Workshop on MRI in Protoplanetary Disks
3rd June 2009, Kobe University, Kobe, Japan

Dust Motion in a Protoplanetary Disk in the Vicinity of an Embedded Planet

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In collaboration with

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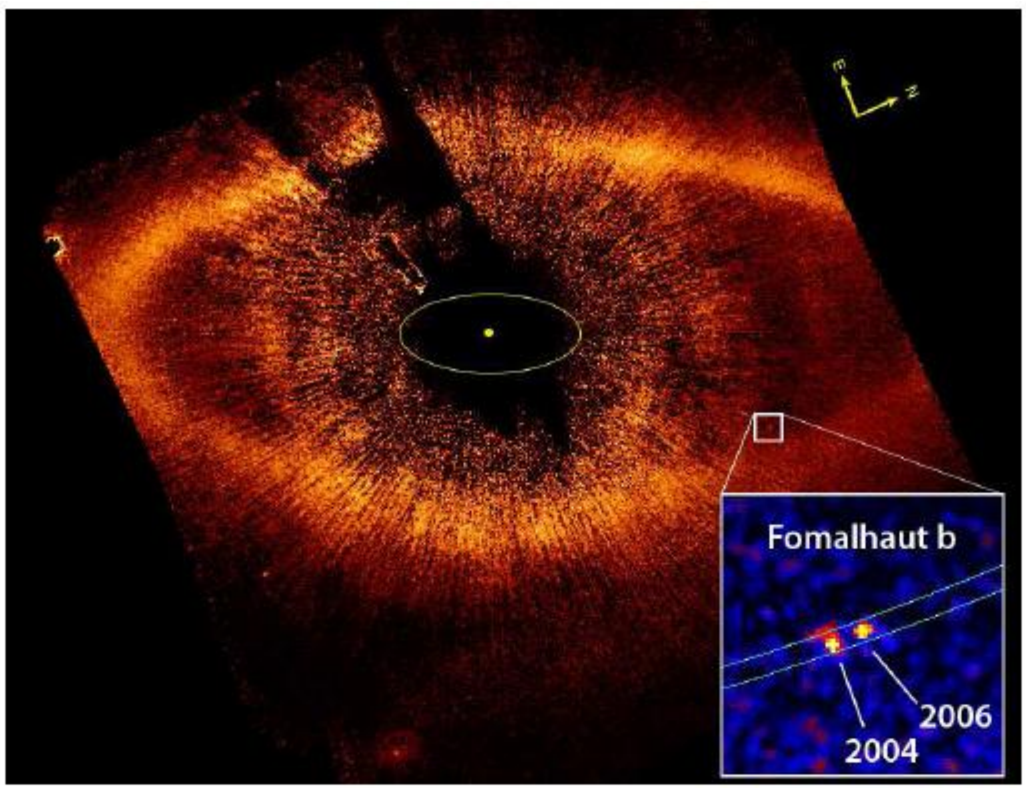
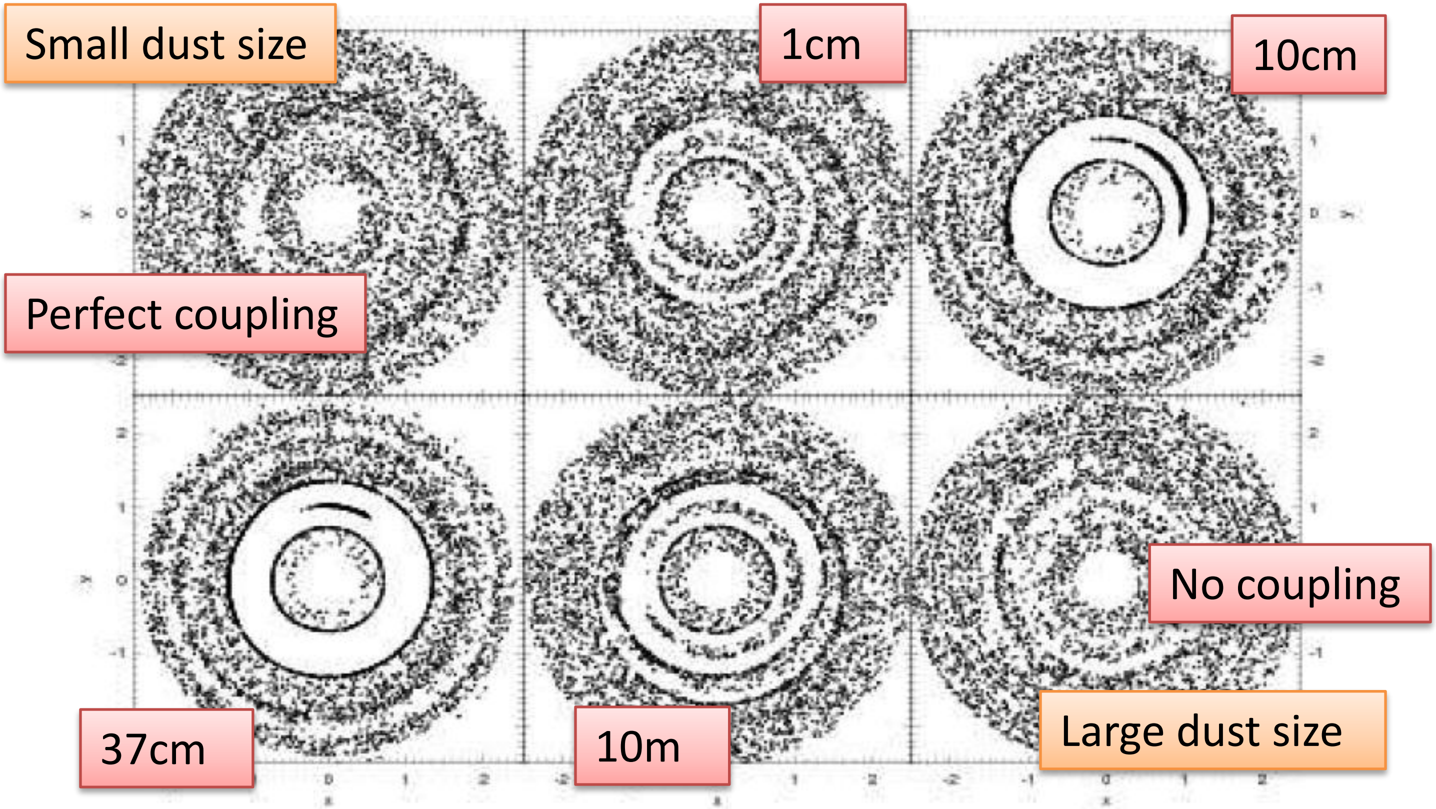
- Introduction
- Analytic Investigation of Dust Motion around a low mass planet
- Application + Discussion

