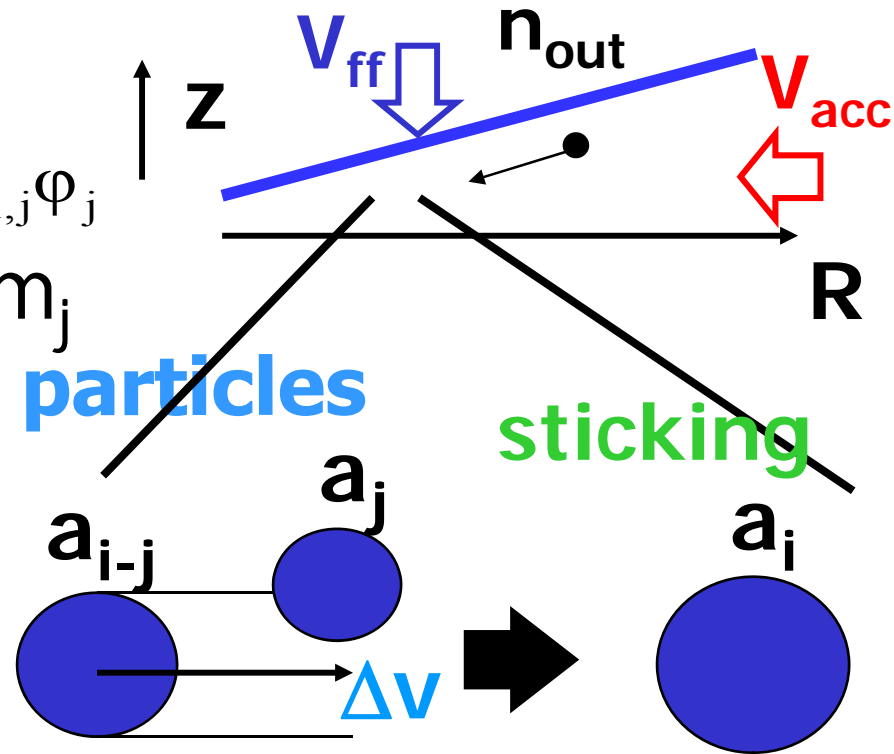
$$\beta_{i-j,j} = \pi (a_{i-j} + a_j)^2 \Delta v \rho_s / m_{i-j} m_j$$

Δv : relative velocity bw. particles

- Brownian motion
- settling & radial velocity differences
- turbulent induced velocity differences

Turbulent mixing $V_z \varphi_i = -(\Omega_z^2 z/D) \varphi_i - D_0 (\partial \varphi_i / \partial z)$

$$D_0 = \alpha c_s H / (1 + \Omega_K / D) \quad D = \rho_{\text{gas}} c_s / a$$



Velocity of Dust particles (V_R & V_Z)

Eq. of motion for dust

a : dust radius

$$\frac{d\mathbf{U}}{dt} = -D(\mathbf{U} - \mathbf{u}) - \frac{GM_*}{R^3} \mathbf{R} \approx 0$$

$$D = \rho_{\text{gas}} c_s / a$$

Eq. of motion for gas

$$\frac{d\mathbf{u}}{dt} = -\frac{\rho_{\text{dust}}}{\rho_{\text{gas}}} D(\mathbf{u} - \mathbf{U}) - \frac{GM_*}{R^3} \mathbf{R} - \frac{\nabla P_{\text{gas}}}{\rho_{\text{gas}}} + \frac{\nabla \cdot \boldsymbol{\sigma}}{\rho_{\text{gas}}} \approx 0$$

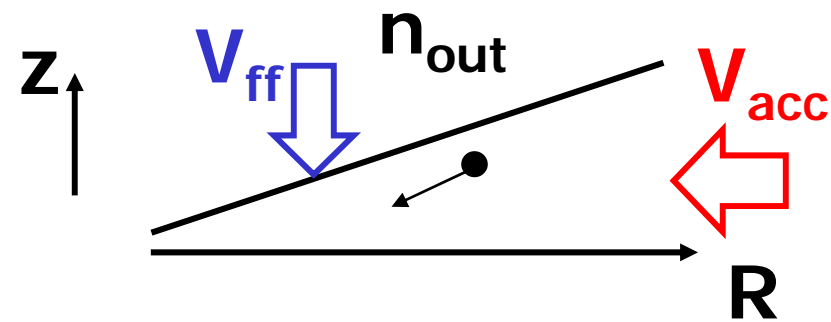
$$V_Z = -(\Omega_K^2 / D) Z$$

$$V = U - v_K, \quad v = u - v_K$$

$$V_R = -\frac{\rho_{\text{gas}}}{\rho_{\text{gas}} + \rho_{\text{dust}}} \left(\frac{2D\Omega_K}{D^2 + \Omega_K^2} \eta + \frac{2D^2}{D^2 + \Omega_K^2} \zeta \right) v_K$$

$$\zeta = -\frac{1}{2} \frac{1}{\rho_{\text{gas}}} \frac{\partial}{R \partial R} (R \rho_{\text{gas}} \alpha c_s h \Omega_K) \frac{1}{R \Omega_K^2}$$

$$\eta = -\frac{1}{2} \frac{1}{\rho_{\text{gas}}} \frac{\partial p_{\text{gas}}}{\partial R} \frac{1}{R \Omega_K^2}$$

