SATELLITE FORMATION

SUPPLY OF SOLID MATERIAL ONTO CIRCUM-PLANETARY DISKS

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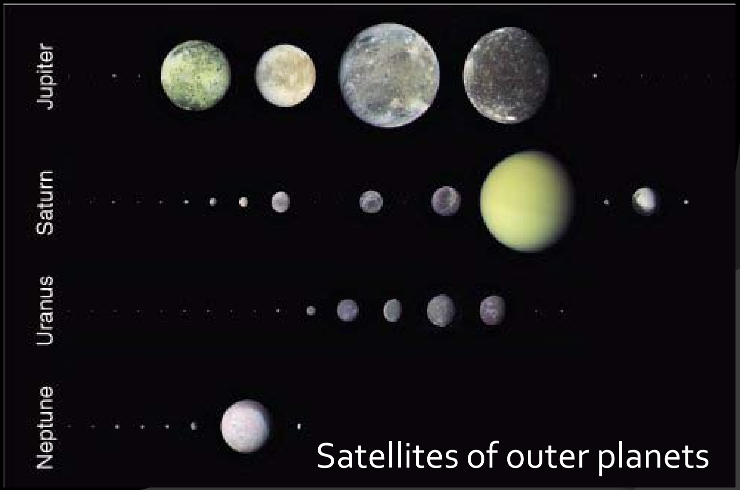
Satellite systems

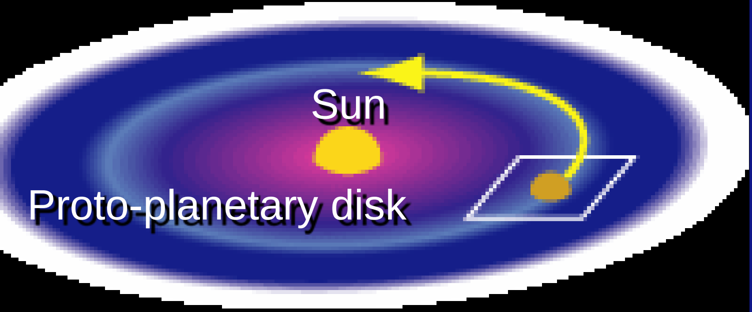
- Regular and irregular satellites
  - Regular satellites:
    - Large fraction of total mass
    - Co-planner and circular orbits
    - → Formed in circum-planetary disks

