慣性重力波にも適用可能な
３次元波動活動度フラックス

宮原　三郎　　（九大・理）

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神戸大学

研究の動機

1980年代初頭　大気大循環における重力波の重要性の認識

Lindzen, 1981: Turbulence and stress owing to gravity wave and tidal breakdown, JGR, 86.

Matsuno, 1982: A quasi one-dimensional model of the middle atmosphere circulation interacting with internal gravity waves, JMSJ, 60.

中間圏界面付近の弱風層および夏冬の温度逆転の成因

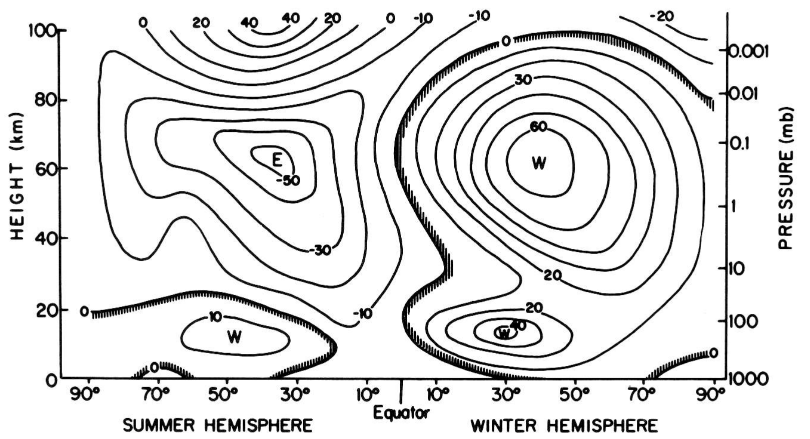


Fig. 1.4. Schematic latitude-height section of zonal mean zonal wind ($m\ s^{-1}$) for solstice conditions; W and E designate centers of westerly (from the west) and easterly (from the east) winds, respectively. (Courtesy of R. J. Reed.)

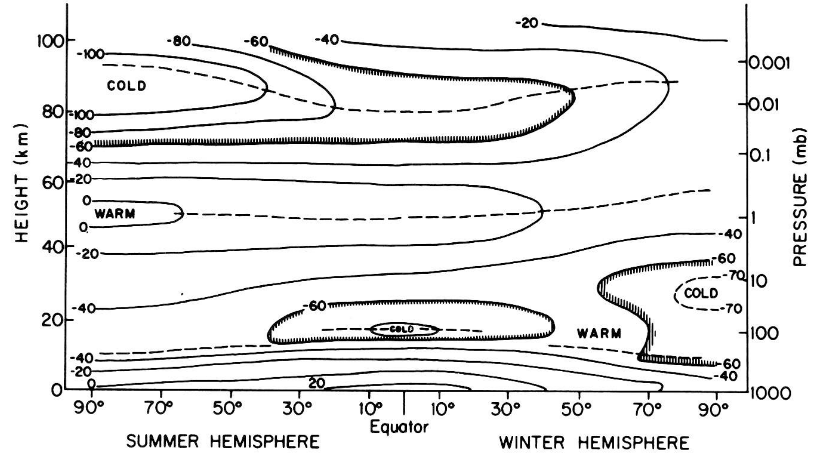


Fig. 1.3. Schematic latitude-height section of zonal mean temperatures (°C) for solstice conditions. Dashed lines indicate tropopause, stratopause, and mesopause levels. (Courtesy of R. J. Reed.)

Andrews et al. 1987: Middle atmosphere Dynamics

Mechanistic models: ２次元モデル＋重力波パラメタリゼーション

たとえば

Holton, 1982: The role of gravity wave induced drag and diffusion in the momentum budget of the mesosphere, JAS, 39.
Miyahara, 1984: A numerical simulation of the zonal mean circulation of the middle atmosphere including effects of solar diurnal tidal waves and internal gravity waves; Solstice condition, Dynamics of the middle atmosphere.

重力波の重要性の確認（重大循環モデルによる力波パラメタルゼーション無し）

Miyahara et al., 1986: Interactions between gravity waves and planetary scale flow simulated by the GFDL “SKYHI” general circulation model, JAS, 43.

帯状平均場のみではなく，プラネタリースケールでの重力波の分布と役割を考察

時間平均場の中の重力波に伴う鉛直運動量fluxと東西風加速
Miyahara et al.(1986)

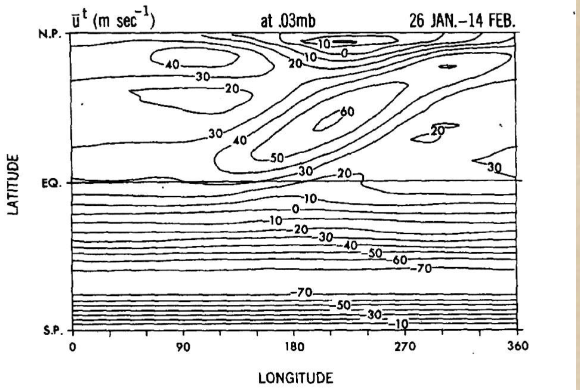


FIG. 15a. Longitude–latitude distribution of the time-mean zonal wind (m s^{-1}) at 0.03 mb, consisting of zonal wavenumbers 0–3 averaged over the PII period.

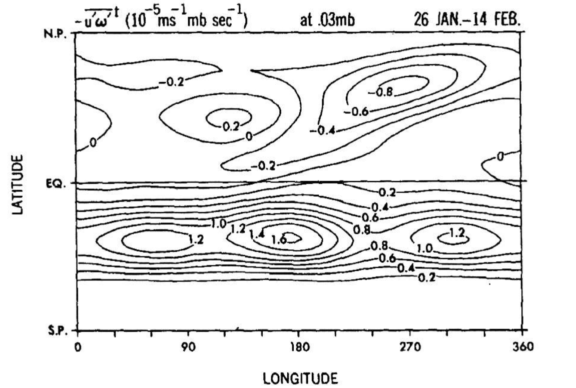


FIG. 15b. Longitude–latitude distribution of spatially smoothed time-mean vertical flux of zonal momentum (10^{-5} m s^{-1} mb s^{-1}) due to higher wavenumber components ($5 \leq k \leq 50$, $5 \leq l \leq 50$) at 0.03 mb. The spatial smoothing is accomplished by a wavenumber filter (k = 0–3).

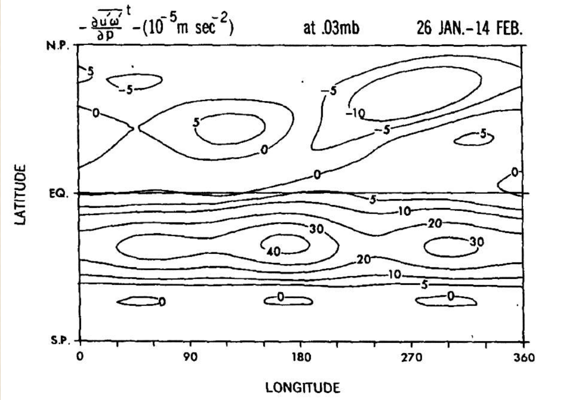


FIG. 15c. Longitude–latitude distribution of the convergence of zonal momentum flux (10^{-5} m s^{-2}) at 0.03 mb.