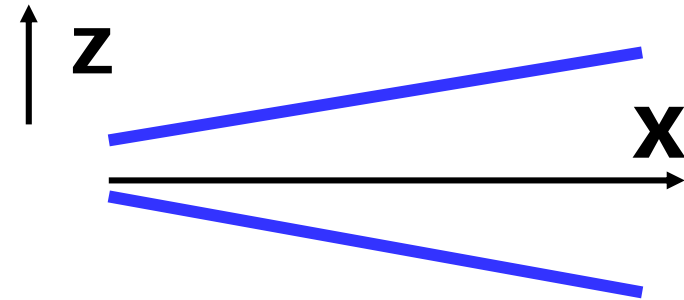


Gas Density Profile

Hydrostatic equilibrium in z-direction

$$\frac{dP}{dz} = -\rho g_z = -\frac{\rho G M_* z}{(x^2 + z^2)^{3/2}}$$

$$P = \rho k T / \mu m_p, \quad M_* = 0.5 M_s$$



Surface density: Σ (←energy balance)

$$\frac{9}{4} \Sigma \alpha c_{s0}^2 \Omega = \frac{3 G M_* \dot{M}}{8 \pi x^3} \left[1 - \left(\frac{R_*}{x} \right)^{1/2} \right]$$

$$\dot{M}_{\text{acc}} = 1 \times 10^{-8} M_s / \text{yr} \quad (= \text{const.}), \quad \alpha = 0.01$$

Gas & Dust Temperature Profile

Gas: Thermal equi. ($\Gamma_x + \Gamma_{pe} + L_{gr} - \Lambda_{line} = 0$)

Γ_x : X-ray heating
(H, H₂ ionization)

Λ_{line} : Rad. cooling
(Ly α , OI, CII, CO lines)

中心星

Γ_{pe} : FUV heating
(grain photoelectric)

L_{gr} : Gas-dust
collisions

Dust: Local radiative equilibrium

$$\int_0^\infty dv \kappa_v \oint I_v d\Omega = 4\pi \int_0^\infty dv \kappa_v B_v(T_{gr})$$

Heating: Irradiation from central star

Cooling: Dust thermal radiation

§ 3

Resulting

Dust Size & Spatial

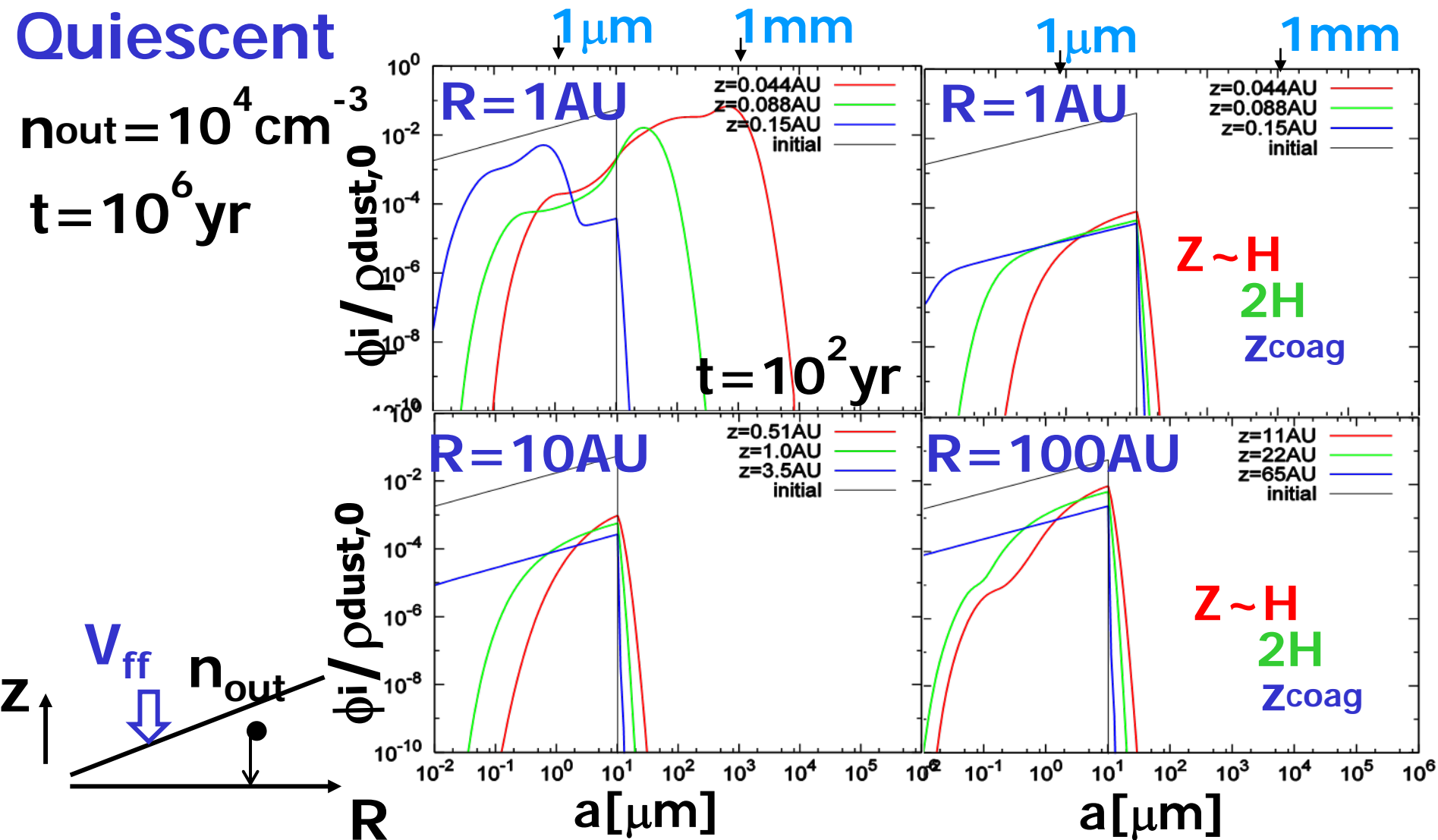
Distributions

Dust size distribution (only V_z)