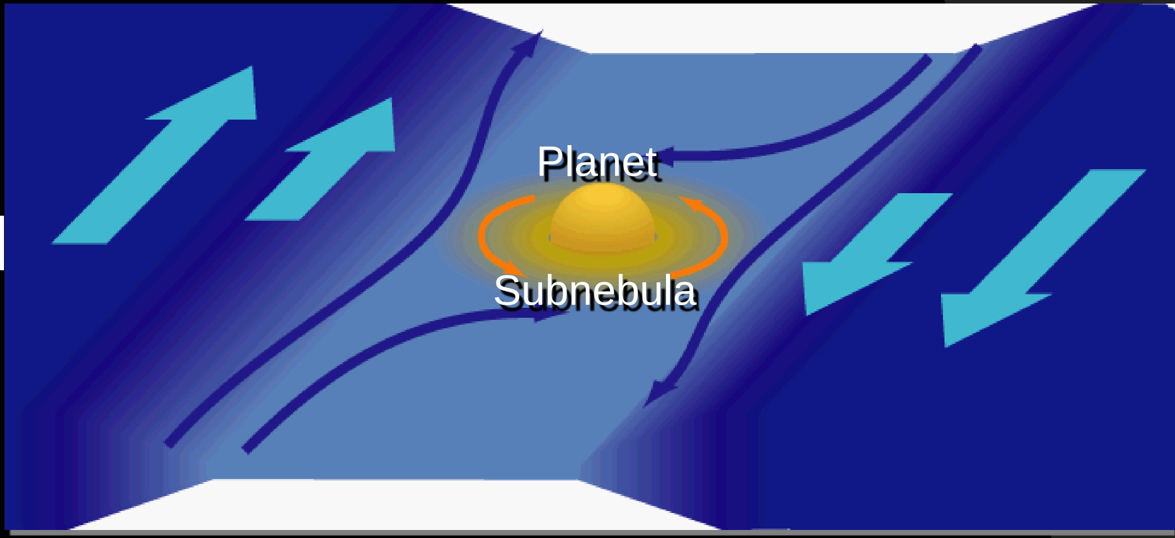

Jupiter and Galilean satellites



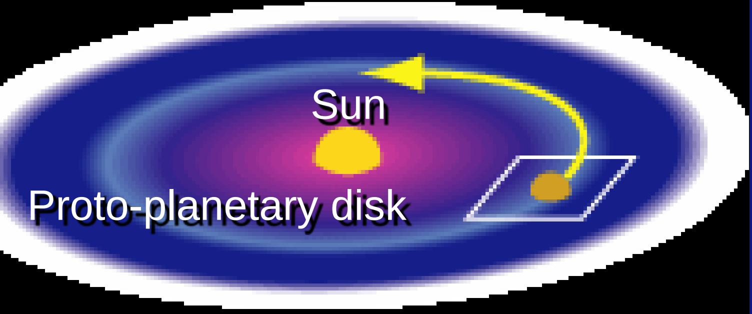
Satellites of outer planets

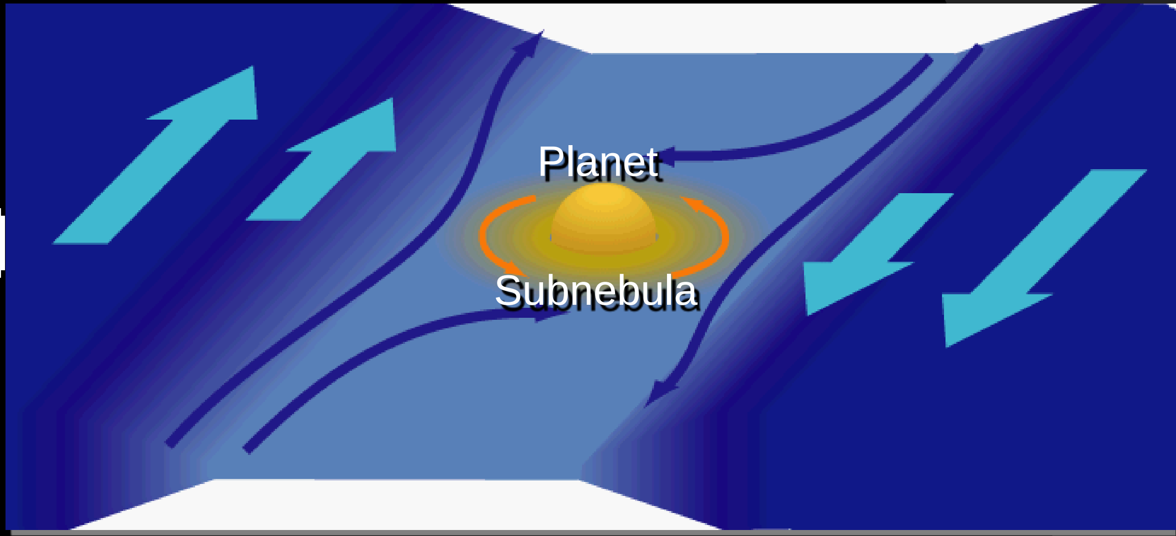
Structure of circum-planetary disks



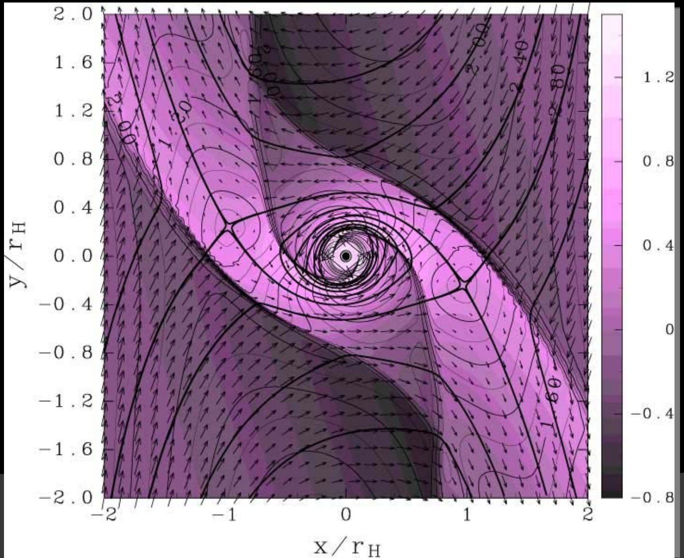


Structure of circum-planetary disks

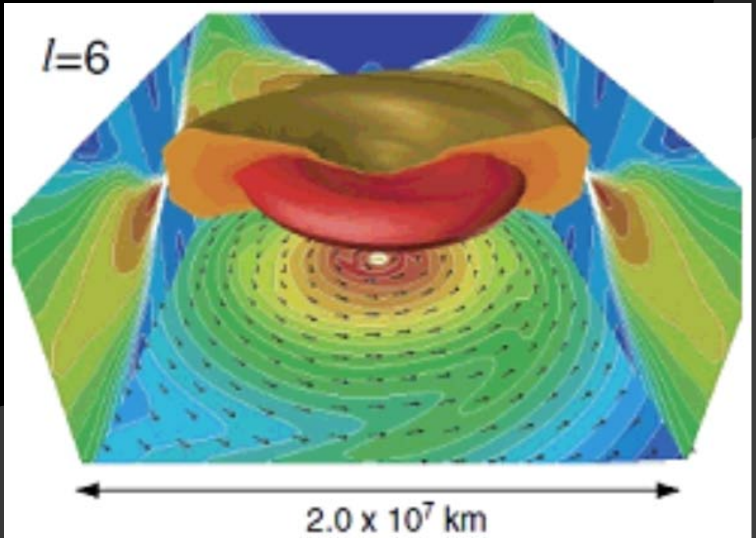




Tanigawa and Watanabe 2002



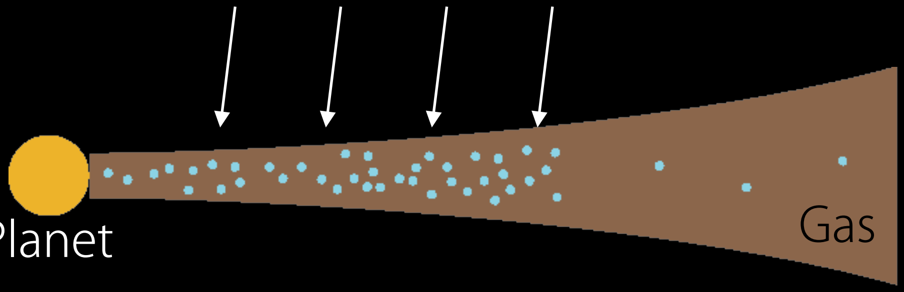
Machida 2009



Previous studies

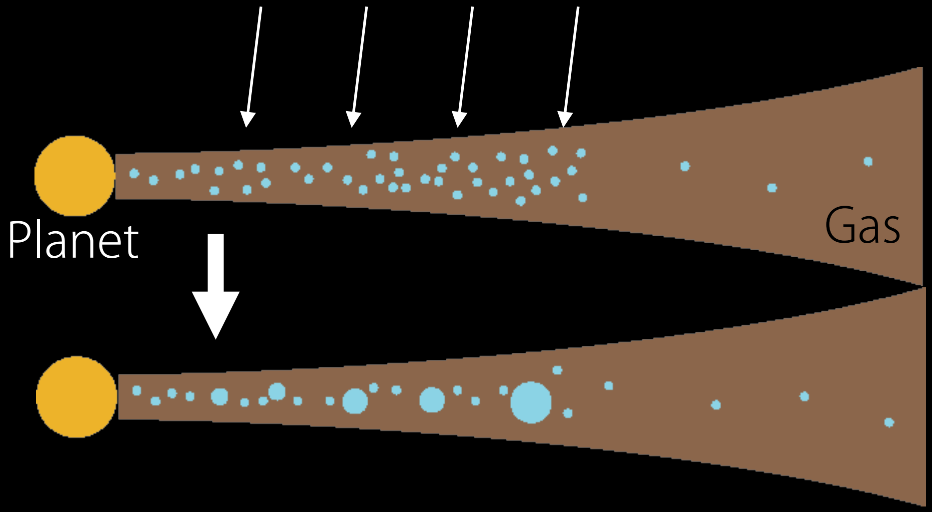
- Traditional model
  - Closed disk model with the “Minimum Mass Sub-Nebula”
  - Several severe problems
    - Temperature, accretion time, type I migration …
- Canup and Ward model (2002, 2006)
  - Open disk model based on the knowledge of gas accretion flow onto gas giant planets
    - Solid material is steadily supplied to circum-planetary disks
    - $M_{satellites} / M_{planet}$ is consistent with the real systems
  - Sasaki et al is trying to explain the difference between Jovian and Saturnian systems.