非地衡風擾乱に対する３次元的flux の理論が確立していないので

$$\overline{u}^t, \quad \overline{u'w'}^t, \quad \frac{\partial \overline{u'w'}^t}{\partial z}$$

の分布を調べた。

この時点で，Plumb (1985), Plumb (1986)のQG-waveに関するflux理論は完成していた.

当時から，慣性重力波に対して適用可能なfluxを研究していたが，未完に終わった.

平均場が帯状平均東西風の場合

帯状平均　wave flux --> E-P Flux

$$\frac{\partial}{\partial y}\left(\overline{u'v'} + \frac{1}{N^2}\frac{\partial \overline{u}}{\partial z}\overline{v'r'}\right) + \frac{\partial}{\partial z}\left(\overline{u'w'} + (f - \frac{\partial \overline{u}}{\partial y})\frac{\overline{v'r'}}{N^2}\right) = 0$$

$$F_y = -\overline{u'v'} - \frac{1}{N^2}\frac{\partial \overline{u}}{\partial z}\overline{v'r'} \quad , \quad F_z = -\overline{u'w'} - (f - \frac{\partial \overline{u}}{\partial y})\frac{\overline{v'r'}}{N^2}$$

エリアッセン・パームflux(E-P flux)は非発散となる

エリアッセン・パームの定理

Eliassen and Palm (1960), Charney and Drazin (1961), Uryu (1973), Andrews and McIntyre (1976) etc.

Transformed Eulerian-Mean (TEM)方程式

$$\frac{\partial \overline{u}}{\partial t} + \overline{v}^* \frac{\partial \overline{u}}{\partial y} + \overline{w}^* \frac{\partial \overline{u}}{\partial z} - f\overline{v}^* = \frac{\partial}{\partial y}\left[-\overline{u'v'} - \frac{1}{N^2}\frac{\partial \overline{u}}{\partial z}\overline{v'r'} \right] + \frac{\partial}{\partial z}\left[-\overline{u'w'} - \left(f - \frac{\partial \overline{u}}{\partial y} \right)\frac{1}{N^2}\overline{v'r'} \right] + \overline{X} \ ,$$

$$\frac{\partial \overline{r}}{\partial t} + \overline{v}^* \frac{\partial \overline{r}}{\partial y} - N^2\overline{w}^* = -\frac{\partial}{\partial z}\left[\overline{r'w'} - \frac{\partial \overline{r}}{\partial y}\frac{\overline{r'v'}}{N^2} \right] + \overline{J} \ ,$$

$$F_y = -\overline{u'v'} - \frac{1}{N^2}\frac{\partial \overline{u}}{\partial z}\overline{v'r'} \quad , \quad F_z = -\overline{u'w'} - (f - \frac{\partial \overline{u}}{\partial y})\frac{\overline{v'r'}}{N^2}$$

$Eliassen - Palm \ (E - P) \ Flux$

x方向の 運動量のy方向，z方向への輸送量(flux)を表現

大気中では，東向き帯状平均運動量の子午面内でのfluxを表現
波動の子午面内での伝播方向を示す
残差循環　物質輸送を近似的に表現

Andrews and McIntyre (1976)

保存量形式

$$\frac{\partial}{\partial t}\left(\frac{\overline{E}}{\overline{u}-c}\right)+\frac{\partial}{\partial y}\left(\overline{-u'v'}-\frac{1}{N^2}\frac{\partial\overline{u}}{\partial z}\overline{v'r'}\right)+\frac{\partial}{\partial z}\left(\overline{-u'w'}-(f-\frac{\partial\overline{u}}{\partial y})\frac{\overline{v'r'}}{N^2}\right)=D$$

E-P Flux は $\left(\frac{E}{c-\overline{u}}\right)$ のFlux を与える

$\left(\frac{E}{\omega-k\overline{u}}\right)$ Wave action

準地衡風擾乱・波動の場合

$$\mathbf{F} = \left(-\overline{u'v'} \ , \ -\frac{f_0}{N_0^{\ 2}}\overline{v'r'} \right)$$

準地衡風渦位擾乱

$$q' = \frac{\partial^2 \psi'}{\partial x^2} + \frac{\partial^2 \psi'}{\partial y^2} + \frac{\partial}{\partial z}\left(\frac{f_0^{\ 2}}{N_0^2} \frac{\partial \psi'}{\partial z} \right)$$

準地衡風渦位擾乱の南北方向輸送(flux)

$$\overline{v'q'} = \frac{\partial}{\partial y}\left(-\overline{u'v'} \right) + \frac{\partial}{\partial z}\left(-\frac{f_0}{N_0^2}\overline{v'r'} \right) = \nabla \cdot \mathbf{F}$$

波動の３次元的伝播　３次元的wave flux

準地衡風的波動の場合

Hoskins et al. (1983), Plumb(1985, 1986),
Trenberth (1986), Takaya and Nakamura (2001) etc.

Planetary wave についての３次元的 wave flux
wave activity flux の大きさと方向を与える
波動の３次元的伝播の解明

Plumb (1986)

準地衡風渦度保存則 $\frac{dq}{dt}=0$， $q=f+\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}+\frac{f^2}{N^2}\frac{\partial^2\psi}{\partial z^2}$

時間平均場 $\overline{u}$, $\overline{v}$ 下での波動

$$\frac{Dq'}{Dt}+\mathbf{u}'\cdot\nabla_H\overline{q}=0 \quad , \qquad \frac{D}{Dt}=\frac{\partial}{\partial t}+\overline{u}\frac{\partial}{\partial x}+\overline{v}\frac{\partial}{\partial y}$$

$$\frac{De}{Dt}+\overline{\mathbf{u}'q'}\cdot\nabla_H\overline{q}=0$$

$e=\frac{1}{2}\overline{q'^2}$ is the eddy potential enstrophy.