Dust distribution in a protoplanetary disk

- Dust motion/distribution in a disk
  - One clue of the presence/mass of an embedded planet (e.g., Kalas et al. 2008 and Chiang et al. 2008 for Fomalhaut debris disk)
  - Formation of the core of gas giant / rocky planet



Previous Numerical Study



- Jupiter mass planet
- Distribution at 20 orbits

Paardekooper 2006

This Work: Analytic Study

- Study low-mass planet case
  - Complementary to previous studies
- General analytic formula of the secular evolution of dust particle's semi-major axis
  - Arbitrary dust size (drag coefficient)
  - Non-axisymmetric gas structure is taken into account
- Application: Long-term evolution of dust particle distribution

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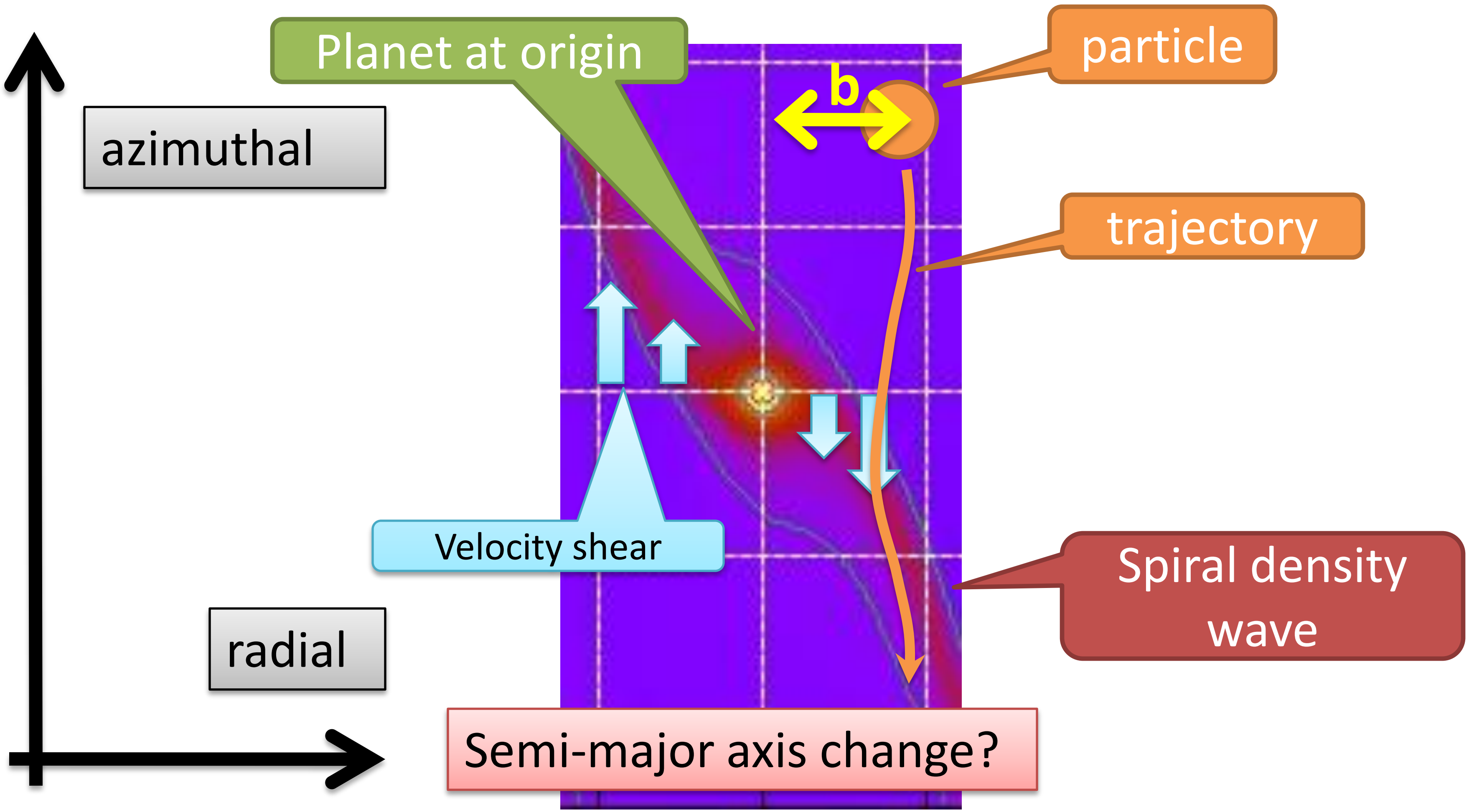
Problem Setup

- How does the dust particle's orbital semi-major axis evolve in the presence of gas + planet?