Canup and Ward model



Steady mass supply

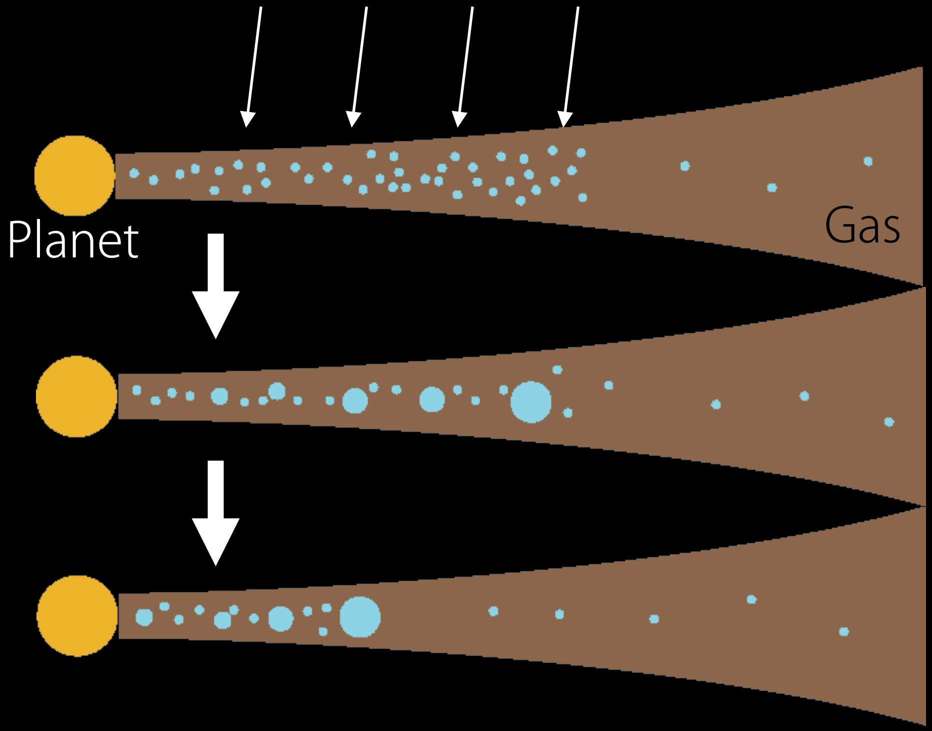
Canup and Ward model



Steady mass supply

Growth from outside

Canup and Ward model



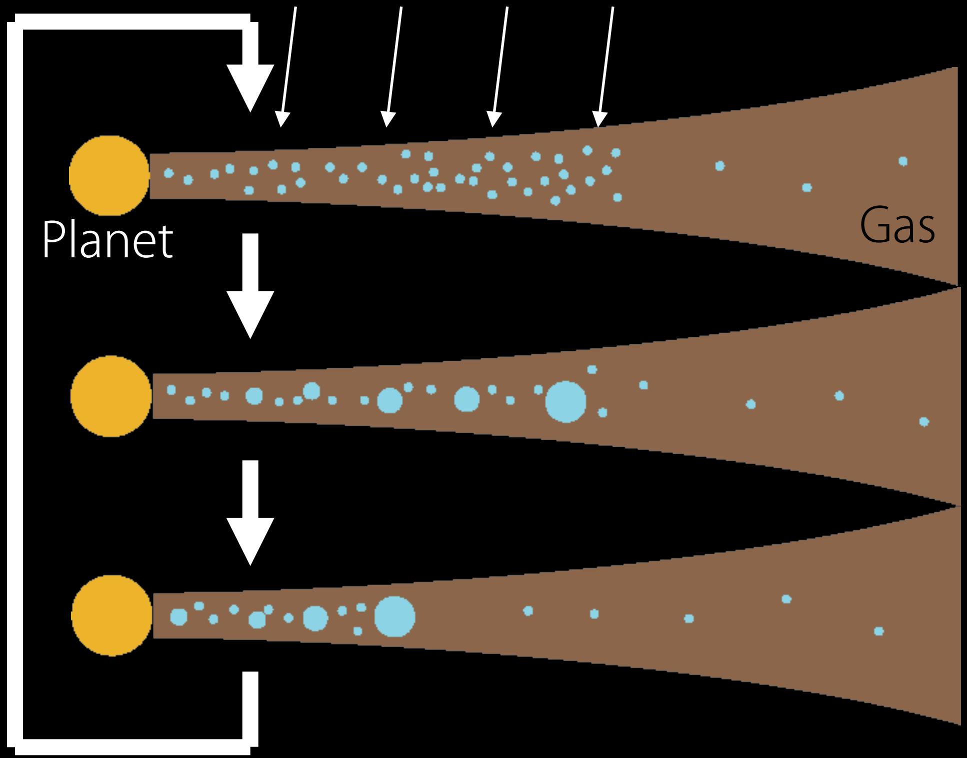
Steady mass supply

Growth from outside

Larger planets move inward

Inner objects are swept

Canup and Ward model



Steady mass supply

Growth from outside

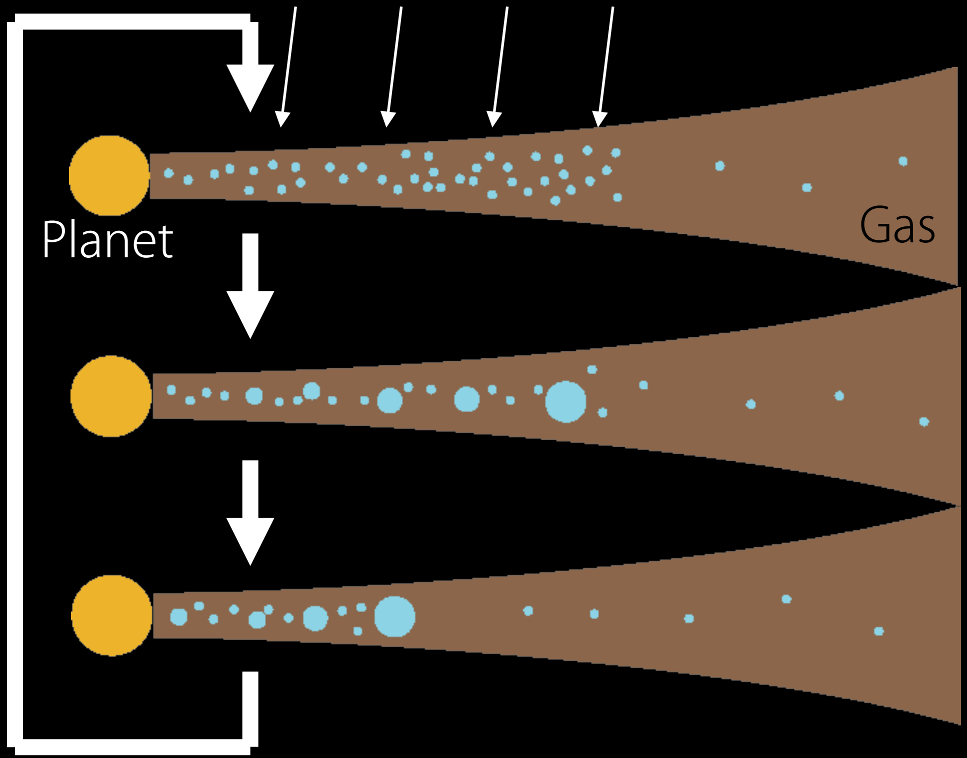
Larger planets move inward

Inner objects are swept

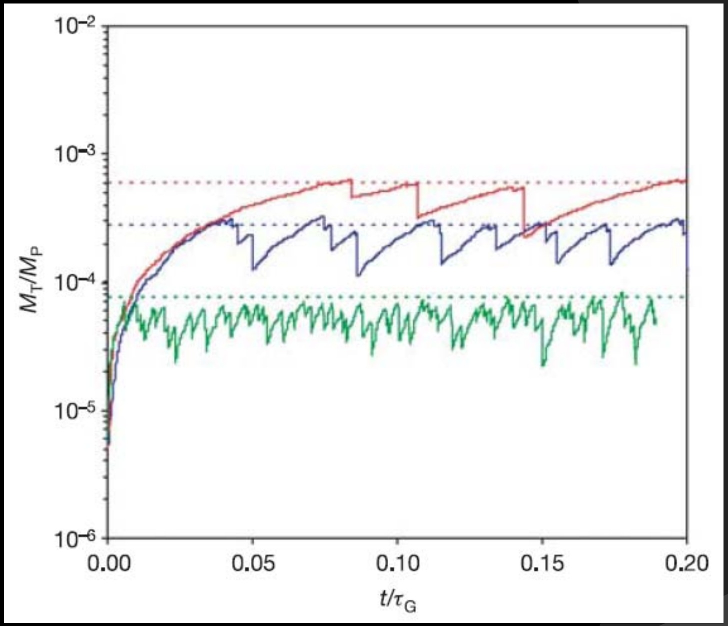
Continue until the mass supply terminates.

Current satellites are the last generation of this cycle

Canup and Ward model



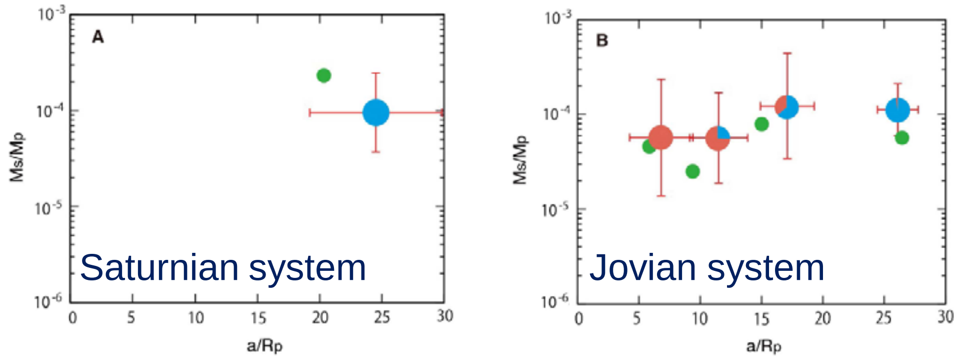
Canup and Ward 2006



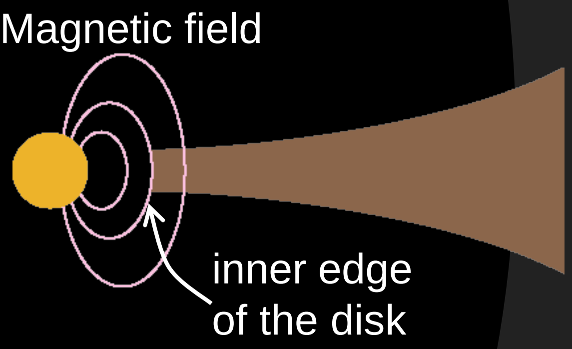
They reproduces total mass of satellite systems, but hard to explain the difference between Jovian and Saturnian systems

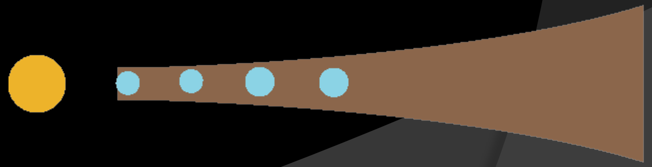
Sasaki, Stewart, and Ida model:

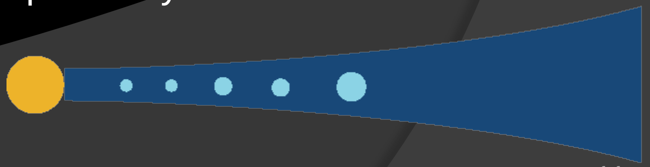
Did inner edge determine the difference between Jovian and Saturnian systems?