$$\overline{\mathbf{u}'q'} = div \begin{bmatrix} \overline{u'v'} & \varepsilon - \overline{u'^2} & \dfrac{f\,\overline{u'\theta'}}{d\bar{\theta}/dz} \\ \overline{v'^2} - \varepsilon & -\overline{u'v'} & \dfrac{f\,\overline{v'\theta'}}{d\bar{\theta}/dz} \\ 0 & 0 & 0 \end{bmatrix} \quad , \quad \varepsilon = \frac{1}{2}\left(\overline{u'^2} + \overline{v'^2} + \alpha \frac{\overline{\theta'^2}}{d\bar{\theta}/dz} \right)$$

$$\alpha = \frac{R}{H}\exp(-\kappa z / H)$$

Slowly varying background field --> WKB Approximation

$$\frac{DM}{Dt} + \nabla \cdot \mathbf{M_R} = 0$$

$$M = e / \left| \nabla_H \bar{q} \right|$$

$$\mathbf{M_R} = \mathbf{n} \cdot B, \quad \mathbf{n} = \nabla_H \bar{q} / \left| \nabla_H \bar{q} \right|$$

$$B \equiv \begin{bmatrix} \overline{u'v'} & \varepsilon - \overline{u'^2} & \dfrac{f\,\overline{u'\theta'}}{d\bar{\theta}/dz} \\ \overline{v'^2} - \varepsilon & -\overline{u'v'} & \dfrac{f\,\overline{v'\theta'}}{d\bar{\theta}/dz} \\ 0 & 0 & 0 \end{bmatrix}$$

$$\frac{DM}{Dt} + \nabla \cdot \mathbf{M_R} = 0$$

More useful form

$$\frac{\partial M}{\partial t} + \nabla \cdot \mathbf{M_T} = 0, \quad \mathbf{M_T} = \mathbf{M_R} + \bar{\mathbf{u}} M$$

$$\mathbf{M_T} = \mathbf{c_g} M$$

$\mathbf{M_T}$ は wave activity $M = e / \left| \nabla_H \bar{q} \right|$ の fluxを与える.

これらの物理量は全て場が与えられていれば計算可能

Trenberth (1986): Time mean equation system under the lnp coordinate

簡単のために静力学平衡ブシネスク系で表現

時間平均場 $\overline{u}$, $\overline{v}$ 下での波動の wave flux

$$\frac{D\overline{u}}{Dt} - f\overline{v}^* = -\frac{\partial\overline{\Phi}}{\partial x} - \nabla\cdot E_x$$

$$\frac{D\overline{v}}{Dt} + f\overline{u}^* = -\frac{\partial\overline{\Phi}}{\partial y} - \nabla\cdot E_y$$

$$\frac{D\overline{r}}{Dt} - N^2\overline{w}^* \cong 0$$

$$\overline{u}^* = \overline{u} + \frac{1}{f}\frac{\partial\overline{K}}{\partial y} + \frac{\partial}{\partial z}\left(\frac{\overline{u'r'}}{N^2}\right)$$

$$\overline{v}^* = \overline{v} - \frac{1}{f}\frac{\partial\overline{K}}{\partial x} + \frac{\partial}{\partial z}\left(\frac{\overline{v'r'}}{N^2}\right)$$

$$\overline{w}^* = \overline{w} - \frac{\partial}{\partial x}\left(\frac{\overline{u'r'}}{N^2}\right) - \frac{\partial}{\partial y}\left(\frac{\overline{v'r'}}{N^2}\right)$$

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \overline{u}\frac{\partial}{\partial x} + \overline{v}\frac{\partial}{\partial y}$$

$$\overline{K} = \frac{1}{2}\left(\overline{u'^2} + \overline{v'^2}\right)$$

ここでテンソルE は

$$E = \begin{bmatrix} \frac{1}{2}\left(\overline{u'^2} - \overline{v'^2}\right) & \overline{u'v'} & \frac{f}{N^2}\overline{v'r'} \\ \overline{u'v'} & \frac{1}{2}\left(\overline{v'^2} - \overline{u'^2}\right) & -\frac{f}{N^2}\overline{u'r'} \\ 0 & 0 & 0 \end{bmatrix}$$

Plumb (1986) との関連などを準地衡風系について議論

Plumb (1985), Takaya and Nakamura (2001)

準地衡風系に適用可能な平均を必要としない３次元
wave flux を提案

３次元ブシネスク系非地衡風的

- 内部重力波・潮汐波・赤道波などにも適用可能な wave activity flux について考える

時間平均場 $\overline{u}, \quad \overline{v}$

下での波動の wave activity flux

$$\frac{D\bar{u}}{Dt} - f\bar{v} = -\frac{\partial \bar{\Phi}}{\partial x} - \frac{\partial \overline{u'u'}}{\partial x} - \frac{\partial \overline{u'v'}}{\partial y} - \frac{\partial \overline{u'w'}}{\partial z} = -\frac{\partial \bar{\Phi}}{\partial x} - \frac{\partial}{\partial x}\frac{1}{2}\left(\overline{u'^2} + \overline{v'^2} + \overline{w'^2} - \frac{\overline{r'^2}}{N^2}\right)$$

$$-\frac{\partial}{\partial x}\frac{1}{2}\left(\overline{u'^2} - \overline{v'^2} - \overline{w'^2} + \frac{\overline{r'^2}}{N^2}\right) - \frac{\partial \overline{u'v'}}{\partial y} - \frac{\partial \overline{u'w'}}{\partial z}$$

$$\frac{D\bar{v}}{Dt} + f\bar{u} = -\frac{\partial \bar{\Phi}}{\partial y} - \frac{\partial \overline{u'v'}}{\partial x} - \frac{\partial \overline{v'v'}}{\partial y} - \frac{\partial \overline{v'w'}}{\partial z} = -\frac{\partial \bar{\Phi}}{\partial y} - \frac{\partial}{\partial y}\frac{1}{2}\left(\overline{u'^2} + \overline{v'^2} + \overline{w'^2} - \frac{\overline{r'^2}}{N^2}\right)$$

$$-\frac{\partial \overline{u'v'}}{\partial x} - \frac{\partial}{\partial y}\frac{1}{2}\left(\overline{v'^2} - \overline{u'^2} - \overline{w'^2} + \frac{\overline{r'^2}}{N^2}\right) - \frac{\partial \overline{v'w'}}{\partial z}$$

$$\frac{\partial \bar{\Phi}}{\partial z} = -\bar{r}$$

$$\frac{D\bar{r}}{Dt} - N^2\bar{w} = -\frac{\partial \overline{r'u'}}{\partial x} - \frac{\partial \overline{r'v'}}{\partial y} - \frac{\partial \overline{r'w'}}{\partial z}$$

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \bar{u}\frac{\partial}{\partial x} + \bar{v}\frac{\partial}{\partial y} + \bar{w}\frac{\partial}{\partial z}$$

右辺はレイノズルストレス項を変形

$$\bar{S} \equiv \frac{1}{2}\left(\overline{u'^2} + \overline{v'^2} + \overline{w'^2} - \frac{\overline{r'^2}}{N^2}\right)$$

$$\frac{D\bar{u}}{Dt} - f\bar{v} = -\frac{\partial\bar{\Phi}}{\partial x} - \frac{\partial\bar{S}}{\partial x} - \frac{\partial}{\partial x}\frac{1}{2}\left(\overline{u'^2} - \overline{v'^2} - \overline{w'^2} + \frac{\overline{r'^2}}{N^2}\right) - \frac{\partial\overline{u'v'}}{\partial y} - \frac{\partial\overline{u'w'}}{\partial z}$$

$$\frac{D\bar{v}}{Dt} + f\bar{u} = -\frac{\partial\bar{\Phi}}{\partial y} - \frac{\partial\bar{S}}{\partial y} - \frac{\partial\overline{u'v'}}{\partial x} - \frac{\partial}{\partial y}\frac{1}{2}\left(\overline{v'^2} - \overline{u'^2} - \overline{w'^2} + \frac{\overline{r'^2}}{N^2}\right) - \frac{\partial\overline{v'w'}}{\partial z}$$

$$\frac{\partial\bar{\Phi}}{\partial z} = -\bar{r}$$

$$\frac{D\bar{r}}{Dt} - N^2\bar{w} = -\frac{\partial\overline{r'u'}}{\partial x} - \frac{\partial\overline{r'v'}}{\partial y} - \frac{\partial\overline{r'w'}}{\partial z}$$

$$\bar{S} \equiv \frac{1}{2}\left(\overline{u'^2} + \overline{v'^2} + \overline{w'^2} - \frac{\overline{r'^2}}{N^2}\right)$$