Basic equations of dust motion

- Consider a dust with semi-major axis close to the planet
  - Hill approx + gas drag

Gas drag

Planet gravity

$$\ddot{x} - 2\Omega_{\rm p}\dot{y} = 3\Omega_{\rm p}^2 x - \nu(\dot{x} - v_{x,{\rm gas}}) - \frac{\partial}{\partial x}\psi_{\rm p}$$

$$\ddot{y} + 2\Omega_{\rm p}\dot{x} = -\nu(\dot{y} - v_{y,{\rm gas}}) - \frac{\partial}{\partial y}\psi_{\rm p}$$

$$\psi_{\rm p} = -\frac{GM_{\rm p}}{\sqrt{x^2+y^2}}$$

ν: drag coefficient (corresponds to dust size)

← assumed to be constant

Approximations

- Laminar Disk
- No back reaction to the gas
- Impulse approximation (distant encounter)
- Dust particle is in a circular orbit initially

Derive **secular evolution** of semi-major axis of the particle

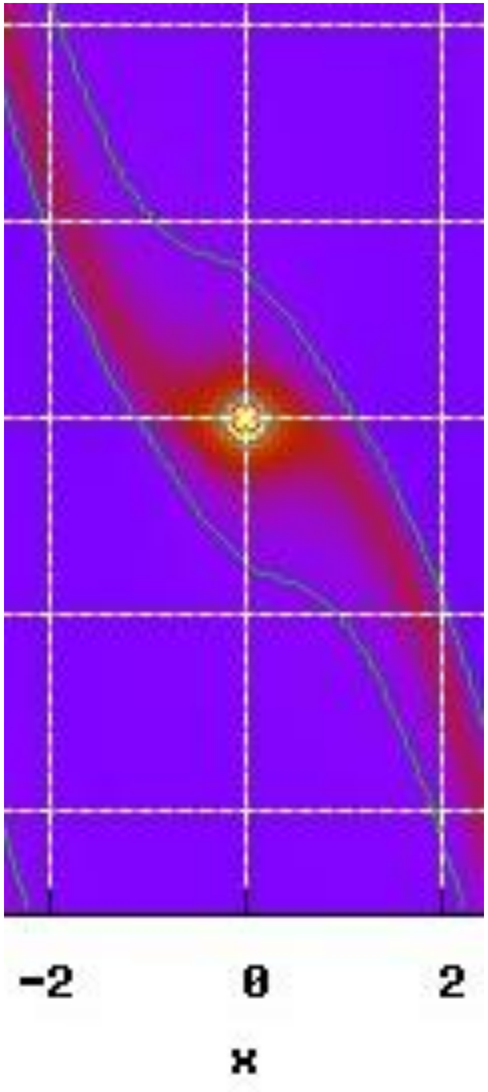
What we can **NOT** derive in this approx:
Resonance, close encounter, turbulence

Gas effects considered

$$\mathbf{v}_{\mathrm{gas}} = \mathbf{v}_{\mathrm{Kepler}} + \delta\mathbf{v}$$

$\delta\mathbf{v}$ includes:

- Effect of radial pressure gradient
- Axisymmetric radial flow
  - e.g., accretion flow
- Spiral density wave
  - Derived by 2nd order perturbation



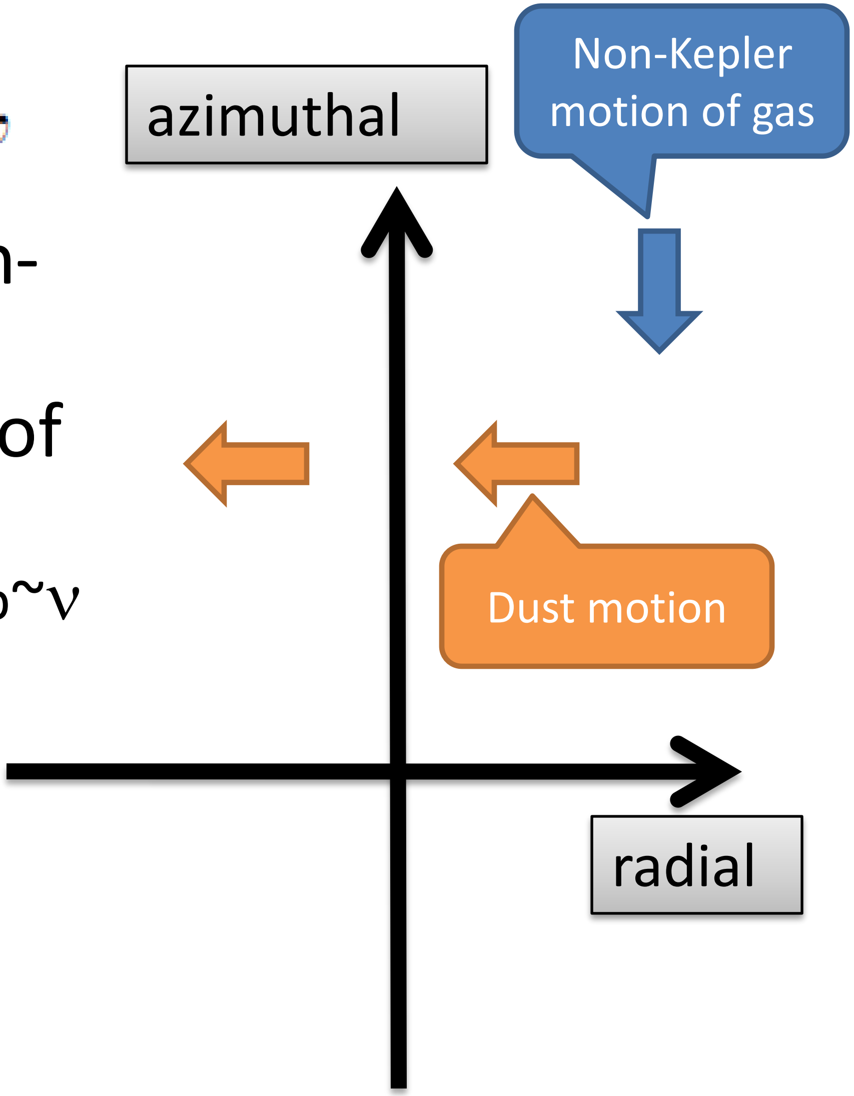
Each contribution is calculated separately, and added up

Global pressure gradient

$$\delta \boldsymbol{v}_{\mathrm{g}} = \eta v_{\mathrm{p}} \boldsymbol{e}_y = \mathrm{const},$$

- Causes gas to rotate at non-Kepler velocity
- Semi-major axis evolution of dust particles:
  - Fastest for particles with $\Omega_p \sim \nu$
- "meter-size barrier" of planetesimal formation

$$\Delta b = 2\eta v_{\mathrm{p}} T \frac{\nu \Omega_{\mathrm{p}}}{\nu^2 + \Omega_{\mathrm{p}}^2},$$