- ◉ Analogy of star formation
  - CTTS stage → strong magnetic field
    - ○ Jupiter?
    - ○ Inner edge exists
  - WTTS stage → magnetic field weakens
    - ○ Saturn?
    - ○ No disk edge?
- ◉ How about gas giant planets?
  - Jupiter can terminate its growth by forming a gap
    - ○ Mass supply suddenly stop
    - ○ Frozen in the stage corresponds to CTTS?
    - ○ Satellites are stacked?
  - Saturn mass is insufficient to form a gap
    - ○ Mass supply gradually decreases with dissipation of proto-planetary disks
    - ○ Evolved through the stage corresponds to WTTS?
    - ○ Satellites fall to the planet easily.
    - ○ Large satellites are likely to be at outer region







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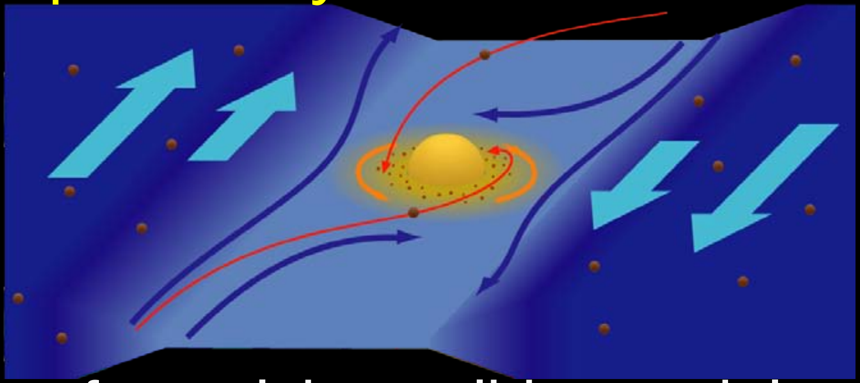
    - Solid material is steadily supplied to circum-planetary disk
    - $M_{\text{satellites}} / M_{\text{planet}}$ is consistent with the real systems.
  - Sasaki et al is trying to explain the difference between Jovian and Saturnian systems.
  - Assumptions
    - Solid material is supplied uniformly on the disks.

Objective

To determine distribution of supplying rate of solid material onto circum-planetary disks from proto-planetary disks.



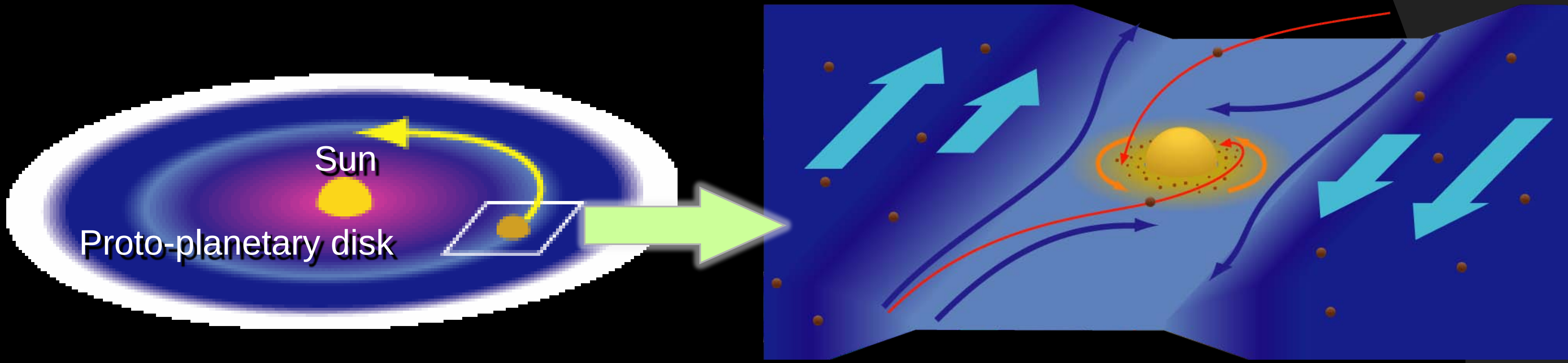
Two manners of supplying solid material

- Smaller size (< m-size)
  - Strongly entrained by gas accretion flow
- Larger size (> m-size)
  - Weakly affected by gas drag

An analytical estimation for larger size is shown.

Analytical model

Setting



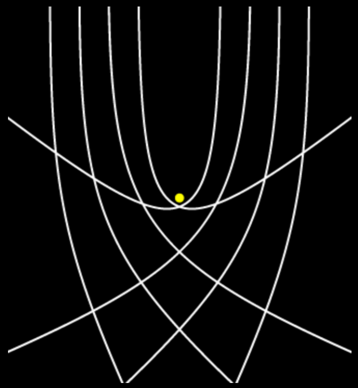
Captured by gas drag with the disks

- Assumptions
  - Axisymmetry of circum-planetary gas disk with power-law surface density distribution
  - Pericenter of orbit just after captured in the Hill sphere does not change in the course of circularization.

Capturing process

Before

Gravitational focusing

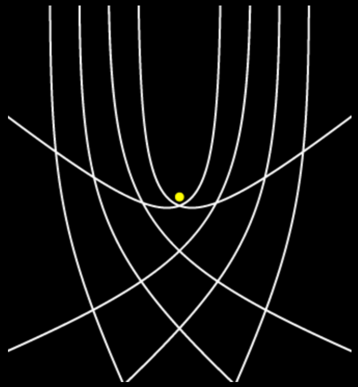


→ $r_{\min} \propto r^2$ （centered near the planet）

Capturing process

Before

Gravitational focusing



→ $r_{\mathrm{min}} \propto r^2$ （centered near the planet）

Critical radius to be captured

$$R_{\mathrm{c}} \propto m^{-1/3(p+1)} \qquad \Sigma_{\mathrm{gas}} = \Sigma_{\mathrm{gas},0}\left(\frac{r}{r_0}\right)^{-p}$$