Quiescent

$$n_{\text{out}} = 10^4 \text{ cm}^{-3}$$

$$t = 10^6 \text{ yr}$$



$t = 10^6 \text{ yr}$: large dust

settle, $R \searrow$

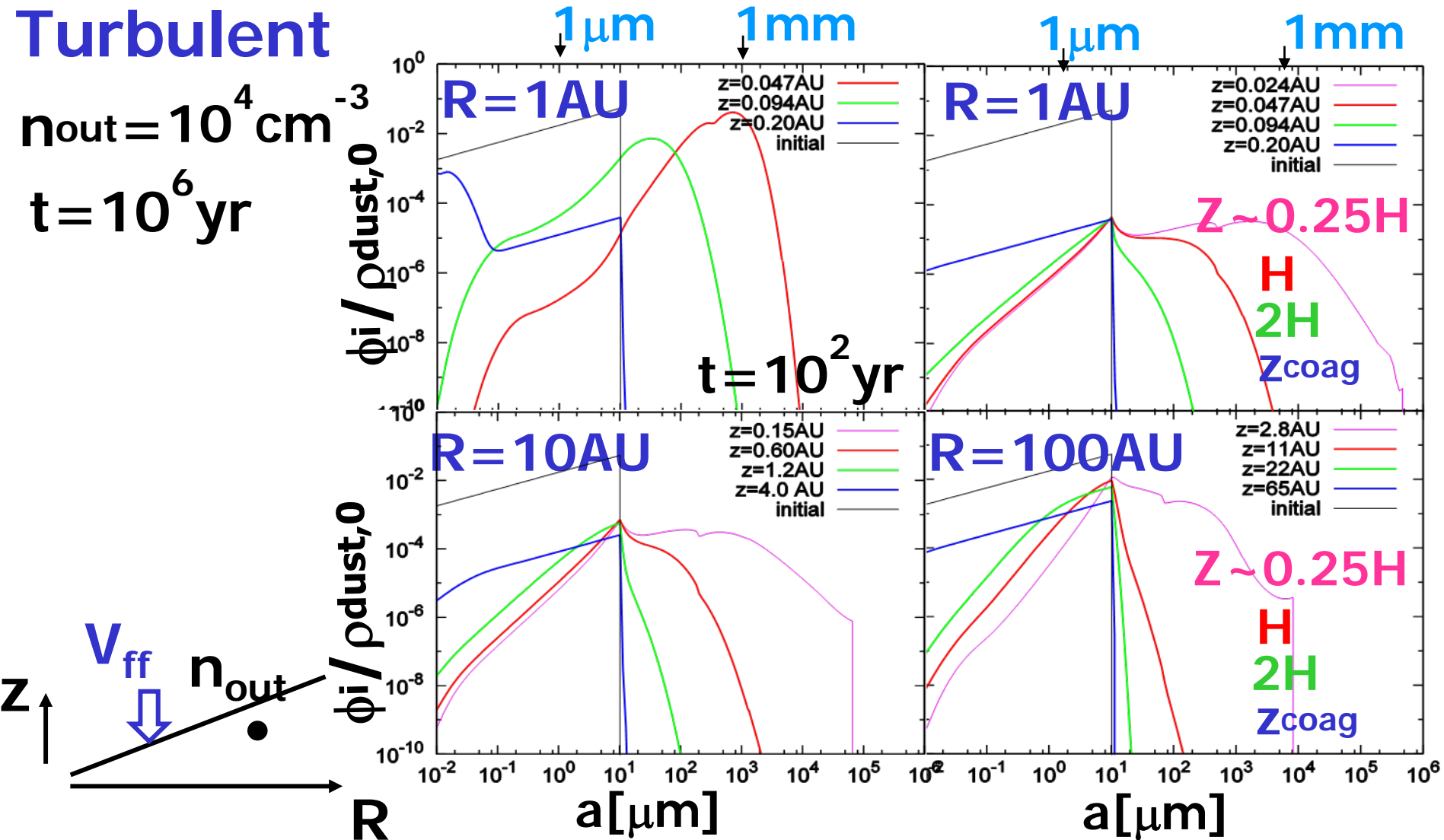
$\phi_i / \rho_{\text{dust},0} \searrow$

Dust size distribution (only V_z)