Plumb(1986) and Trenberth(1986) を参照して変形

残差循環を参照して変形

Plumb(1986),Trenberth(1986) では

$\overline{S}$ に対応する項も残差循環に繰り込んでいたが，ここではとりあえず残している

$$\overline{u}^* = \overline{u} + \frac{\partial}{\partial z}\left(\frac{\overline{u'r'}}{N^2}\right)$$

$$\overline{v}^* = \overline{v} + \frac{\partial}{\partial z}\left(\frac{\overline{v'r'}}{N^2}\right)$$

$$\overline{w}^* = \overline{w} - \frac{\partial}{\partial x}\left(\frac{\overline{u'r'}}{N^2}\right) - \frac{\partial}{\partial y}\left(\frac{\overline{v'r'}}{N^2}\right)$$

$$\frac{D\overline{u}}{Dt} - f\overline{v}^* = -\frac{\partial\overline{\Phi}}{\partial x} - \frac{\partial\overline{S}}{\partial x} - \nabla\cdot F_x$$

$$\frac{D\overline{v}}{Dt} + f\overline{u}^* = -\frac{\partial\overline{\Phi}}{\partial y} - \frac{\partial\overline{S}}{\partial y} - \nabla\cdot F_y$$

$$\frac{D\overline{r}}{Dt} - N^2\overline{w}^* = -\frac{\partial\overline{r'w'}}{\partial z}$$

$$F \equiv \begin{bmatrix} \frac{1}{2}\left(\overline{u'^2} - \overline{v'^2} - \overline{w'^2} + \frac{\overline{r'^2}}{N^2}\right) & \overline{u'v'} & \overline{u'w'} + \frac{f}{N^2}\overline{v'r'} \\ \overline{u'v'} & \frac{1}{2}\left(\overline{v'^2} - \overline{u'^2} - \overline{w'^2} + \frac{\overline{r'^2}}{N^2}\right) & \overline{v'w'} - \frac{f}{N^2}\overline{u'r'} \\ 0 & 0 & 0 \end{bmatrix}$$

F, $\overline{S}$ は何か意味ある物理量と関係しているのか？

Weak shear limit: 慣性重力波のWKB 的平面波解

$$a'(x,y,z,t) = A(X,Y,Z,T)\exp\{i(kx+ly+mz-\omega t)\}$$

X,Y,Z,T, slow variables

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \overline{u}\frac{\partial}{\partial x} + \overline{v}\frac{\partial}{\partial y} + \overline{w}\frac{\partial}{\partial z} = -i\omega + ik\overline{u} + il\overline{v} + im\overline{w} \equiv -i\hat{\omega}$$

$\hat{\omega} = \omega - k\overline{u} - l\overline{v}$ Intrinsic Frequency

WKB 的：ドップラーシフトのみ考慮した
local plane wave 解

WKB的な摂動方程式

$$\frac{Du'}{Dt} - fv' = -\frac{\partial \phi'}{\partial x}$$

$$\frac{Dv'}{Dt} + fu' = -\frac{\partial \phi'}{\partial y}$$

$$\frac{Dw'}{Dt} = -\frac{\partial \phi'}{\partial z} - r'$$

$$\frac{Dr'}{Dt} - N^2 w' = 0$$

$$\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} + \frac{\partial w'}{\partial z} = 0$$

分散公式

$$\hat{\omega}^2 = \frac{(k^2 + l^2)N^2 + f^2 m^2}{k^2 + l^2 + m^2}$$

WKB的平面波解

$$-i\hat{\omega}U - fV = -ik\Phi$$

$$-i\hat{\omega}V + fU = -il\Phi$$

$$-i\hat{\omega}W = -im\Phi - R$$

$$-i\hat{\omega}R - N^2 W = 0$$

$$ikU + ilV + imW = 0$$

$$U = \frac{k\hat{\omega} + ilf}{\hat{\omega}^2 - f^2}\Phi \quad , \quad V = \frac{l\hat{\omega} - ikf}{\hat{\omega}^2 - f^2}\Phi$$

$$W = \frac{m\hat{\omega}}{\hat{\omega}^2 - N^2}\Phi \quad , \quad R = \frac{imN^2}{\hat{\omega}^2 - N^2}\Phi$$

Intrinsic Group velocity

$$\hat{c}_{gx} = \frac{\partial \hat{\omega}}{\partial k} = \frac{k}{\hat{\omega}} \frac{N^2 - \hat{\omega}^2}{k^2 + l^2 + m^2}$$

$$\hat{c}_{gy} = \frac{\partial \hat{\omega}}{\partial l} = \frac{l}{\hat{\omega}} \frac{N^2 - \hat{\omega}^2}{k^2 + l^2 + m^2}$$

$$\hat{c}_{gz} = \frac{\partial \hat{\omega}}{\partial m} = \frac{m}{\hat{\omega}} \frac{f^2 - \hat{\omega}^2}{k^2 + l^2 + m^2}$$

Intrinsic phase velocity

$$\hat{c}_x = \frac{\hat{\omega}}{k}$$

$$\hat{c}_y = \frac{\hat{\omega}}{l}$$

$$\hat{c}_z = \frac{\hat{\omega}}{m}$$

Wave energy

$$E = \frac{1}{2}\left(\overline{u'^2} + \overline{v'^2} + \overline{w'^2} + \frac{\overline{r'^2}}{N^2}\right) = \frac{1}{2} \frac{\left(k^2 + l^2\right)\left(f^2 - N^2\right)\hat{\omega}^2}{\left(\hat{\omega}^2 - f^2\right)^2\left(\hat{\omega}^2 - N^2\right)} \left|\Phi\right|^2$$

$$F \equiv \begin{bmatrix} \frac{1}{2}\left(\overline{u'^2} - \overline{v'^2} - \overline{w'^2} + \frac{\overline{r'^2}}{N^2}\right) & \overline{u'v'} & \overline{u'w'} + \frac{f}{N^2}\overline{v'r'} \\ \overline{u'v'} & \frac{1}{2}\left(\overline{v'^2} - \overline{u'^2} - \overline{w'^2} + \frac{\overline{r'^2}}{N^2}\right) & \overline{v'w'} - \frac{f}{N^2}\overline{u'r'} \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} \hat{c}_{gx}\frac{E}{\hat{c}_x} & \hat{c}_{gy}\frac{E}{\hat{c}_x} & \hat{c}_{gz}\frac{E}{\hat{c}_x} \\ \hat{c}_{gx}\frac{E}{\hat{c}_y} & \hat{c}_{gy}\frac{E}{\hat{c}_y} & \hat{c}_{gz}\frac{E}{\hat{c}_y} \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} (c_{gx} - \overline{u})k & (c_{gy} - \overline{v})k & c_{gz}k \\ (c_{gx} - \overline{u})l & (c_{gy} - \overline{v})l & c_{gz}l \\ 0 & 0 & 0 \end{bmatrix} \cdot \frac{E}{\hat{\omega}}$$