Dissipation energy due to gas drag

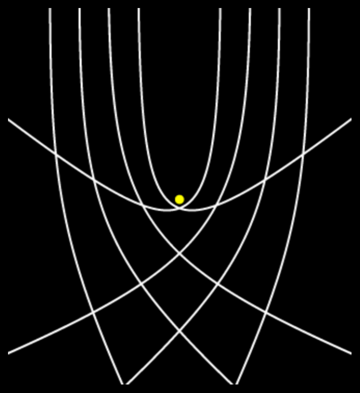
||

Energy necessary to be captured
by the gravitational potential

Capturing process

Before

Gravitational focusing



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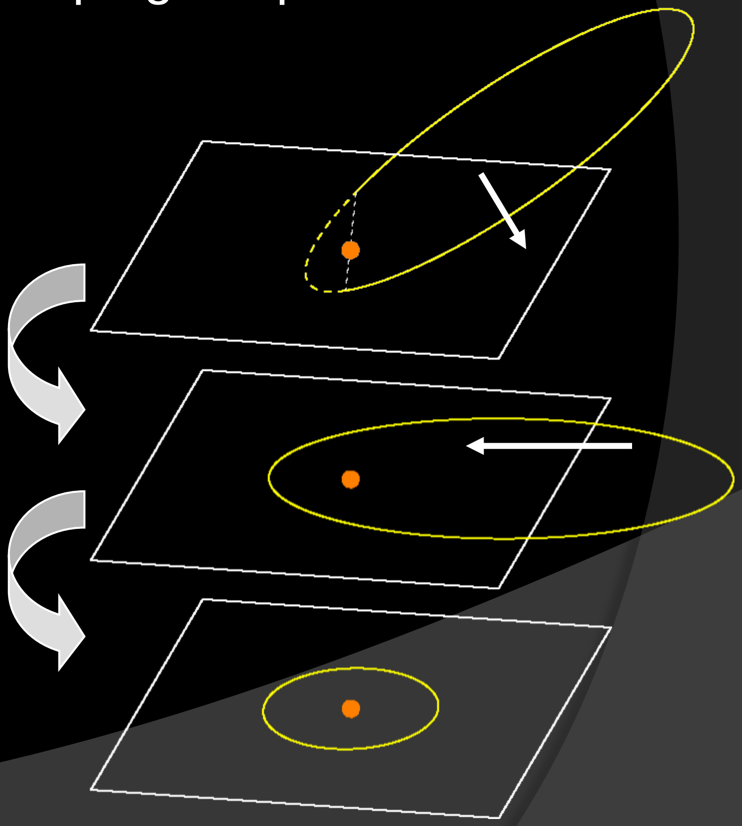
Dissipation energy due to gas drag

||

Energy necessary to be captured
by the gravitational potential

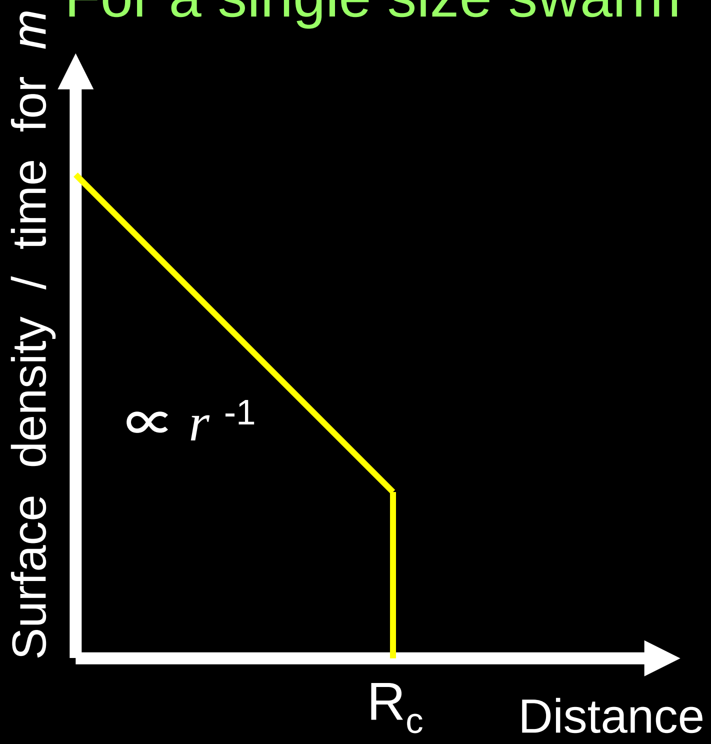
After

Eccentricity and inclination decrease
with keeping the pericenter



Supplying rate of solid material

For a single size swarm



（R_c = Critical radius to be captured）

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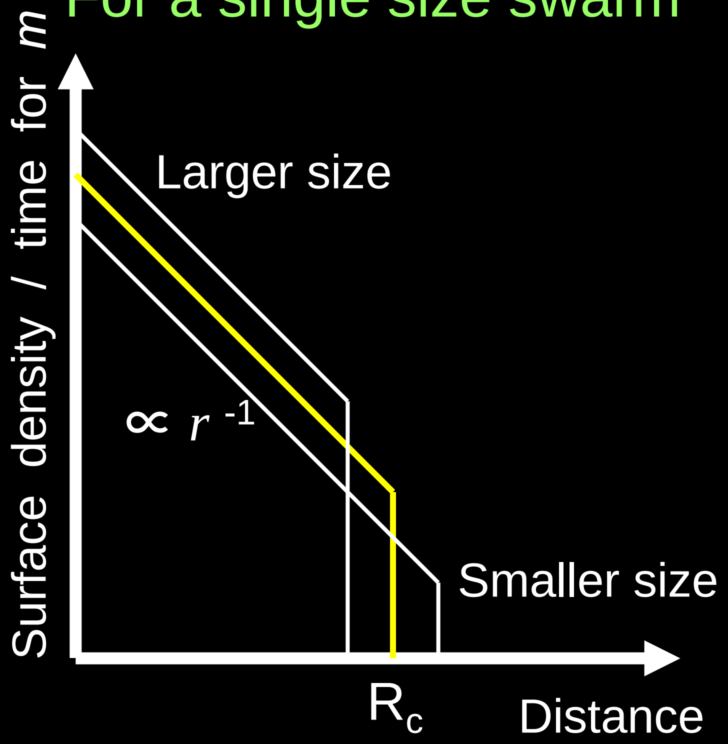

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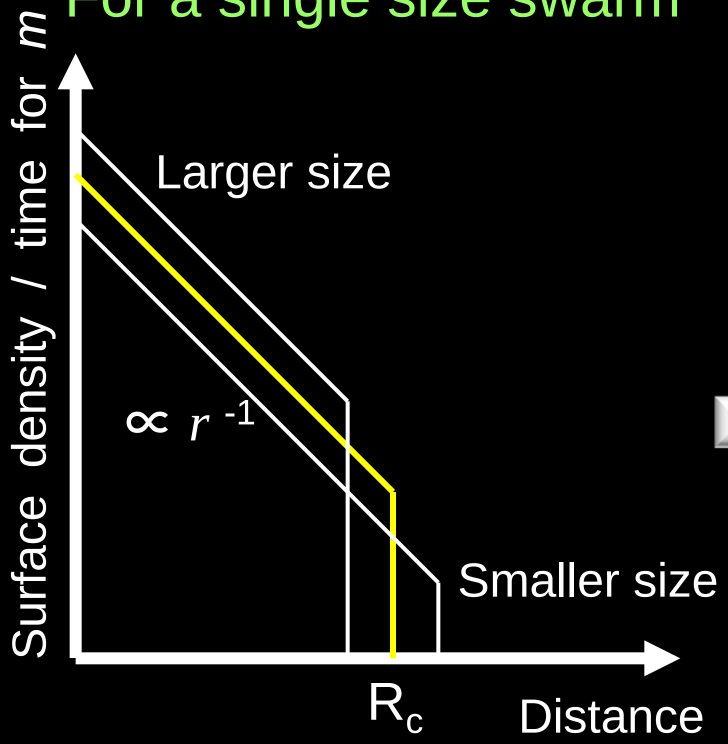