Turbulent

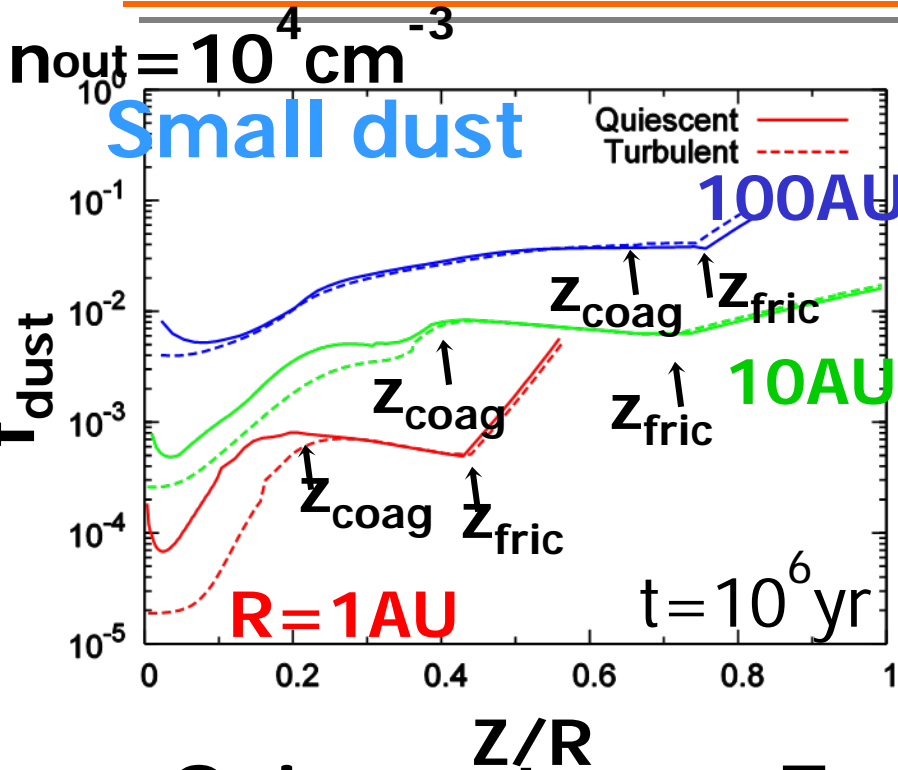
$$n_{\text{out}} = 10^4 \text{ cm}^{-3}$$

$$t = 10^6 \text{ yr}$$



Large grains exist due to turbulent mixing

Small-dust/Gas Ratio (only V_z)



$$A(R, z) = \int \pi a^2 \frac{dn(R, z)}{da} da$$

$$f_{dust}(R, z) = A(R, z) / A_0(R, z)$$

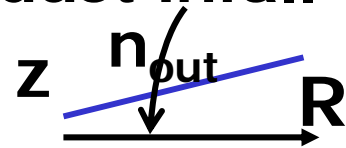
Vertical velocity of dust

$$\frac{dV_z}{dz} = -DV_z - \frac{GM_*}{r} z$$

gas drag gravity

$$D = \rho_{gas} c_s / a$$

dust infall



———— Quiescent Turbulent

Surface layer ($z_{fric} < z$, ρ_{gas} : small)

V_z : free-fall (only gravity)

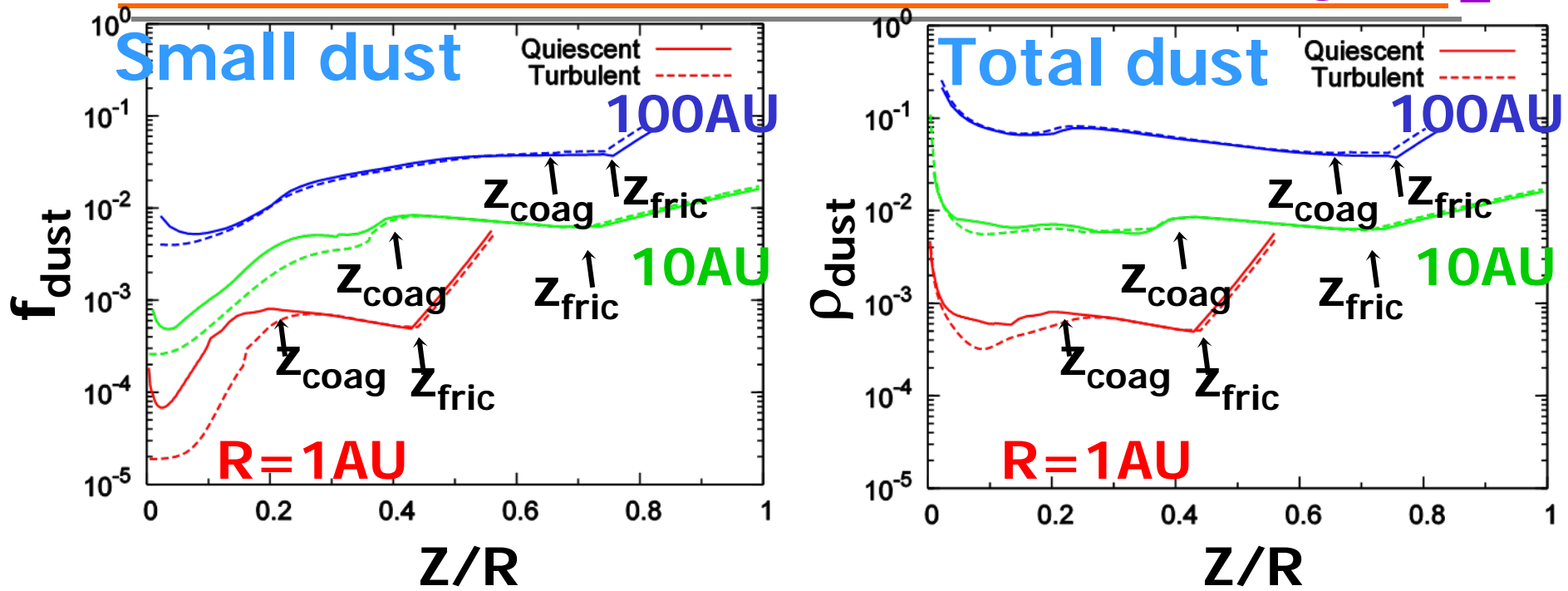
$$f_{dust} \quad 1 / \rho_{gas}$$

Middle layer ($z_{coag} < z < z_{fric}$)

gravity ~ gas drag $\rightarrow V_z = \frac{GM_*}{Dr} z$

$$f_{dust} \quad 1 / z$$

Small-dust/Gas Ratio (only V_z)