Zonal および meridional 擬運動量の平均流に相対的な輸送 flux を与える.

$$\frac{D\bar{u}}{Dt} - f\bar{v}^* = -\frac{\partial\bar{\Phi}}{\partial x} - \frac{\partial\bar{S}}{\partial x} - \nabla\cdot F_x$$

$$\frac{D\bar{v}}{Dt} + f\bar{u}^* = -\frac{\partial\bar{\Phi}}{\partial y} - \frac{\partial\bar{S}}{\partial y} - \nabla\cdot F_y$$

$$\frac{D\bar{r}}{Dt} - N^2\bar{w}^* = -\frac{\partial\overline{r'w'}}{\partial z}$$

$$\bar{u}^* = \bar{u} + \frac{\partial}{\partial z}\left(\frac{\overline{u'r'}}{N^2}\right)$$

$$\bar{v}^* = \bar{v} + \frac{\partial}{\partial z}\left(\frac{\overline{v'r'}}{N^2}\right)$$

$$\bar{w}^* = \bar{w} - \frac{\partial}{\partial x}\left(\frac{\overline{u'r'}}{N^2}\right) - \frac{\partial}{\partial y}\left(\frac{\overline{v'r'}}{N^2}\right)$$

$$\frac{\partial\bar{S}}{\partial x},\quad \frac{\partial\bar{S}}{\partial y}\qquad \bar{S} = \frac{1}{2}\left(\overline{u'^2} + \overline{v'^2} + \overline{w'^2} - \frac{\overline{r'^2}}{N^2}\right)$$

の物理的意味

$$\bar{u}^* = \bar{u} + \frac{\partial}{\partial z}\left(\frac{\overline{u'r'}}{N^2}\right) + \frac{1}{f}\frac{\partial\bar{S}}{\partial y}$$

$$\bar{S} = \frac{1}{2}\frac{\left(k^2 + l^2\right)f^2}{\left(\hat{\omega}^2 - f^2\right)^2}|\Phi|^2$$

$$\bar{v}^* = \bar{v} + \frac{\partial}{\partial z}\left(\frac{\overline{v'r'}}{N^2}\right) - \frac{1}{f}\frac{\partial\bar{S}}{\partial x}$$

$\beta - effect$

$$\bar{w}^* = \bar{w} - \frac{\partial}{\partial x}\left(\frac{\overline{u'r'}}{N^2}\right) - \frac{\partial}{\partial y}\left(\frac{\overline{v'r'}}{N^2}\right)$$

連続の式を満たさない

流体粒子の変位と速度の関係

変位ベクトル：$(\xi', \ \eta', \ \varsigma')$

$$\frac{D\xi'}{Dt} = \frac{\partial \xi'}{\partial t} + \bar{u}\frac{\partial \xi'}{\partial x} + \bar{v}\frac{\partial \xi'}{\partial y} = u^L = u' + \xi'\frac{\partial \bar{u}}{\partial x} + \eta'\frac{\partial \bar{u}}{\partial y} + \varsigma'\frac{\partial \bar{u}}{\partial z} \approx u'$$

$$\frac{D\eta'}{Dt} = v^L = v' + \xi'\frac{\partial \bar{v}}{\partial x} + \eta'\frac{\partial \bar{v}}{\partial y} + \varsigma'\frac{\partial \bar{v}}{\partial z} \approx v'$$

$$\frac{D\varsigma'}{Dt} = w^L = w'$$

$$\xi' \cong \frac{i}{\hat{\omega}}u'$$

$$\eta' \cong \frac{i}{\hat{\omega}}v'$$

$$\varsigma' = \frac{i}{\hat{\omega}}w'$$

連続の式より

$$\frac{\partial \xi'}{\partial x} + \frac{\partial \eta'}{\partial y} + \frac{\partial \varsigma'}{\partial z} = 0$$

ストークスドリフト

$$\overline{u_S} = \overline{\xi'\frac{\partial u'}{\partial x}} + \overline{\eta'\frac{\partial u'}{\partial y}} + \overline{\varsigma'\frac{\partial u'}{\partial z}} = \frac{\partial \overline{\xi' u'}}{\partial x} + \frac{\partial \overline{\eta' u'}}{\partial y} + \frac{\partial \overline{\varsigma' u'}}{\partial z}$$

$$\overline{v_S} = \frac{\partial \overline{\xi' v'}}{\partial x} + \frac{\partial \overline{\eta' v'}}{\partial y} + \frac{\partial \overline{\varsigma' v'}}{\partial z}$$

$$\overline{w_S} = \frac{\partial \overline{\xi' w'}}{\partial x} + \frac{\partial \overline{\eta' w'}}{\partial y} + \frac{\partial \overline{\varsigma' w'}}{\partial z}$$

$$\overline{u_S} \approx \frac{\partial \overline{\xi' u'}}{\partial x} + \frac{\partial \overline{\eta' u'}}{\partial y} + \frac{\partial \overline{\varsigma' u'}}{\partial z} = \frac{1}{2} \frac{f\left(k^2 + l^2\right)}{\left(\hat{\omega}^2 - f^2\right)^2} \frac{\partial |\Phi|^2}{\partial y} + \frac{1}{2} \frac{mlf}{\left(\hat{\omega}^2 - N^2\right)\left(\hat{\omega}^2 - f^2\right)} \frac{\partial |\Phi|^2}{\partial z}$$

$$\overline{u}^* = \overline{u} + \frac{1}{f} \frac{\partial \overline{S}}{\partial y} + \frac{\partial}{\partial z}\left(\frac{\overline{u'r'}}{N^2}\right)$$

$$\frac{1}{f} \frac{\partial \overline{S}}{\partial y} + \frac{\partial}{\partial z}\left(\frac{\overline{u'r'}}{N^2}\right) \approx \frac{1}{2} \frac{f\left(k^2 + l^2\right)}{\left(\hat{\omega}^2 - f^2\right)^2} \frac{\partial |\Phi|^2}{\partial y} + \frac{1}{2} \frac{mlf}{\left(\hat{\omega}^2 - N^2\right)\left(\hat{\omega}^2 - f^2\right)} \frac{\partial |\Phi|^2}{\partial z}$$

$$\overline{u}^* = \overline{u} + \frac{1}{f} \frac{\partial \overline{S}}{\partial y} + \frac{\partial}{\partial z}\left(\frac{\overline{u'r'}}{N^2}\right) \approx \overline{u} + \overline{u_S}$$

$$\overline{v}^* = \overline{v} - \frac{1}{f} \frac{\partial \overline{S}}{\partial x} + \frac{\partial}{\partial z}\left(\frac{\overline{v'r'}}{N^2}\right) \approx \overline{v} + \overline{v_S}$$

$$\overline{w}^* = \overline{w} - \frac{\partial}{\partial x}\left(\frac{\overline{u'r'}}{N^2}\right) - \frac{\partial}{\partial y}\left(\frac{\overline{v'r'}}{N^2}\right) \approx \overline{w} + \overline{w_S}$$