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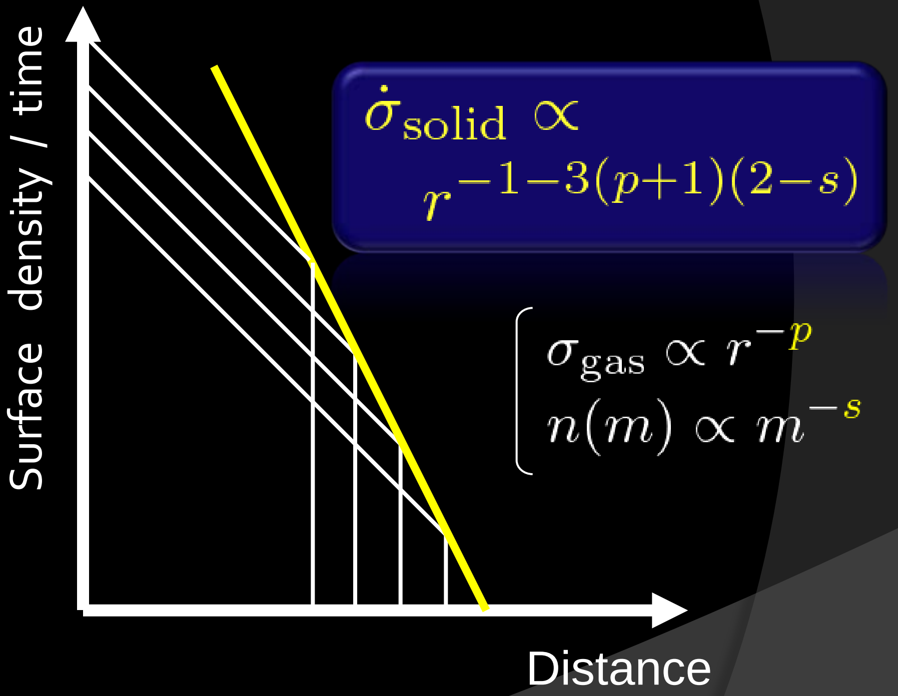
For a power-law size distribution



$$\dot{\sigma}_{\mathrm{solid}} \propto r^{-1-3(p+1)(2-s)}$$

$$\begin{cases} \sigma_{\mathrm{gas}} \propto r^{-p} \\ n(m) \propto m^{-s} \end{cases}$$

A typical case （ p=1, s=11/6 ）

$$\dot{\sigma}_{\mathrm{solid}} \propto r^{-2}$$

Supplying rate of solid

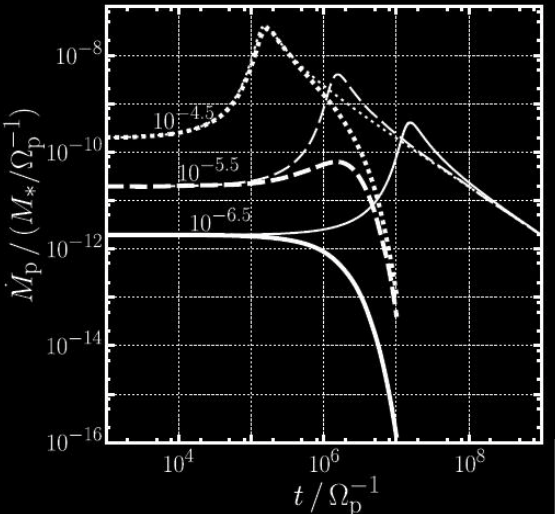
$$\dot{\sigma}(\tilde{r}) = \frac{1}{2\pi}\left(\frac{9\pi}{128}C_{\mathrm{D}}^{3}(A(i))^{3}\right)^{2-\alpha}\left(\frac{\sigma_{\mathrm{g},0}^{3}}{\rho_{\mathrm{s}}^{2}m_{\max}}\right)^{2-\alpha}\left(\frac{\Sigma_{\mathrm{d}}}{\Omega_{\mathrm{K}}^{-1}}\right)\tilde{r}^{-1-3(p+1)(2-\alpha)}\frac{d}{d\tilde{r}}P_{\mathrm{col}}(\tilde{r}),$$

$$A(i) = \frac{2(3-2\sqrt{2}\cos i)^{1/2}}{3\sin i}(\sqrt{2}-\cos i)$$

Typically （α＝11/6, a=5AU）

$$\dot{\sigma}(r) \sim 10^{3}\mathrm{g\,cm}^{-2}\,\mathrm{yr}^{-1}\left(\frac{\sigma_{\mathrm{g},0}}{10^{3}\mathrm{g\,cm}^{2}}\right)^{1/2}\left(\frac{m_{\max}}{10^{18}\mathrm{g}}\right)^{-1/6}\left(\frac{\Sigma_{\mathrm{d}}}{10\mathrm{g\,cm}^{-2}}\right)\left(\frac{\rho_{\mathrm{s}}}{1\mathrm{g\,cm}^{-3}}\right)^{-1/3}\left(\frac{r}{10R_{\mathrm{J}}}\right)^{-2}$$

Tanigawa and Ikoma 2007



Mass supplying rate $\propto$ (gas surface density)$^{1/2}$

Dust/gas ratio increases with decreasing disk gas?

Satellite formation promotes late stage of formation of gas giant?

Migration due to gas drag?

- After circularization with short timescale, objects slowly spiral toward the planets by gas drag

Migration velocity due to the gas drag with disk gas:

$$v_{r,\mathrm{s}} = -2\eta\Gamma v_{\mathrm{K}}$$

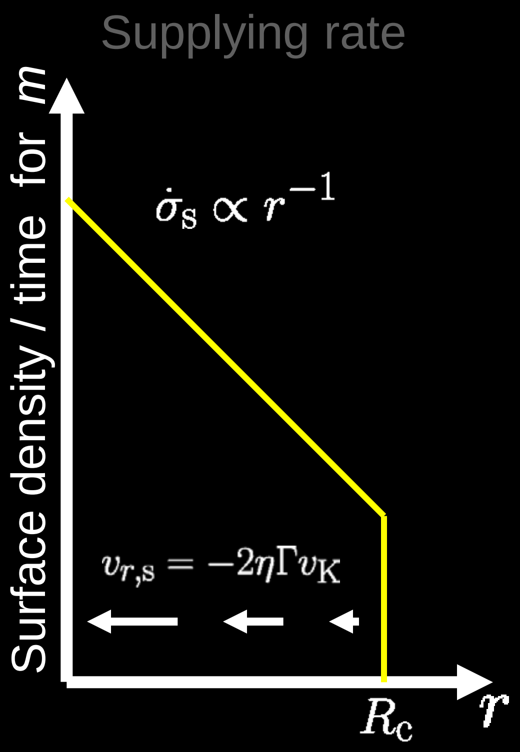
$$\Gamma = \frac{\tau_{\mathrm{K}}}{\tau_{\mathrm{stop}}} \qquad \eta \sim \left(\frac{c}{v_{\mathrm{K}}}\right)^2$$

⟶ How about the steady state distribution?

Steady state distribution

considering radial migration due to gas drag

For a single size swarm



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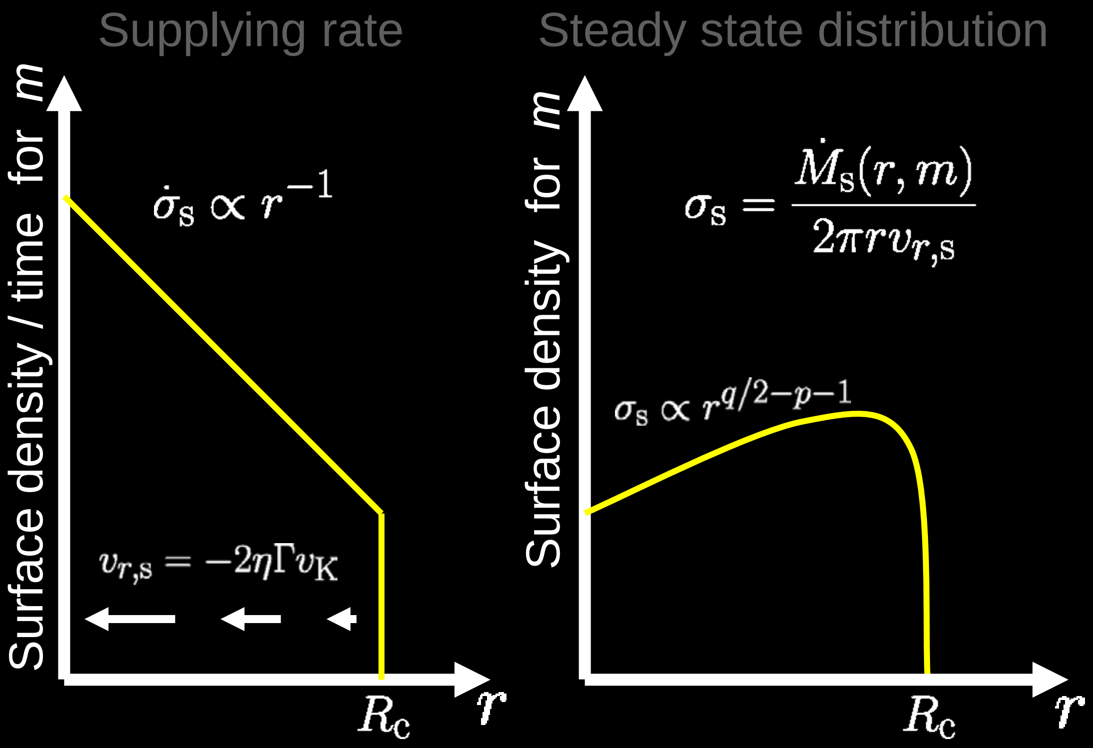