—— Quiescent Turbulent (Nomura et al. 2007)

Midplane ($z < z_{\text{coag}}$, ρ_{dust} : large)

$$\tau_{\text{coag}} < \tau_{\text{sed}}, \quad \tau_{\text{coag}} \sim 1 / \left(n_{\text{dust}} \pi a^2 \Delta V_z \right), \quad \tau_{\text{sed}} \sim z / V_z$$

f_{dust} : small (smaller in turbulent disk)

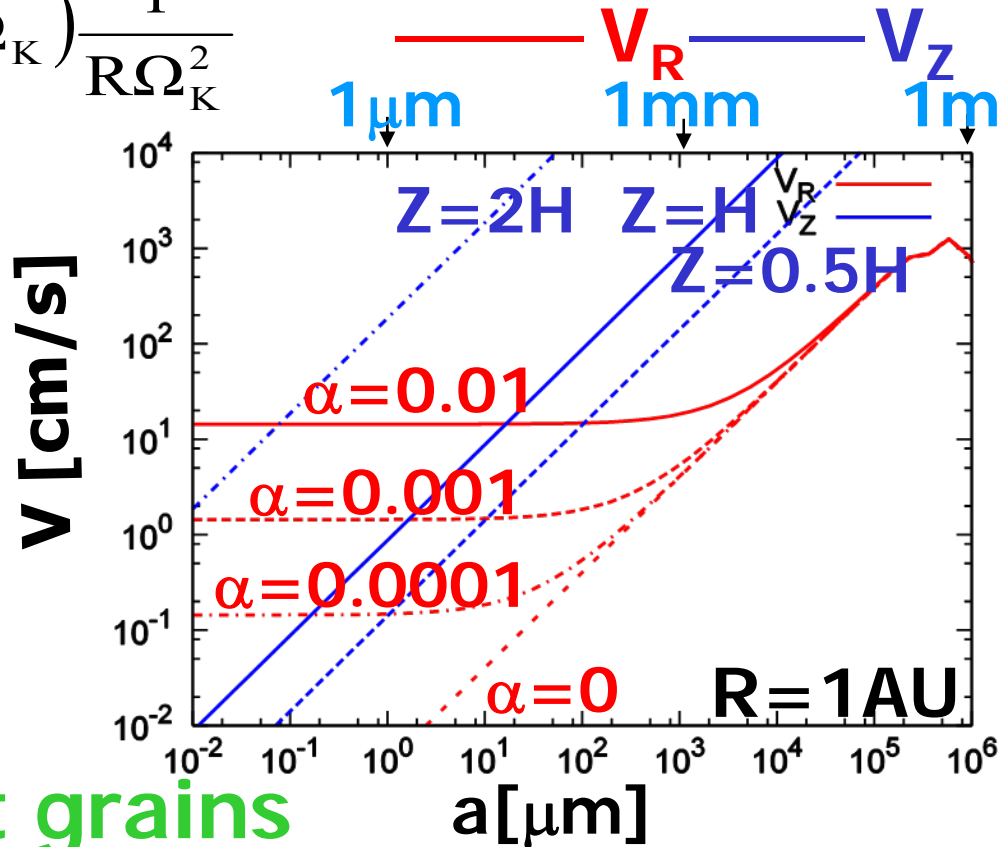
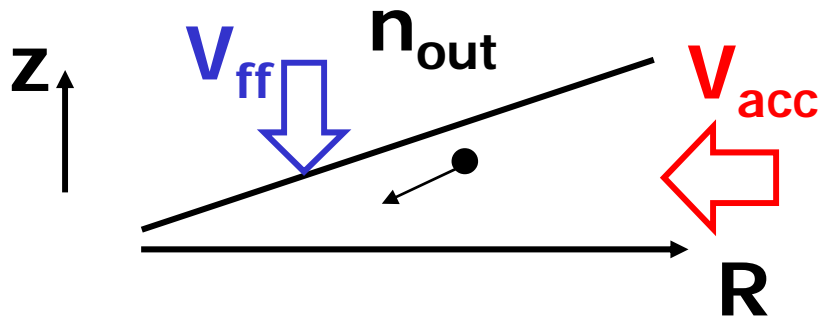
Effect of radial migration: V_R vs. V_Z

$$V_R = - \frac{\rho_{\text{gas}}}{\rho_{\text{gas}} + \rho_{\text{dust}}} \left(\frac{2D\Omega_K}{D^2 + \Omega_K^2} \eta + \frac{2D^2}{D^2 + \Omega_K^2} \zeta \right) v_K \quad D = \rho_{\text{gas}} c_s / a$$

$$\zeta = - \frac{1}{2} \frac{1}{\rho_{\text{gas}}} \frac{\partial}{R \partial R} (R \rho_{\text{gas}} \alpha c_s h \Omega_K) \frac{1}{R \Omega_K^2}$$

$$\eta = - \frac{1}{2} \frac{1}{\rho_{\text{gas}}} \frac{\partial p_{\text{gas}}}{\partial R} \frac{1}{R \Omega_K^2}$$

$$V_Z = - (\Omega_K^2 / D) Z$$



Amount of small dust grains

dust inflow in vertical & radial directions

n_{out} & α

Dust size distribution (V_z & V_z)