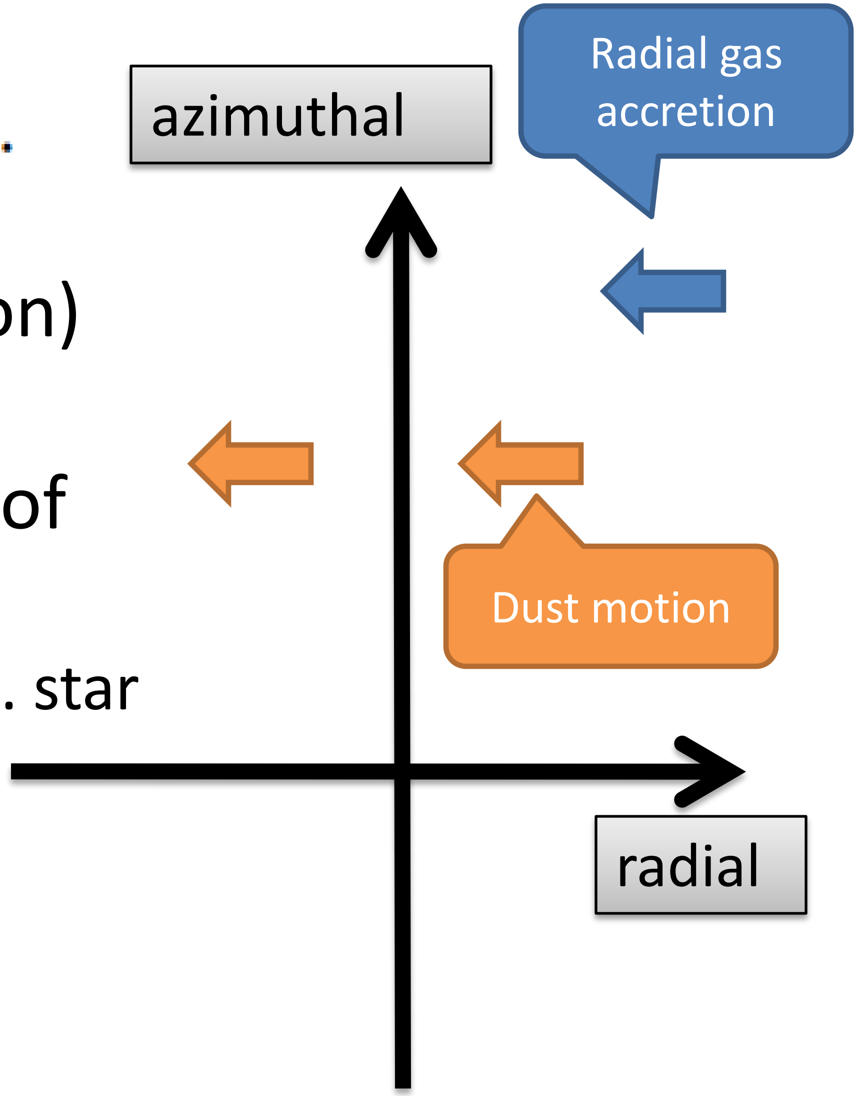


Axisymmetric radial motion

$$\delta \boldsymbol{v}_{\mathrm{g}} = \zeta v_{\mathrm{p}} \boldsymbol{e}_x = \mathrm{const.}$$

- Gas accretion (or deccretion) onto cent. star
- Semi-major axis evolution of dust particles:
  - Dust accretes onto the cent. star for $\Omega_{\mathrm{p}} << \nu$

$$\Delta b = \zeta v_{\mathrm{p}} T \frac{\nu^2}{\nu^2 + \Omega_{\mathrm{p}}^2}$$



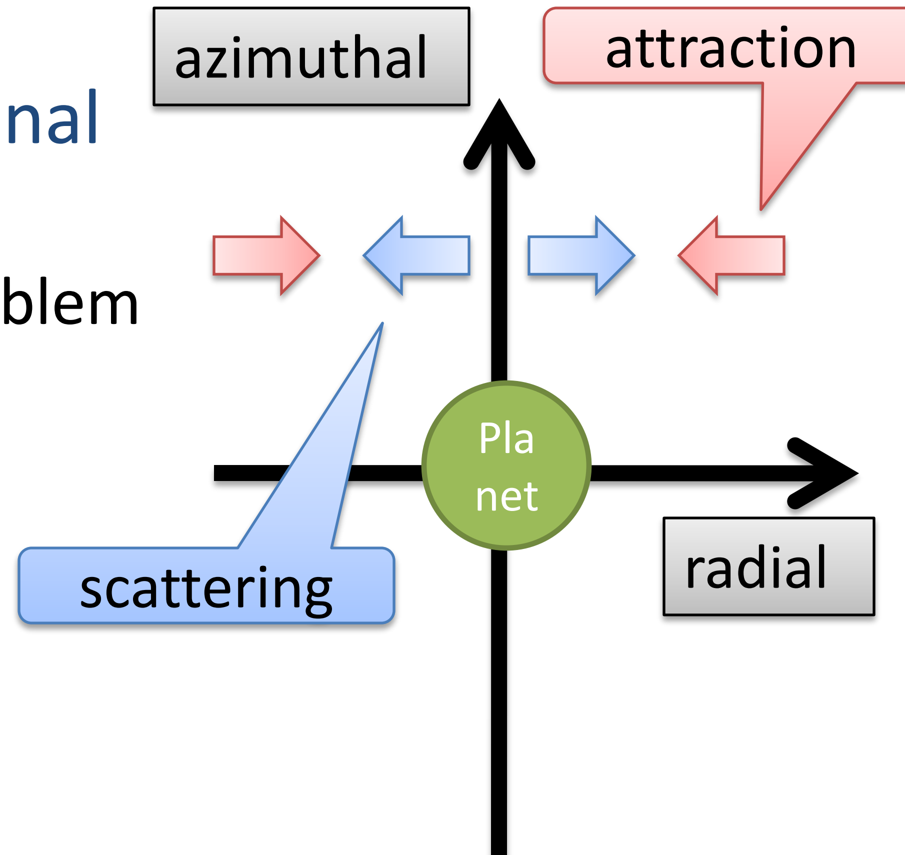
Planet encounter

attraction

scattering

$$\Delta b = -\mathrm{sgn}(b)4\frac{r_{\mathrm{H}}^3}{b^2}\frac{\nu\Omega_{\mathrm{p}}}{\nu^2+\Omega_{\mathrm{p}}^2} + \alpha\frac{r_{\mathrm{H}}^6}{b^5}\frac{\Omega_{\mathrm{p}}^2}{\nu^2+\Omega_{\mathrm{p}}^2}$$