Supplying rate

Steady state distribution

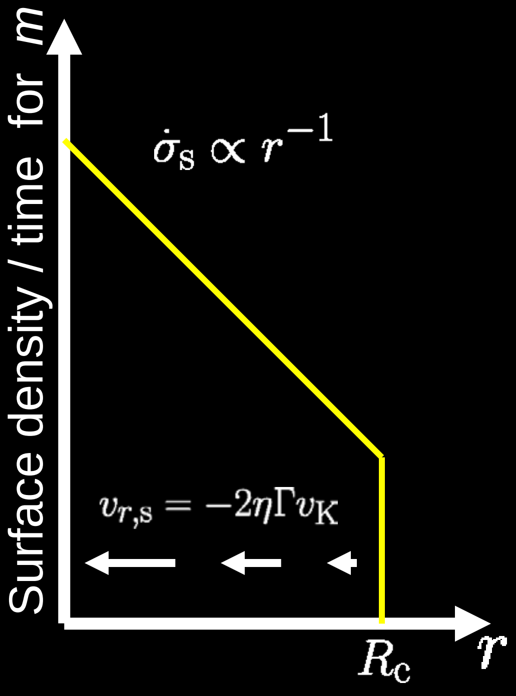
$$\dot{M}_{\rm s}(r,m) = \int_r^{R_{\rm c}} 2\pi r' \dot{\sigma}_{\rm s}(r',m)dr'$$

Steady state distribution

considering radial migration due to gas drag

For a single size swarm

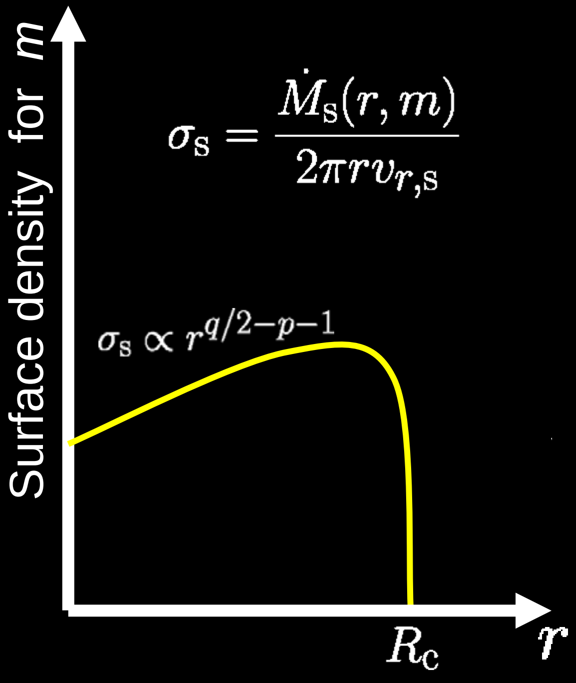
Supplying rate



$\dot{\sigma}_\mathrm{s} \propto r^{-1}$

$v_{r,\mathrm{s}} = -2\eta\Gamma v_\mathrm{K}$

Steady state distribution



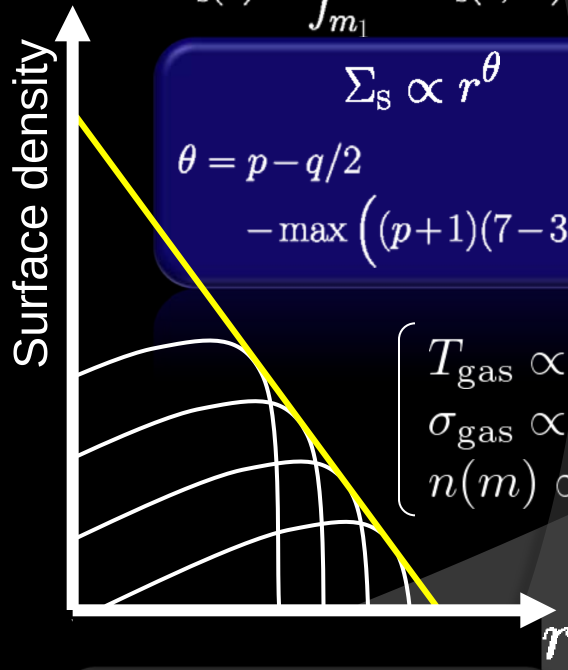
$$\sigma_\mathrm{s} = \frac{\dot{M}_\mathrm{s}(r,m)}{2\pi r v_{r,\mathrm{s}}}$$

$\sigma_\mathrm{s} \propto r^{q/2-p-1}$

$$\dot{M}_\mathrm{s}(r,m) = \int_r^{R_\mathrm{c}} 2\pi r' \dot{\sigma}_\mathrm{s}(r',m)dr'$$

For a power-law size distribution

$$\Sigma_\mathrm{s}(r) = \int_{m_1}^{m_\mathrm{c}(r)} \sigma_\mathrm{s}(r,m)dm$$



$$\Sigma_\mathrm{s} \propto r^{\theta}$$

$$\theta = p - q/2 - \max\Big((p+1)(7-3s),0\Big)$$

$$\left\{\begin{array}{l} T_\mathrm{gas} \propto r^{-q} \\ \sigma_\mathrm{gas} \propto r^{-p} \\ n(m) \propto m^{-s} \end{array}\right.$$

p =1, q =1/2, s =11/6:

$$\Sigma_\mathrm{s} \propto r^{-9/4}$$

Test orbital calculations for captured satellitesimals

Basic equations

Equation of motion

$$\frac{d\tilde{\boldsymbol{v}}}{d\tilde{t}} = -\nabla\tilde{\Phi}_{\mathrm{hill}} - 2\boldsymbol{e}_z\times\tilde{\boldsymbol{v}} + \tilde{\boldsymbol{a}}_{\mathrm{drag}}$$

Hill's potential $$\tilde{\Phi}_{\mathrm{hill}} = -\frac{3}{\tilde{r}} - \frac{3}{2}\tilde{x}^2 + \frac{1}{2}\tilde{z}^2 + \frac{9}{2}$$

Gas drag term

$$\tilde{\boldsymbol{a}}_{\mathrm{drag}} = -\frac{3}{8}C_{\mathrm{D}}\left(\frac{\rho_{\mathrm{g}}}{\rho_{\mathrm{s}}}\right)\tilde{r}_{\mathrm{s}}^{-1}\Delta\tilde{u}\Delta\tilde{\boldsymbol{u}}$$

（Only inside the Hill's sphere）

Hydrostatic equilibrium in *z*-direction and axisymmetric

$$\rho_g(r,\ z) = \rho_0 r_{\mathrm{AU}}^{p}\exp\left(-\frac{z^2}{2h_g^2}\right)$$