残差循環：

ストークスドリフト補正

物質輸送を近似的に表現

Hydrostaticの場合

$$F \equiv \begin{bmatrix} \frac{1}{2}\left(\overline{u'^2} - \overline{v'^2} + \frac{\overline{r'^2}}{N^2}\right) & \overline{u'v'} & \overline{u'w'} + \frac{f}{N^2}\overline{v'r'} \\ \overline{u'v'} & \frac{1}{2}\left(\overline{v'^2} - \overline{u'^2} + \frac{\overline{r'^2}}{N^2}\right) & \overline{v'w'} - \frac{f}{N^2}\overline{u'r'} \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} \hat{c}_{gx}\frac{E}{\hat{c}_x} & \hat{c}_{gy}\frac{E}{\hat{c}_x} & \hat{c}_{gz}\frac{E}{\hat{c}_x} \\ \hat{c}_{gx}\frac{E}{\hat{c}_y} & \hat{c}_{gy}\frac{E}{\hat{c}_y} & \hat{c}_{gz}\frac{E}{\hat{c}_y} \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} \left(c_{gx} - \overline{u}\right)k & \left(c_{gy} - \overline{v}\right)k & c_{gz}k \\ \left(c_{gx} - \overline{u}\right)l & \left(c_{gy} - \overline{v}\right)l & c_{gz}l \\ 0 & 0 & 0 \end{bmatrix} \cdot \frac{E}{\hat{\omega}}$$

$\ln p$ 球面座標系の場合

$$F \equiv \begin{bmatrix} \frac{1}{2}\left(\overline{u'^2} - \overline{v'^2} + \alpha \frac{\overline{\theta'^2}}{\partial\bar{\theta}/\partial z}\right) & \overline{u'v'} & \overline{u'w'} - \frac{2\Omega\sin\varphi}{\partial\bar{\theta}/\partial z}\overline{v'\theta'} \\ \overline{u'v'} & \frac{1}{2}\left(\overline{v'^2} - \overline{u'^2} + \alpha \frac{\overline{\theta'^2}}{\partial\bar{\theta}/\partial z}\right) & \overline{v'w'} + \frac{2\Omega\sin\varphi}{\partial\bar{\theta}/\partial z}\overline{u'\theta'} \\ 0 & 0 & 0 \end{bmatrix} pa\cos\varphi$$

$$= \begin{bmatrix} \hat{c}_{g\lambda}\frac{E}{\hat{c}_\lambda} & \hat{c}_{g\varphi}\frac{E}{\hat{c}_\lambda} & \hat{c}_{gz}\frac{E}{\hat{c}_\lambda} \\ \hat{c}_{g\lambda}\frac{E}{\hat{c}_\varphi} & \hat{c}_{g\varphi}\frac{E}{\hat{c}_\varphi} & \hat{c}_{gz}\frac{E}{\hat{c}_\varphi} \\ 0 & 0 & 0 \end{bmatrix} pa\cos\varphi$$

$$\alpha = \frac{R}{H}\exp(-\kappa z / H)$$

$$\frac{\partial \bar{u}}{\partial t}+\frac{\bar{u}}{a\cos\varphi}\frac{\partial \bar{u}}{\partial \lambda}+\frac{\bar{v}}{a\cos\varphi}\frac{\partial}{\partial \varphi}(\bar{u}\cos\varphi)+\bar{w}\frac{\partial \bar{u}}{\partial z}-2\Omega sin\varphi\overline{v^{*}}=-\frac{1}{a\cos\varphi}\frac{\partial \bar{\phi}}{\partial \lambda}-(pa\cos\varphi)^{-1}\nabla\cdot F_{\lambda}$$

$$\frac{\partial \bar{v}}{\partial t}+\frac{\bar{u}}{a\cos\varphi}\frac{\partial \bar{v}}{\partial \lambda}+\frac{\bar{v}}{a}\frac{\partial \bar{v}}{\partial \varphi}+\bar{w}\frac{\partial \bar{u}}{\partial z}+\frac{\bar{u}^{2}}{a}tan\varphi+2\Omega sin\varphi\overline{u^{*}}=-\frac{1}{a}\frac{\partial \bar{\phi}}{\partial \varphi}-(pa\cos\varphi)^{-1}\nabla\cdot F_{\varphi}$$

$$\frac{\partial \bar{\theta}}{\partial t}+\frac{\bar{u}}{a\cos\varphi}\frac{\partial \bar{\theta}}{\partial \lambda}+\frac{\bar{v}}{a}\frac{\partial \bar{\theta}}{\partial \varphi}+\overline{w^{*}}\frac{\partial \bar{\theta}}{\partial z}=G$$

$$\nabla\cdot(\;)=\frac{1}{a\cos\varphi}\frac{\partial(\;)}{\partial \lambda}+\frac{1}{a\cos\varphi}\frac{\partial}{\partial \varphi}(\qquad\cos\varphi)+\frac{1}{p}\frac{\partial(\qquad p)}{\partial z}$$

$$\overline{u^{*}}=\bar{u}+\frac{1}{2\Omega sin\varphi}\left\{\frac{\partial}{\partial \varphi}\frac{1}{2}\left(\overline{u'^{2}}+\overline{v'^{2}}\right)-\frac{1}{\cos^{2}\varphi}\frac{\partial}{\partial \varphi}\frac{1}{2}\left(\alpha\frac{\overline{\theta'^{2}}\cos\varphi}{\partial\bar{\theta}/\partial z}\right)\right\}-\frac{1}{p}\frac{\partial}{\partial z}\left(\frac{p\overline{u'\theta'}}{\partial\bar{\theta}/\partial z}\right)$$

$$\overline{v^{*}}=\bar{v}-\frac{1}{2\Omega sin\varphi}\frac{1}{a\cos^{2}\varphi}\frac{\partial}{\partial \lambda}\left\{\frac{1}{2}\left(\overline{u'^{2}}+\overline{v'^{2}}-\alpha\frac{\overline{\theta'^{2}}}{\partial\bar{\theta}/\partial z}\right)\cos\varphi\right\}-\frac{1}{p}\frac{\partial}{\partial z}\left(\frac{p\overline{v'\theta'}}{\partial\bar{\theta}/\partial z}\right)$$

$$\overline{w^{*}}=\bar{w}+\frac{1}{a\cos\varphi}\frac{\partial}{\partial \lambda}\left(\frac{\overline{u'\theta'}}{\partial\bar{\theta}/\partial z}\right)+\frac{1}{a\cos\varphi}\frac{\partial}{\partial \varphi}\left(\frac{p\overline{v'\theta'}}{\partial\bar{\theta}/\partial z}\cos\varphi\right)$$

WKB近似

コリオリ力の緯度変化無視
Local inertio-gravity wave

$$\nabla\cdot(\ \ ) \cong \frac{1}{a\cos\varphi}\frac{\partial(\ \ )}{\partial\lambda} + \frac{1}{a}\frac{\partial}{\partial\varphi}(\qquad) + \frac{1}{p}\frac{\partial(\qquad p)}{\partial z}$$

Zonal および meridional 擬運動量 の
平均流に相対的な輸送 flux を与える.