- Modification of gravitational scattering due to gas
  - Coincides with 3-body problem without gas for $\Omega_p >> \nu$
- Drag-induced attraction towards the planet
  - Peaks at $\Omega_p \sim \nu$



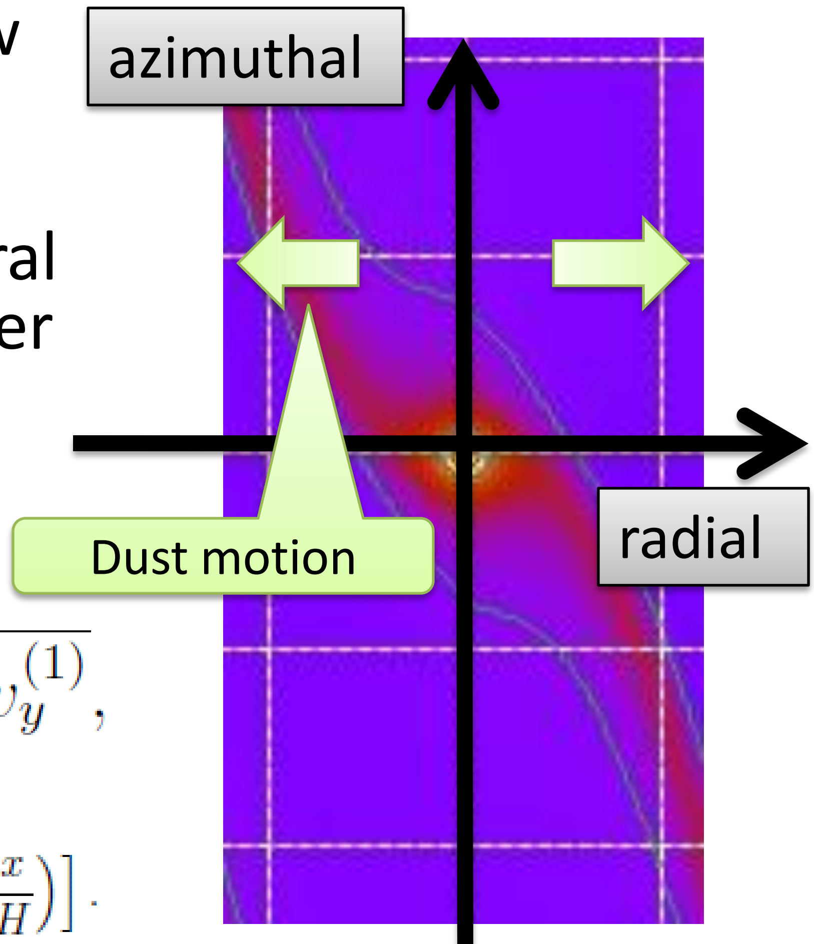
Gas flow modified by planet gravity

$$\delta \mathbf{v}_{\mathrm{g}} = \delta \mathbf{v}^{(1)} + \delta \mathbf{v}^{(2)}$$

1st order, propto Mp

2nd order, propto Mp^2

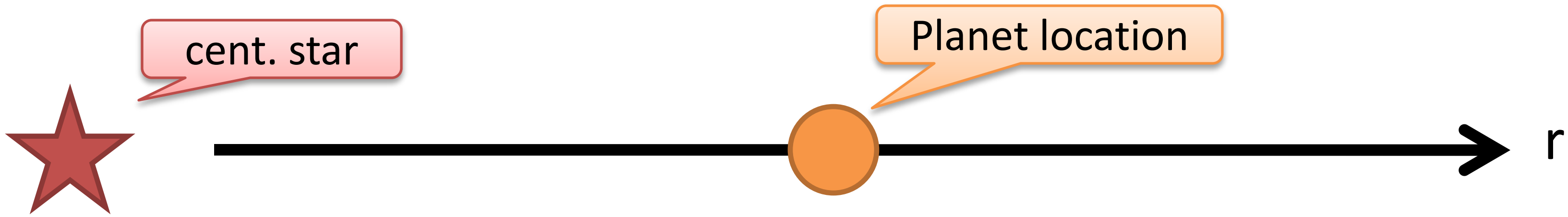
- Only 1st-order axisymmetric flow structure contributes
- Axisymmetric mode and non-axisymmetric contributions (spiral density wave) cancel when higher order terms are considered
  - Assumption: No vortensity formation



$$\Delta b = \mathrm{sgn}(b) \frac{4}{3} \frac{1}{b\Omega_{\mathrm{p}}} \frac{\nu \Omega_{\mathrm{p}}}{\nu^2 + \Omega_{\mathrm{p}}^2} L_y \overline{\delta v_y^{(1)}},$$

$$\overline{\delta v_y^{(1)}} = \frac{H^2 \Omega_{\mathrm{p}}}{2L_y} \frac{GM_{\mathrm{p}}}{Hc^2} \left[e^{-(x/H)} \mathrm{Ei}\left(\frac{x}{H}\right) - e^{x/H} \mathrm{Ei}\left(-\frac{x}{H}\right) \right].$$

Gas Effects on Particle Motion



	Inside planet orbit	Outside planet orbit
Pressure grad.	← *Depends on sign	← *Depends on sign
Radial gas flow	← *Depends on sign	← *Depends on sign
Encounter with planet	→ ← direction change at intermediate distance	→ ← direction change at intermediate distance
Spiral density wave	←	→