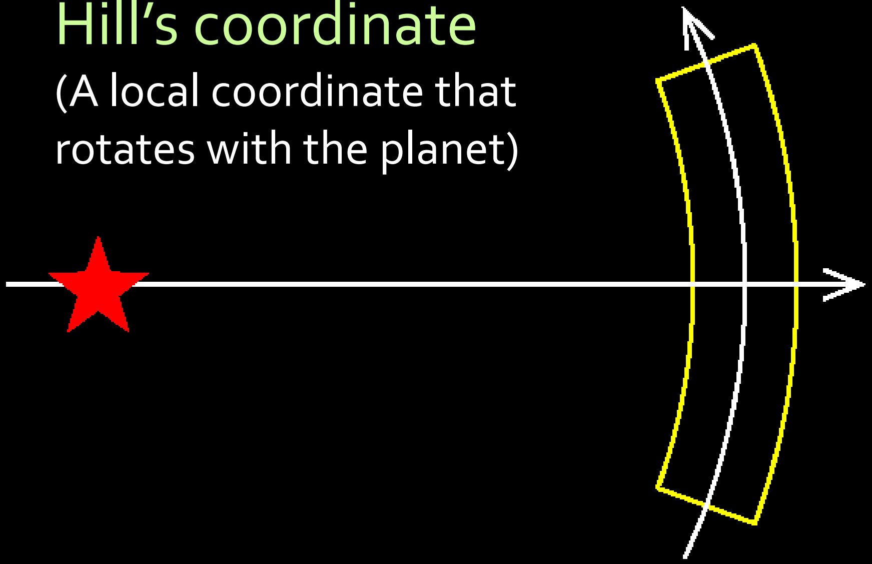
$$\Omega_g(r,\ z) = \Omega_{\mathrm{K,mid}}\left[1+\frac{1}{2}\left(\frac{h_g}{r}\right)^2\left(p+q+\frac{q}{2}\frac{z^2}{h_g^2}\right)\right]$$

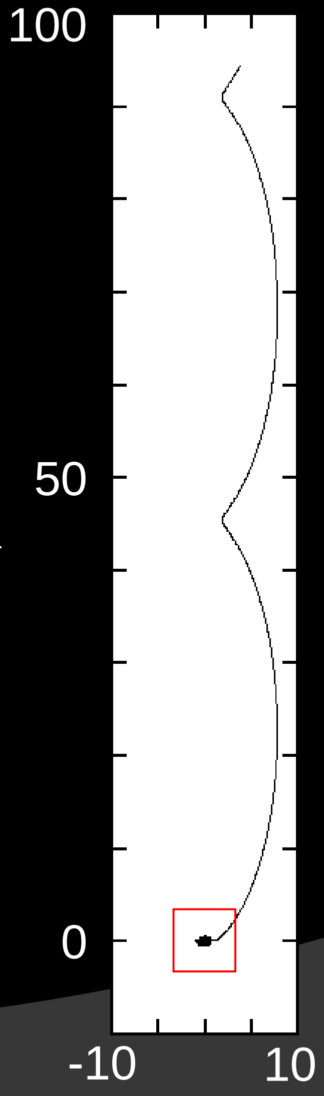
$$c^2(r) = c_0^2 r_{\mathrm{AU}}^{q}$$

Example orbits

Hill's coordinate

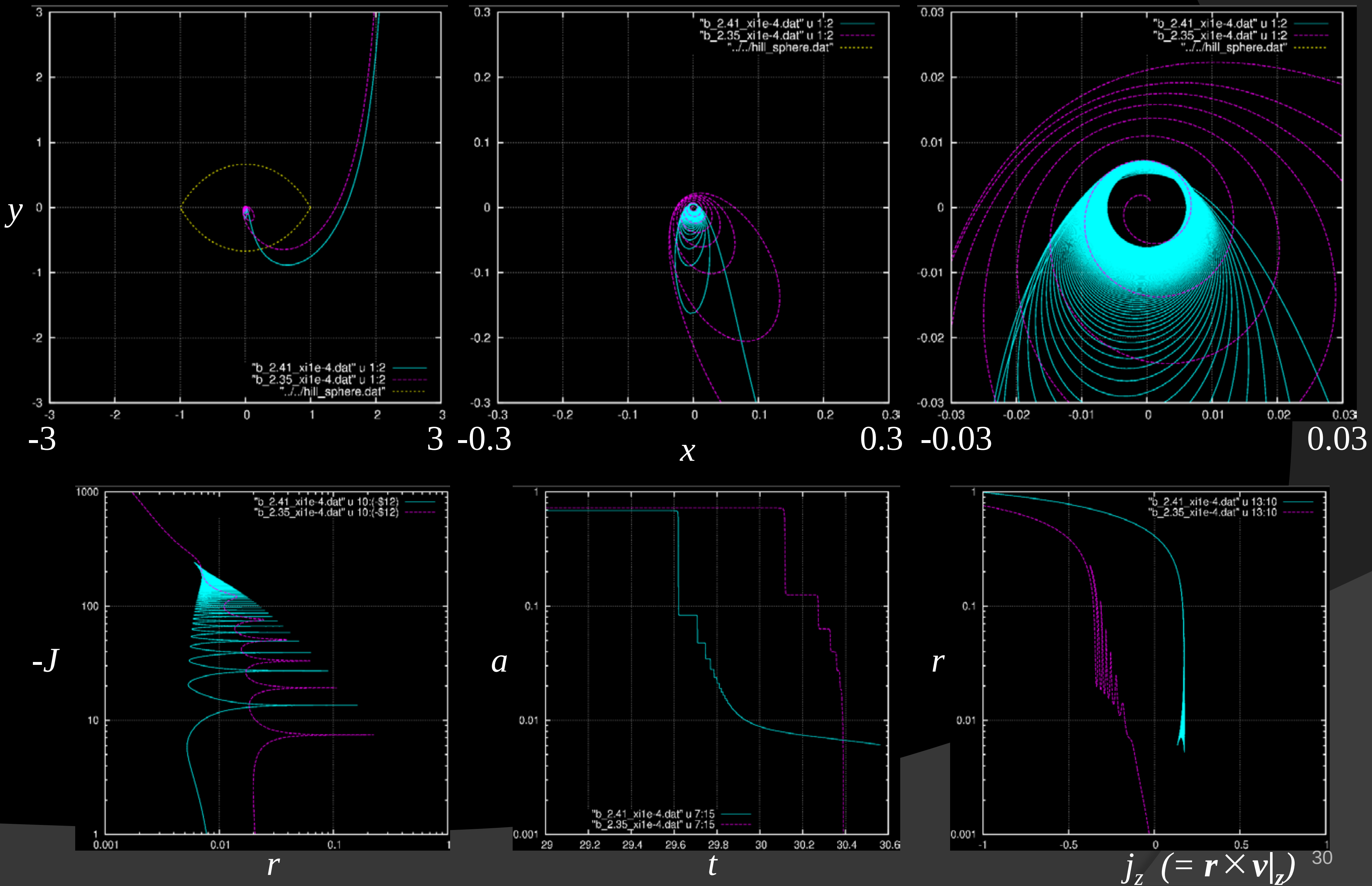
(A local coordinate that rotates with the planet)





Example orbits (e=i=0, b=2.35, 2.41)

Prograde ——— Retrograde ———



Summary

- Solid supply onto circum-planetary disks
  - Capture of planetesimals by gas drag with circum-planetary disks
  - Analytical estimation
    - Distribution of solid supplying rate

$$\dot{\sigma}_{\rm solid} \propto r^{-1-3(p+1)(2-s)} \quad \textit{cf.}\ \sigma_{\rm gas} \propto r^{-p}$$

Typical case

$\dot{\sigma}_{\rm solid} \propto r^{-2}$ for m – km size（s=11/6）

$\dot{\sigma}_{\rm solid} \propto r^{-1}$ for larger than 1km size（s=8/3）

- Gradients of solid and gas surface density is generally different.
  - Dust/gas ratio is a function of radius

- Dependence of solid supplying rate on gas surface density
  - Proportional to (gas surface density)$^{1/2}$
  - → Dust/gas ratio increases in the late stage