残差循環　ストークスの補正を含む
物質輸送を近似的に表現

この定式化の問題点は？

TEM 方程式系：移流項まで含めて残差流にする.

$$\frac{\partial \bar{u}}{\partial t} + \overline{v^*}\frac{\partial \bar{u}}{\partial y} + \overline{w^*}\frac{\partial \bar{u}}{\partial z} - f\overline{v^*} = -\frac{\partial}{\partial y}\left[\overline{u'v'} + \frac{1}{N^2}\frac{\partial \bar{u}}{\partial z}\overline{v'q'}\right] - \frac{\partial}{\partial z}\left[\overline{u'w'} + \left(f - \frac{\partial \bar{u}}{\partial y}\right)\frac{1}{N^2}\overline{v'q'}\right] + \overline{X}$$

時間平均の場合でも可能である.

残差流が連続の式を満足しない.

Kinoshita et al., 2010:
On the three-dimensional residual mean circulation and wave-activity flux of the primitive equations. JMSJ, 88.

時間平均流ではなく地球に固定した座標系に対して相対的なfluxを求める方法?

$$F=\begin{bmatrix} \hat{c}_{gx}\dfrac{E}{\hat{c}_x} & \hat{c}_{gy}\dfrac{E}{\hat{c}_x} & \hat{c}_{gz}\dfrac{E}{\hat{c}_x} \\ \hat{c}_{gx}\dfrac{E}{\hat{c}_y} & \hat{c}_{gy}\dfrac{E}{\hat{c}_y} & \hat{c}_{gz}\dfrac{E}{\hat{c}_y} \\ 0 & 0 & 0 \end{bmatrix}=\begin{bmatrix} \left(c_{gx}-\bar{u}\right)k & \left(c_{gy}-\bar{v}\right)k & c_{gz}k \\ \left(c_{gx}-\bar{u}\right)l & \left(c_{gy}-\bar{v}\right)l & c_{gz}l \\ 0 & 0 & 0 \end{bmatrix}\cdot\frac{E}{\hat{\omega}}$$

Plumb (1986) の場合

$$M = \frac{1}{2}\frac{\overline{q'^2}}{|\nabla_H \bar{q}|}$$

$$\frac{DM}{Dt} + \nabla \cdot \mathbf{M_R} = 0$$

$$\mathbf{M}_R \cong \frac{1}{|\bar{\mathbf{u}}|}\begin{bmatrix} \bar{u}\left(\overline{v'^2} - \bar{E}\right) - \bar{v}\,\overline{u'v'} \\ \bar{v}\left(\overline{u'^2} - \bar{E}\right) - \bar{u}\,\overline{u'v'} \\ \frac{f_0^2}{N_0^2}\left(\bar{v}\,\overline{u'r'} - \bar{u}\,\overline{v'r'}\right) \end{bmatrix}$$

More useful form

$$\frac{\partial M}{\partial t} + \nabla \cdot \mathbf{M_T} = 0, \quad \mathbf{M_T} = \mathbf{M_R} + \bar{\mathbf{u}}M$$

$$\mathbf{M_T} = \mathbf{c_g}M \qquad \mathbf{M}_R = \mathbf{M}_T - \bar{\mathbf{u}}M = \left(\mathbf{c}_g - \bar{\mathbf{u}}\right)M = \hat{\mathbf{c}}_g M$$

$\mathbf{M_T}$ は wave activity $M = e/|\nabla_H \bar{q}|$ の fluxを与える.

これらの物理量は全て場が与えられていれば計算可能

３次元TEM方程式の右辺の加速項に対応するのは，$\mathrm{M_R}$

Takaya and Nakamura (2001)

$$\frac{\partial \tilde{A}}{\partial t} + \nabla \cdot \mathbf{W} = D \ ,$$

$$\tilde{A} \equiv \frac{1}{2}\left(M + \tilde{E}\right) = \frac{1}{2}\left(\frac{1}{2}\frac{q'^2}{|\nabla_H Q|} + \frac{e}{|\mathbf{U}| - C_P}\right), \quad \mathbf{C}_U = \left(C_P \frac{U}{|\mathbf{U}|},\ C_P \frac{V}{|\mathbf{U}|}\right)$$

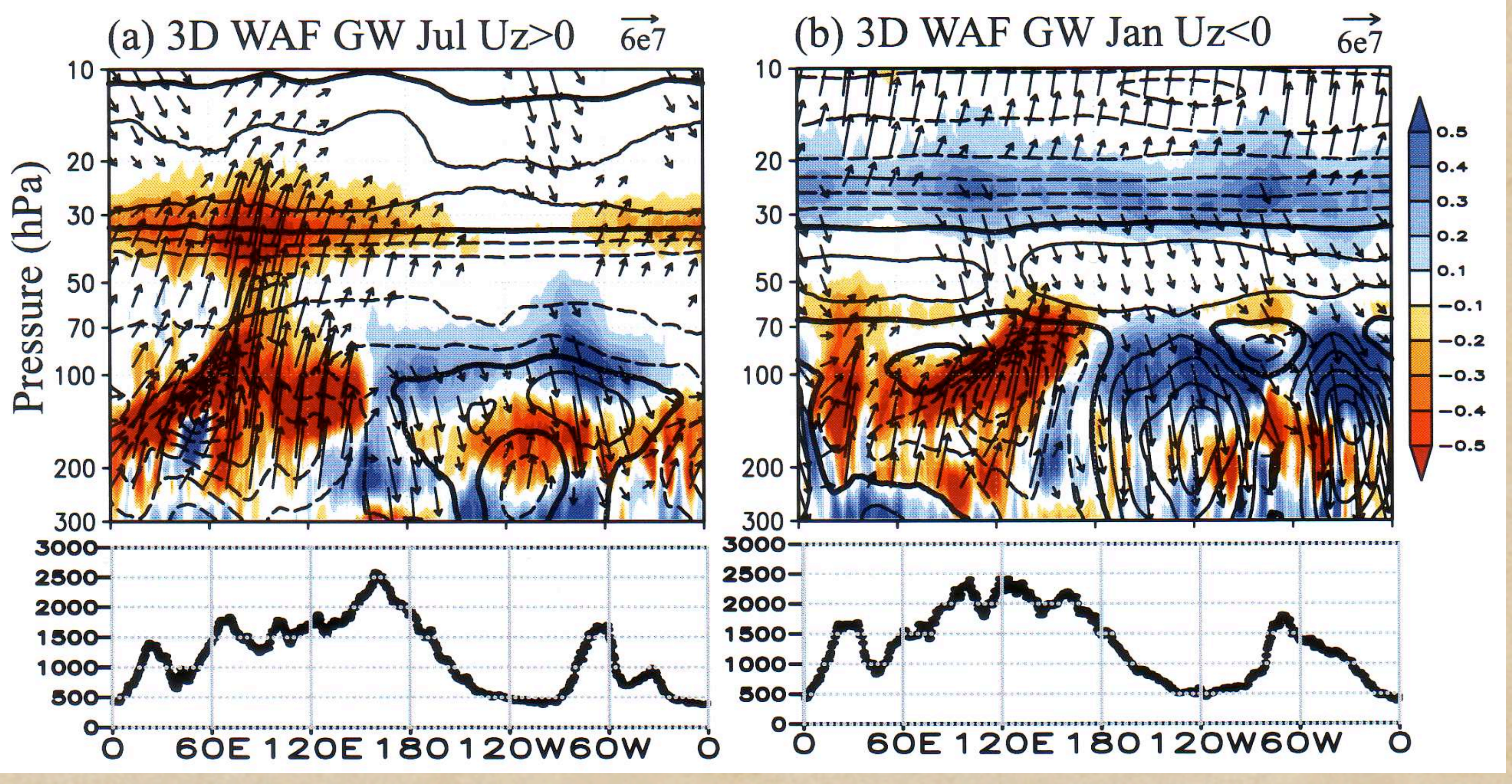
$$\mathbf{W} \equiv \mathbf{W}_R + \mathbf{C}_U \tilde{A},$$