Semi-major axis change of the particle

Pressure gradient

Mass accretion

Gravitational scattering and attraction

$$\frac{\Delta b}{T} = 2\eta v_\mathrm{p} \frac{\nu \Omega_\mathrm{p}}{\nu^2+\Omega_\mathrm{p}^2} + \zeta v_\mathrm{p} \frac{\nu^2}{\nu^2+\Omega_\mathrm{p}^2}$$
$$-\mathrm{sgn}(b)\frac{4}{T}\frac{r_\mathrm{H}^3}{b^2}\frac{\nu\Omega_\mathrm{p}}{\nu^2+\Omega_\mathrm{p}^2} + \frac{\alpha}{T}\frac{r_\mathrm{H}^6}{b^5}\frac{\Omega_\mathrm{p}^2}{\nu^2+\Omega_\mathrm{p}^2}$$
$$+\mathrm{sgn}(b)\frac{2}{T}\frac{r_\mathrm{H}^3}{bH}\left[e^{-(b/H)}\mathrm{Ei}\left(\frac{b}{H}\right) - e^{b/H}\mathrm{Ei}\left(-\frac{b}{H}\right)\right]\frac{\nu\Omega_\mathrm{p}}{\nu^2+\Omega_\mathrm{p}^2},$$

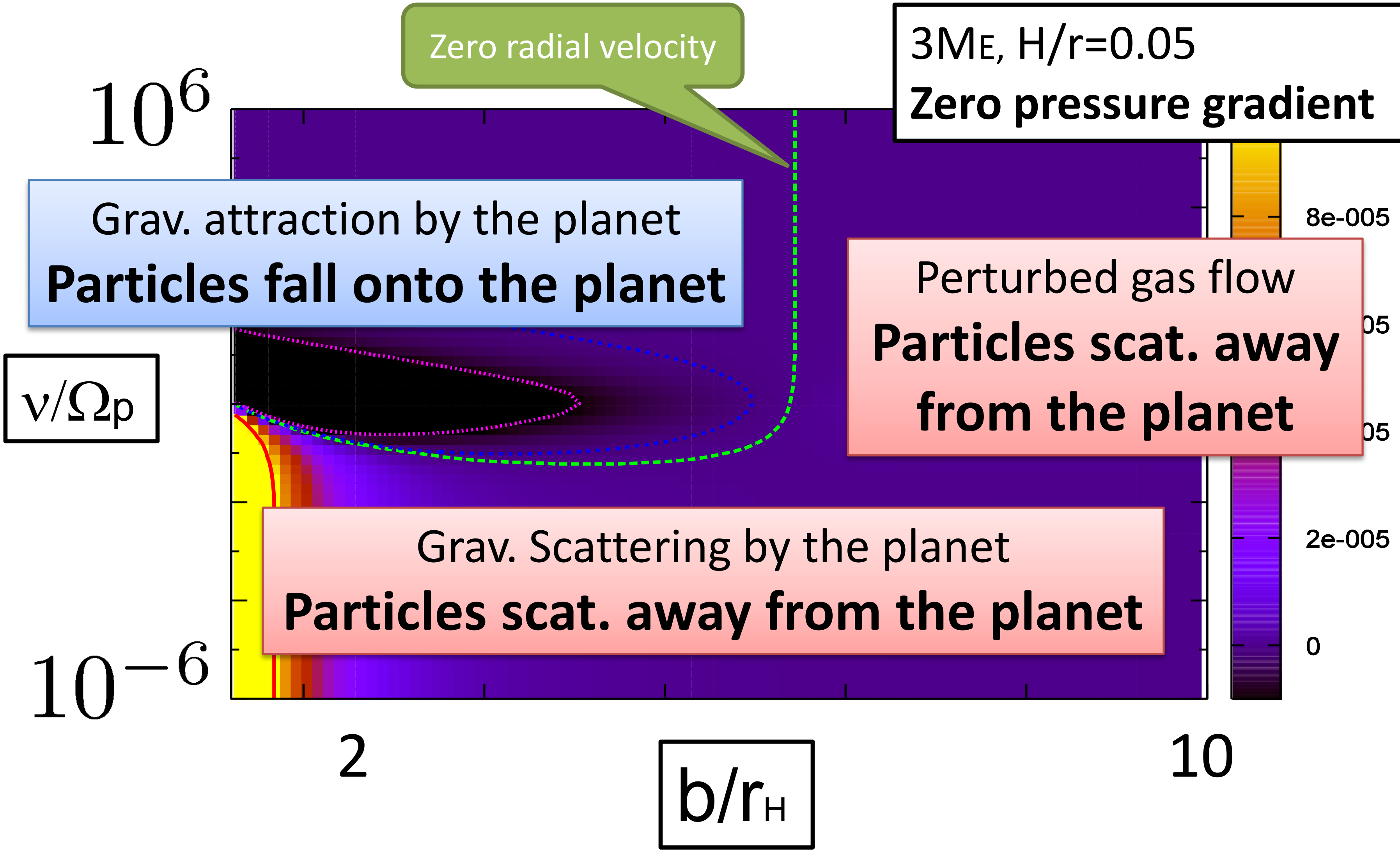
Spiral density wave

$$\alpha \equiv \frac{128}{27}\left[K_1\left(\frac{2}{3}\right) + 2K_0\left(\frac{2}{3}\right)\right]^2 = 30.094$$