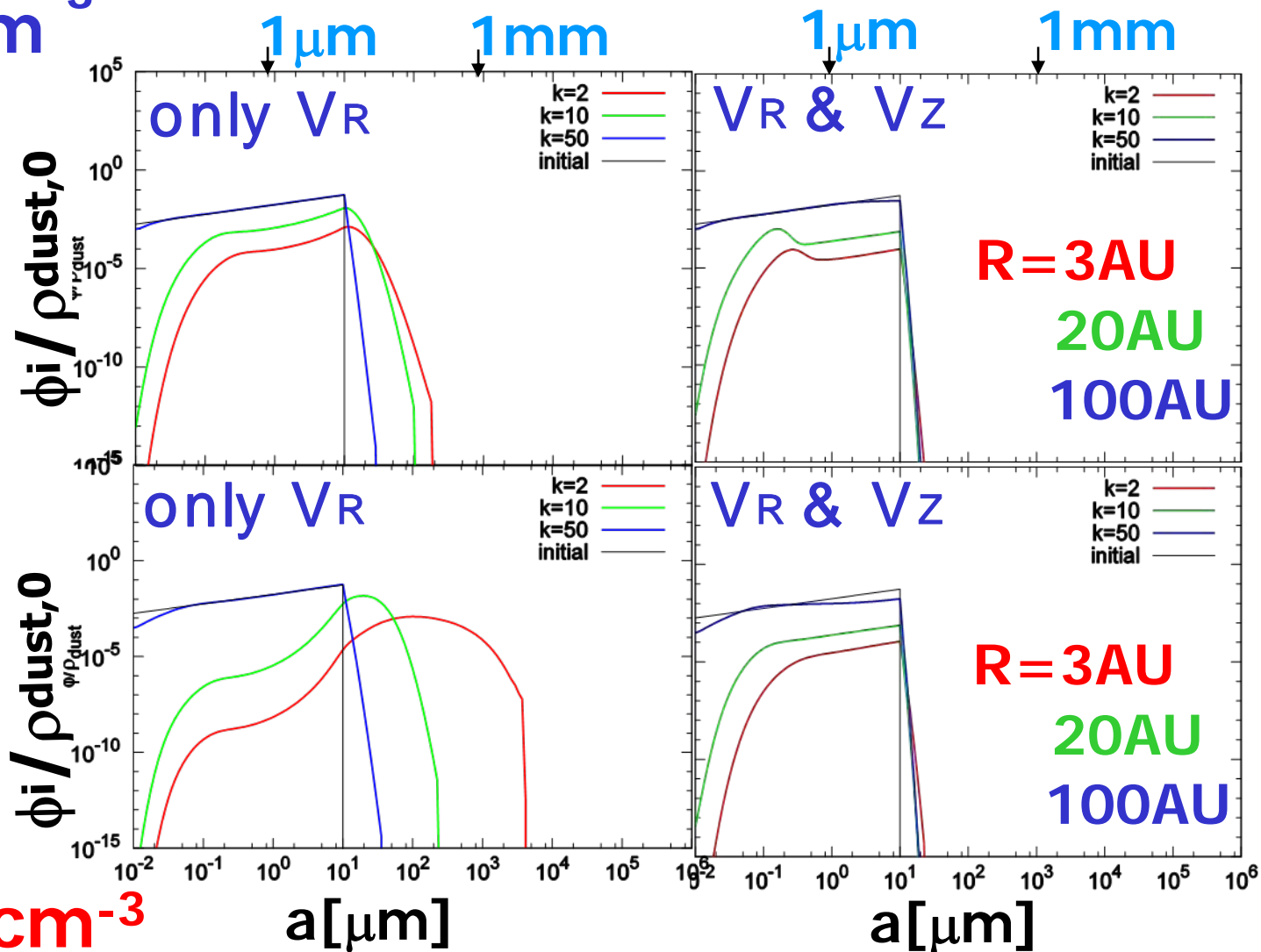
$$n_{\text{out}} = 10^4 \text{ cm}^{-3}$$