$$\mathbf{W} = \mathbf{C}_g \tilde{A}. \qquad \mathbf{W}_R = \mathbf{W} - \mathbf{C}_U \tilde{A} = \left(\mathbf{C}_g - \mathbf{C}_U\right)\tilde{A}$$

$$\mathbf{W}_R = \frac{1}{2|\mathbf{U}|}\begin{bmatrix} U\left(\psi_x'^{\,2} - \psi'\psi_{xx}'\right) + V\left(\psi_x'\psi_y' - \psi'\psi_{xy}'\right) \\ U\left(\psi_x'\psi_y' - \psi'\psi_{xy}'\right) + V\left(\psi_y'^{\,2} - \psi'\psi_{yy}'\right) \\ \frac{f_0^2}{N_0^2}\left[U\left(\psi_x'\psi_z' - \psi'\psi_{xz}'\right) + V\left(\psi_y'\psi' - \psi'\psi_{yz}'\right)\right] \end{bmatrix}$$

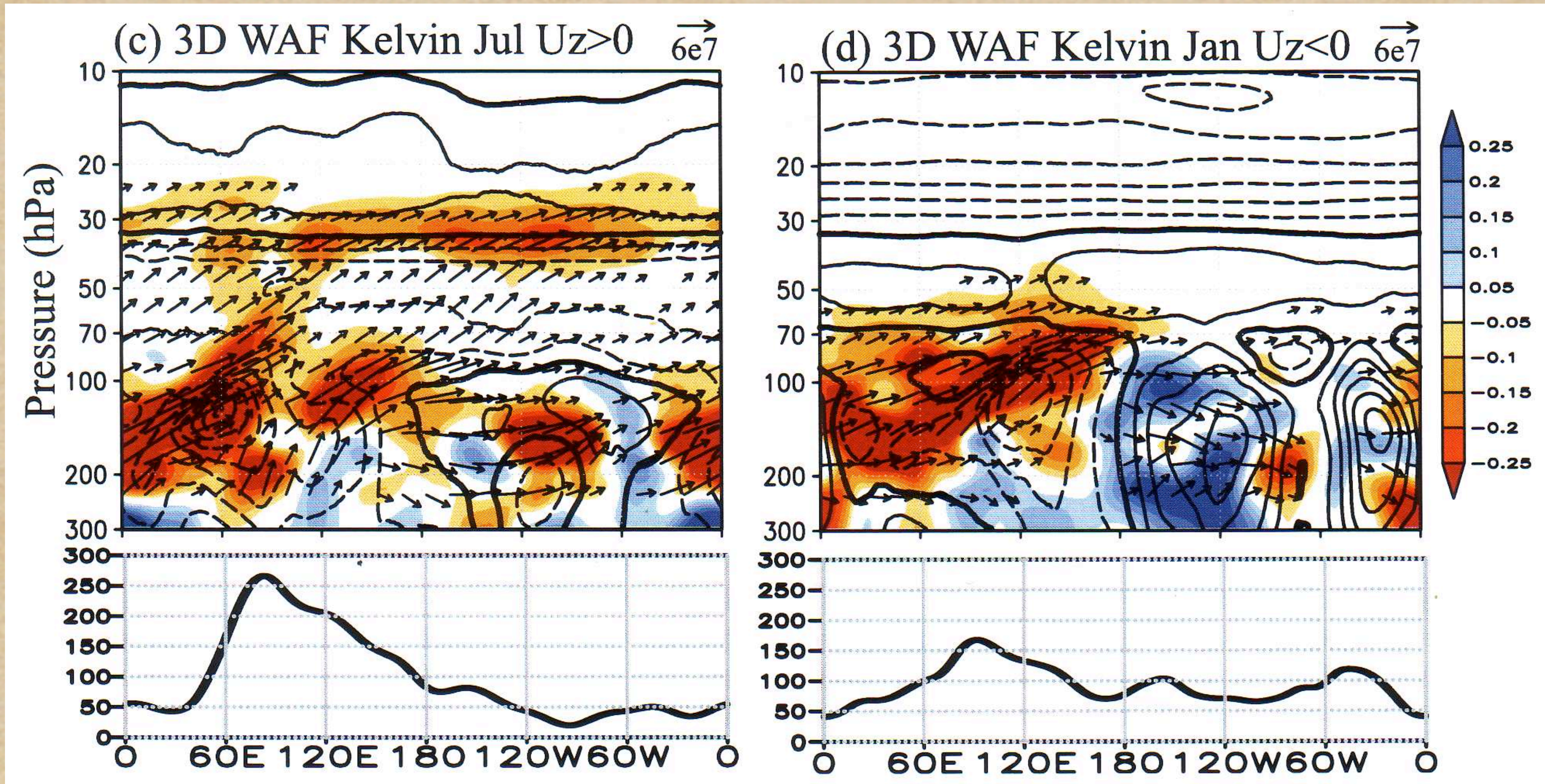
３次元TEM方程式の右辺の加速項に対応するのは，W_R

3D fluxの適用例



Gravity waves

Kawatani et al., 2010, JAS, 67, 981-997.



Kelvin waves

Kawatani et al., 2010, JAS, 67, 981-997.