The most general result for non-turbulent, non-self-gravitating gas disk

Muto and Inutsuka, 2009

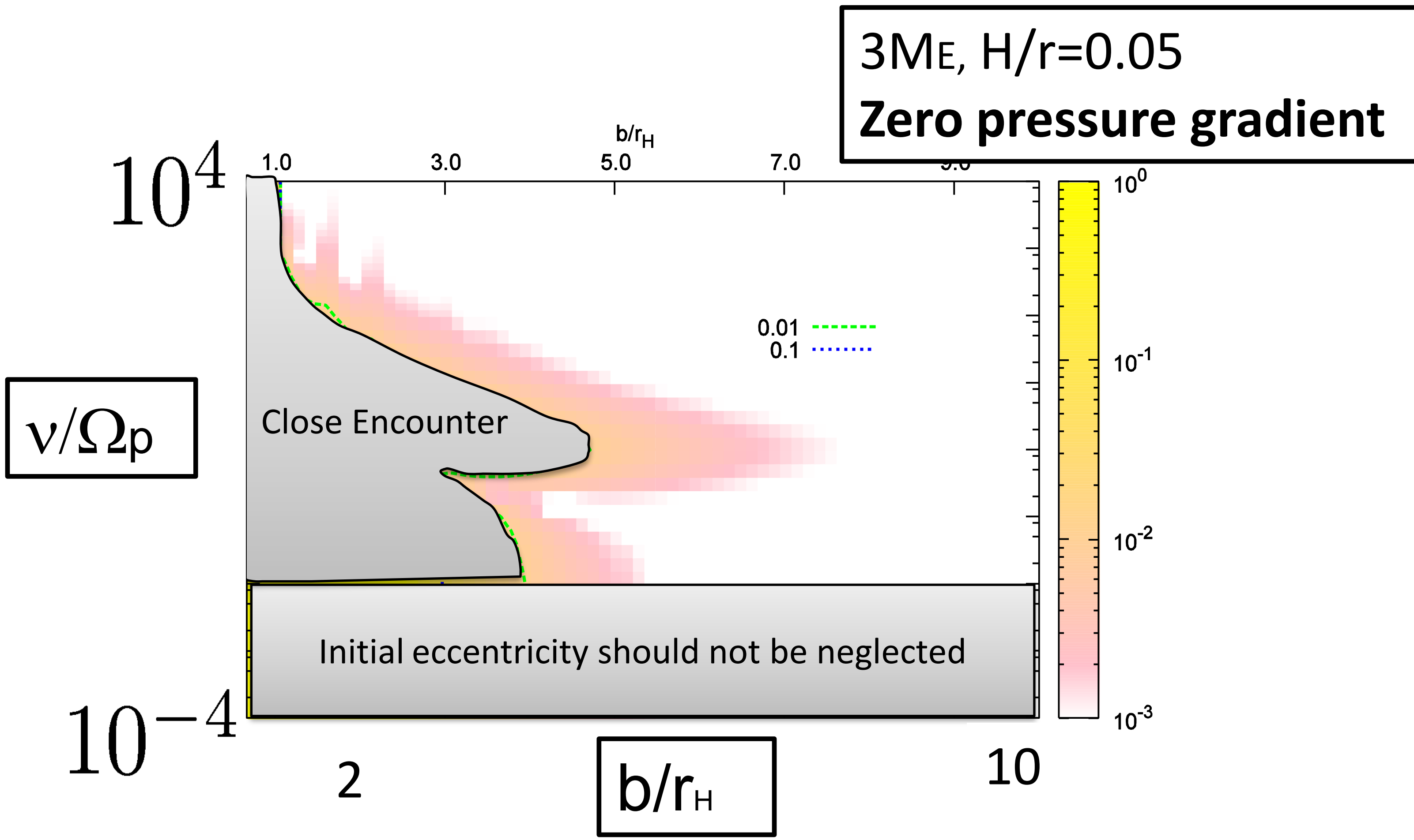
Radial velocity of the particle: example



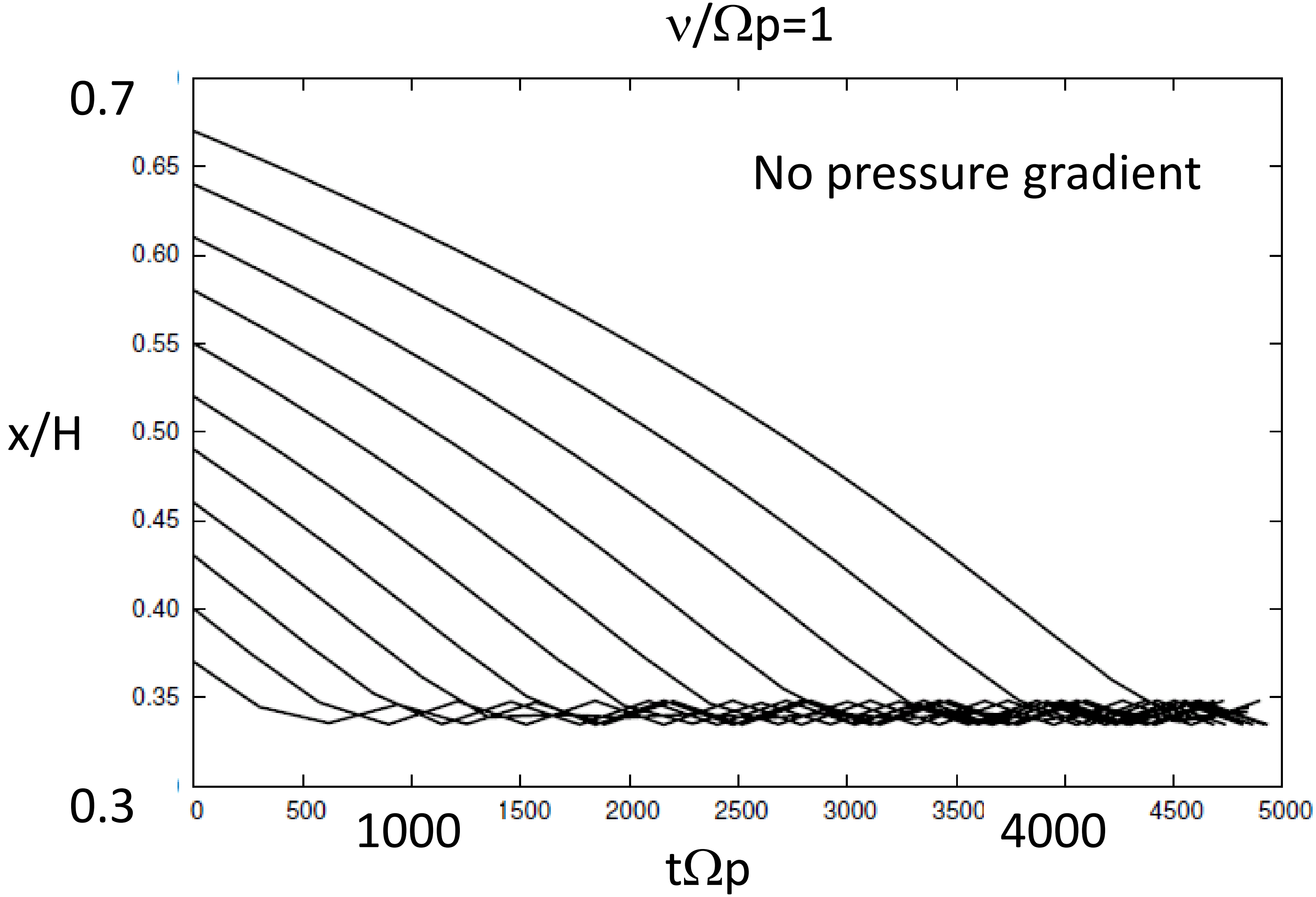
Applicability of analytic formula

- Compare analytic results with numerical calculation
- Analytic results
  - well describe motions of particles with large drag
  - qualitatively good approx. of motions of particles with small drag

Validity diagram of the formula



Example of Semi-major Axis Evolution



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  - Model of long-term evolution of dust particle distribution
  - Is it possible to detect a low-mass planet embedded in a disk?