$$t = 10^6 \text{ yr}$$

$$z = H$$

$$\alpha = 0.01$$

$$\alpha = 0.001$$



$$n_{\text{out}} = 10^4 \text{ cm}^{-3}$$

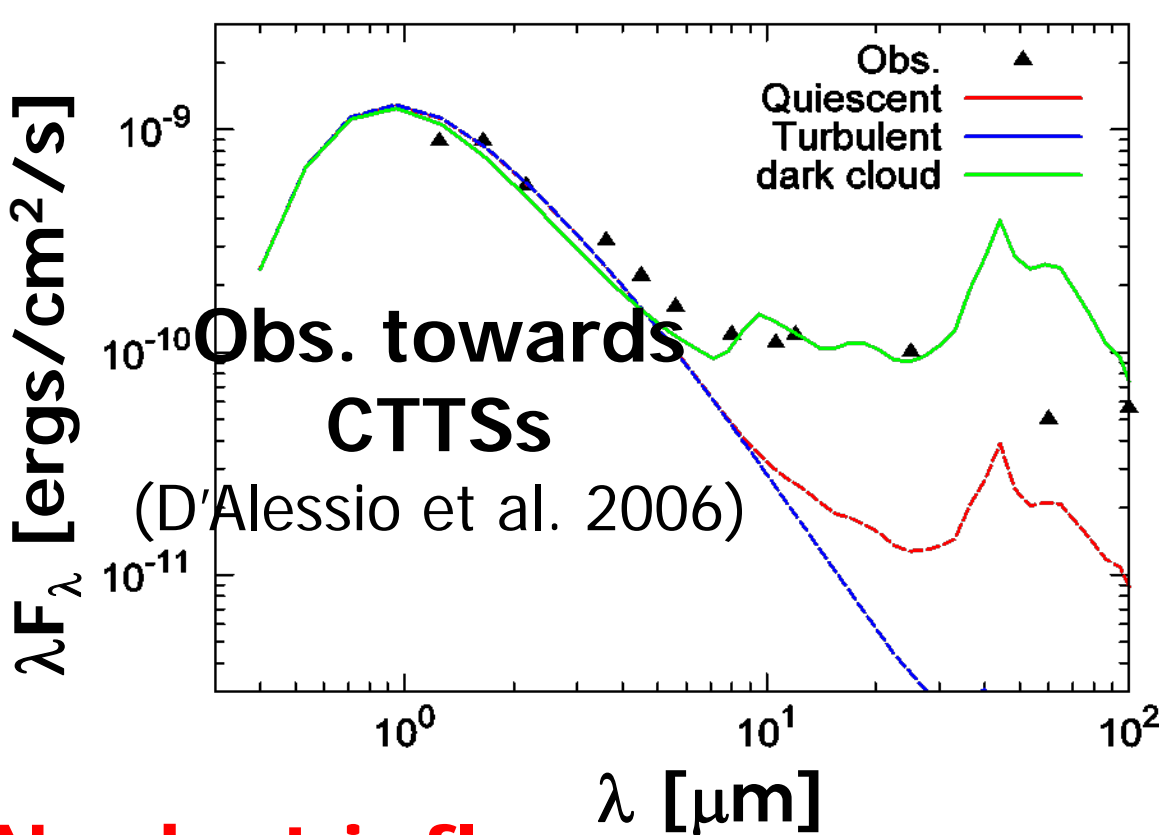
accretion flow dominant if $\alpha > 10^{-2} \sim -3$

§ 4

Effects on Dust Continuum Emission

Effect of dust inflow on SED

Disk temp. & density + Dust evolution
+ Dust opacity + Rad. transfer **SED**



$t = 1 \times 10^6$ yr

Dark clouds dust

Quiescent

Turbulent

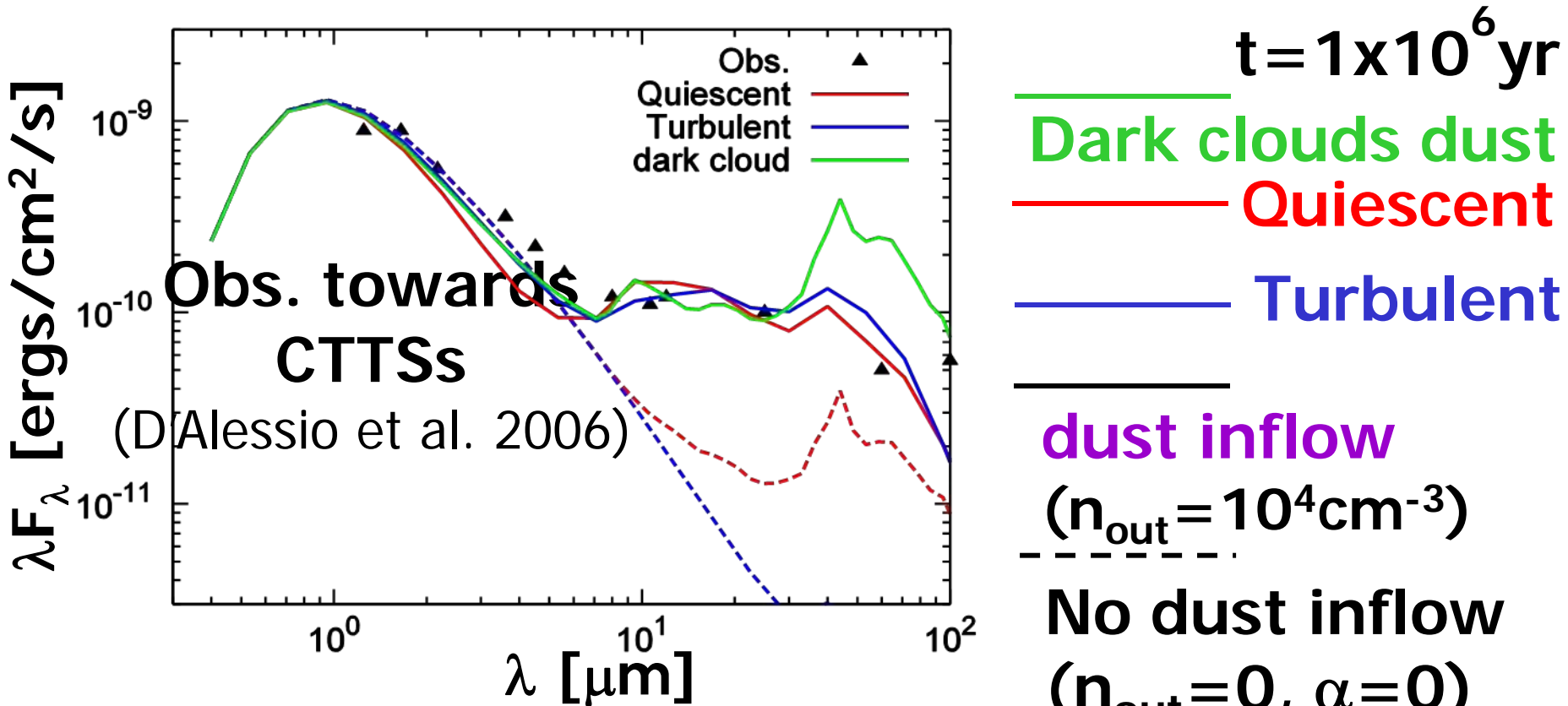
No dust inflow
($n_{\text{out}} = 0, \alpha = 0$)