Model of long-term evolution of dust particle distribution

1-dimensional model: only *radial* distribution

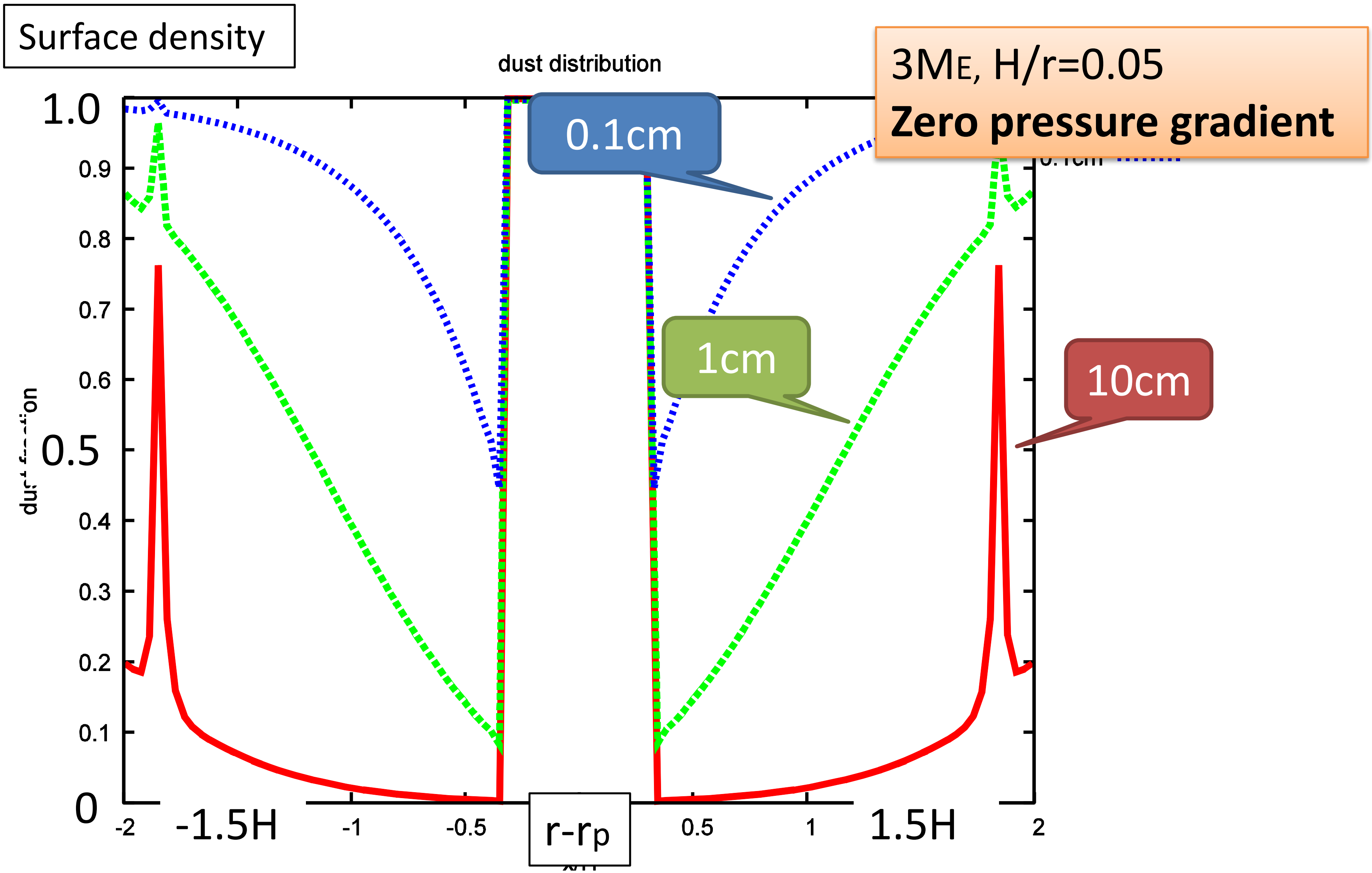
$$\frac{\partial N(t,b)}{\partial t} + \frac{\partial}{\partial b}\left[v_b(b) N(t,b)\right] = 0$$

Dust radial velocity

Make use of the analytic results of dust semi-major axis evolution

Easily follow the evolution of ~10^6 years

Distribution of various size dust @ t=10^6yr



Is it possible to detect a low-mass planet embedded in a disk?

- Gap width of ~H for ~0.1-1cm particles
  - Local pressure gradient should be close to zero
- For H/r_p=0.05 and $3M_E$@30AU, gap with ~1-2AU
- **0.01" @ 100pc with λ>1cm**
- Possibly at shorter wavelength if small particles are depleted.

- Maybe possible with ALMA, higher possibility with SKA?

Summary

- Analytic formula of dust particle’s semi-major axis evolution is derived
- General results including the effects of
  - Embedded low-mass planet
  - Effect of radial pressure gradient
  - Axisymmetric accretion flow onto the central star
  - Spiral density wave
- Results with arbitrary dust size (stopping time)
  - The formula is especially useful for small particles
- Model of lomg-term evolution of dust surface density
  - Gap width with ~H
  - Direct imaging with ALMA/SKA can be used to detect an embedded low-mass planet (but very close to detection limit…)