No dust inflow

Model cannot reproduce observations

Effect of dust inflow on SED

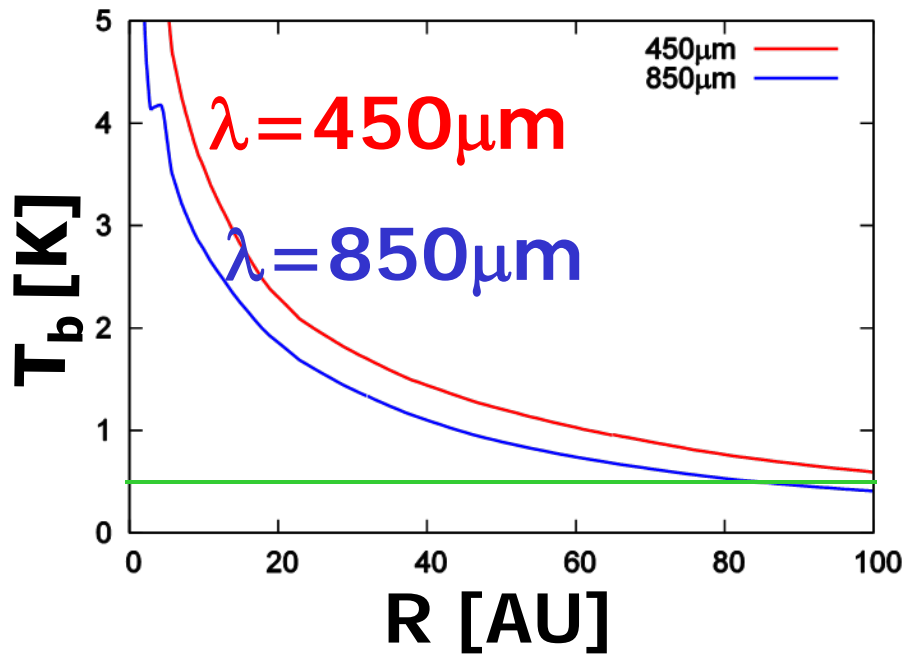
Disk temp. & density + Dust evolution
+ Dust opacity + Rad. transfer **SED**



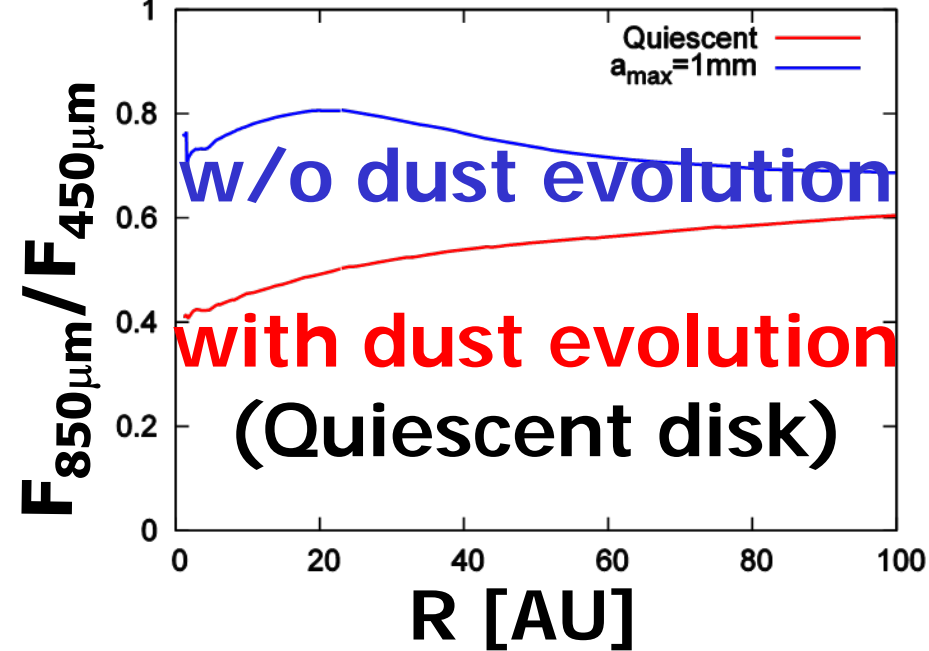
$n_{\text{out}} > 10^4 \text{cm}^{-3}$ or $\alpha > 10^{-2} \sim -3$

consistent with observations

Spatial distri. of dust emission



ALMA 5σ detection limit
50 antennas, $0''.1$, 600s



Dust evolution

$F_{850 \mu\text{m}} / F_{450 \mu\text{m}}$
@ inner disk

Dependence of spatial distribution of
dust flux ratio on dust evolution

Observable by ALMA

§ 5 Summary

Dust size growth, settling, and radial migration in protoplanetary disks

Supply of small dust grains to inner disk

{ Vertical: cloud → disk midplane n_{out}
{ Radial: migrate with gas accretion α

SED model calculations

$n_{\text{out}} > 10^4 \text{cm}^{-3}$ or $\alpha > 10^{-2 \sim -3}$

consistent with observations

Effects on spatial distri. of dust emission

: $F_{850\mu\text{m}} / F_{450\mu\text{m}} \searrow$ @ inner disk

Observational diagnostics by ALMA